Simultaneous Signaling and Channel Estimation for In-Band Full-Duplex
Communications Employing Adaptive Spatial Protection

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ABSTRACT

In-band full-duplex relays are envisioned as promising solution to increase the throughput of next generation wireless communications. Full-duplex relays, being able to transmit and receive at same carrier frequency, offers increased spectral efficiency compared to half-duplex relays that transmit and receive at different frequencies or times. The practical implementation of full-duplex relays is limited by the strong self-interference caused by the coupling of relay’s own transit signals to its desired received signals. Several techniques have been proposed in literature to mitigate the relay self-interference. In this thesis, the performance of in-band full-duplex multiple-input multiple-output (MIMO) relays is considered in the context of simultaneous communications and channel estimation. In particular, adaptive spatial transmit techniques is considered to protect the full-duplex radio’s receive array. It is assumed that relay’s transmit and receive antenna phase centers are physically distinct. This allows the radio to employ adaptive spatial transmit and receive processing to mitigate self-interference. The performance of this protection is dependent upon numerous factors, including channel estimation accuracy, which is the focus of this thesis. In particular, the concentration is on estimating the self-interference channel. A novel approach of simultaneous signaling to estimate the self-interference channel in MIMO full-duplex relays is proposed. To achieve this simultaneous communications and channel estimation, a full-rank pilot signal at a reduced relative power is transmitted simultaneously with a low rank communication waveform. The self-interference mitigation is investigated in the context of eigenvalue spread of spatial relay receive covariance matrix. Performance is demonstrated by using simulations, in which orthogonal-frequency division-multiplexing communications and pilot sequences are employed.
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Next generation wireless communications are expected to provide high data rate, seamless coverage and more reliable wireless transmission. The random nature of the wireless transmission channel due to multipath, fading and pathloss imposes a major challenge in designing these wireless systems. We are living in a world, where the available spectrum is limited but the demand for the spectrum is growing at a rapid rate. The high data rates envisioned by fourth generation (4G) networks require higher bandwidths, which are available only above 2 GHz [1]. The high pathloss incurred by the radio waves propagating at this high frequency and transmit power constraints imposed by FCC, limits the coverage area of the base station (BS). To provide seamless coverage, one solution is to deploy several base stations which scales the deployment costs. Incorporation of advanced signal processing techniques associated with multiple-input multiple-output (MIMO) and orthogonal-frequency-division-modulation (OFDM) can overcome these challenges to a large extent, however there may be some situations where the end user quality-of-service (QoS) cannot be guaranteed. For example, small form factor of mobile devices limits the number of multiple antenna. The quest for achieving high QoS requires a novel signalling approach in addition to these advanced signal processing techniques.

Relaying is seen as a promising solution to increase both the capacity and coverage area of future wireless networks where direct communication between source and destination is not feasible due to pathloss, fading and shadowing effects. A relay is a low-power intermediate node that improves the end-to-end communication between a source node and a destination node by helping in the transfer of information bearing
signals between them. They receive signals on one end, do some processing based on the underlying relaying protocol and retransmit the signals on the other end. This way of communication through relays is known as relay-assisted communication and has been standardized in technical specification group j in IEEE 802.11j [2].

Figure 1.1 shows a simple scenario where a source node (S) is communicating with a destination node (D). The direct path between the source and destination is blocked by the presence of obstacles such as buildings. Also the source and destination nodes are separated by very large distances. Instead of transmitting on the direct path and encountering severe fading and pathloss, the source can make use of unobstructed line of sight through the relay (R). If the node R operates in half-duplex mode, then it would need to switch between receiving from source (S) and forwarding it to destination (D). On the other hand, if the relay R operates in full-duplex mode, then it could receive and transmit simultaneously doubling the spectral efficiency measured in bits/second/Hz. In-band full-duplex radio indicates a node that transmits and receives at the same frequency. Note that here only relay operates in full-duplex.
mode, it is not necessary for the source and destination to simultaneously transmit and receive.

Relays in wireless communication are typically half-duplex (HD) relays or out-of-band relays. By HD, we mean that the relay transmits and receives either at different time slots, or over different frequency bands. This result in a significant loss of throughput as it requires two time slots for transmission. The spectral efficiency can be doubled if the relay operates in full-duplex mode i.e they transmit and receive over the same frequency and is the topic of interest at present.

Historically, in-band full-duplex (IBFD) relays have not been widely employed due to the inherent self-interference or loop interference. The self-interference arises from the coupling of relay’s own transmit signal to its own receive signal as shown in Figure 1.2. This self-interference causes significant degradation in the system performance.

Figure 1.2: Self-Interference in In-Band Relay.

As the distance between relay receive and transmit antenna is usually small, the signal from relay’s transmitter is several orders of magnitude stronger than the signal received from the source. Hence, the relay received signal cannot be decoded correctly until the self-interference is mitigated. Also, the large self-interference signal saturates the analog-to-digital converter in relay receiver which increases the quantization noise
of the desired signal. IBFD offers significant advantages over HD relays, if the self-interference is mitigated. Recently, there has been considerable research interests in this area and several techniques to mitigate the self-interference in IBFD relays have been proposed in literature. Research carried out in IBFD relays that is based on OFDM is of particular interest, because of its robustness against multi-path fading and inter-symbol interference.

1.1 Advantages Of IBFD Relays

Incorporation of IBFD relays into current architecture results in higher throughputs. The advantages of IBFD relays can be well understood by looking at these scenarios: a) relay scenario and b) bi-directional scenario. Consider the example relay scenario shown in Figure 1.3a, where the relay is in connection with two users. User $U_1$ sends data on uplink to relay and user $U_2$ is receiving data on downlink from relay. If the relay operates in HD mode, then it has to switch between two data
flows i.e., receiving data on uplink from $U_1$ and transmitting data on downlink to $U_2$. IBFD operation enabled relay can support simultaneous uplink and downlink connection doubling the spectral efficiency.

In bidirectional scenario shown in Figure 1.3b, data exchange occurs between two nodes A and B. If nodes A and B operate in HD mode, then data flow A-B and B-A cannot take place at the same time. The two communications occur in either different slots or different frequency. On the other hand, if the nodes are able to operate in IBFD mode, the data exchange between nodes A and B in both directions can occur simultaneously. Thus equipping nodes with IBFD capability doubles the spectral efficiency compared to HD mode.

IBFD relays apart from increasing spectral efficiency can improve MAC layer throughput. Nodes with IBFD capability can transmit a signal to another nodes while simultaneously probing the network to detect any possible collisions or can receive feedback from other nodes regarding state information such as their power, acknowledgements.

1.2 Literature Survey on Relays

Relays have a long history in wireless communication systems. Van der Meulen [3] first introduced the idea of three terminal classic relay channel, consisting of source, relay and destination in 1968. Cover et al. [4], 1979 considered discrete memoryless additive white gaussian noise (AWGN) SISO relay channels and proposed upper and lower bounds on the capacity. Thanks to Cover and El Gamal, for their significant contribution which fuelled further research in this field. Later, Gatspar et al [5] studied capacity bounds with a particular relay traffic pattern in a fading environment.

Sendonaris et al. [6], [7] introduced the concept of user cooperation, a new method of transmit diversity in cellular communication particularly for CDMA. The informa-
tion theoretical concepts of user cooperation is analysed and they demonstrated that user cooperation results in higher data rate and decreased sensitivity to channel variations over non-cooperative strategy. Also they showed that user cooperation results in decreased transmit powers to achieve same data rates as that of non-cooperative strategy. However, these advantages come at the expense of increased complexity at the receiver circuitry. In [8], they proposed various cooperative diversity relaying schemes such as amplify and forward (AF), decode and forward (DF), selection relaying, incremental relaying and analysed their performance in multipath fading environment. The performance is evaluated by determining outage events and outage probabilities. Cooperative diversity has proven to increase the capacity of rank-deficient MIMO channels [9]. In [10], the authors considered relays in the context of cellular communications. They have shown that multi-hop communication through a cluster of relays increases the coverage area and capacity of cellular networks. The authors also addressed various problems associated with relay assisted communication such as radio resource management and routing.

Relay can operate in either of two modes: HD or IBFD. HD relays employs different frequencies or different time slots for transmission and reception. IBFD relays, on the other hand uses same frequency for both transmission and reception. The theoretical advantages of full-duplex relays over half-duplex relays are studied in [11]. The practical implementation of full-duplex relays is limited by the heavy self-interference or loop interference caused by the coupling of relay’s own transit signals to its desired received signals. Due to the inherent self-interference associated with IBFD relays, most of the early works discussed above focussed on HD relays.

The earlier techniques to mitigate the self-interference are limited to provide natural isolation between transmit and receive antenna by increasing the physical separation between the antenna [12], [13], [14], using different polarizations [14], [15], for
transmitter and receiver. Later, various interference mitigation techniques such as circuit-domain interference cancellation: analog domain, digital domain [16], [17], [18], [19], [20] were proposed. In [21], the self-interference or echo observed in on-channel repeaters are cancelled by the use of finite impulse response (FIR) filters in feedback loop. With advances in digital signal processing, various spatial domain techniques; null space projection [22], minimum mean square filtering [22], optimal eigenbeam-forming [23] were proposed to mitigate the self-interference. Aforementioned techniques assumed perfect cancellation of self-interference, they neglected the effects of residual self-interference. The effects of residual self-interference is considered in [24]. The effects of hardware non-linearities such as IQ imbalance, third order non-linearities, oscillator phase noise, frequency offset on the performance of relays have been considered in [25], [26].

IBFD relays are widely studied, only after several academic and corporate laboratories [13], [27], [28], [26], [29] demonstrated the feasibility of IBFD over short ranges. In [30], the information theoretical bounds on upper and lower capacity of MIMO IBFD relay under gaussian and fading environments are evaluated. Bliss et al. [27], demonstrated the feasibility of simultaneous transmission and reception in case of MIMO IBFD. In that paper, the interference is mitigated through a combination of time domain and adaptive transmit spatial processing and adaptive spatial receive processing techniques. In WiFi, the average power of the transmit signal is at 20 dBm, and the noise floor is around -90 dBm. The self-interference signal power has to be reduced by 110 dB to reduce it to the noise floor. The first working model of SISO IBFD relays for WiFi radios is demonstrated in [31], where the self-interference is cancelled to that of receiver noise floor.
1.3 Research Problem and Contribution

The potential benefits of MIMO IBFD relays in increasing the network throughput, encourages one to explore them further to make them practically viable. In this thesis, we aim to optimise the performance of IBFD relay in the context of self-interference suppression by employing spatial domain techniques such as transmit beamforming. The transmit beamforming is carried out such that relay is spatially protected by creating nulls at the receiver. The null points do exists because of destructive interference between transmitted signals from multiple transmit antenna. Transmit beamforming techniques require channel state information (CSI) such as number of delay lines, channel gains of each tap at relay-transmitter. The performance of this approach to reduce self-interference depends on the accuracy of the CSI. In MIMO-OFDM system, multiple frequency selective channels challenge the accuracy of the CSI.

The conventional methods of channel estimation, with separate training and communication periods, reduce the goodput due to additional overhead involved in transmission of pilot signals. Also with rapidly varying channel, channel has to be estimated more often leading to decreased goodput. Goodput is nothing but the useful data rate. In this work, we propose a novel approach to estimate the self-interference channel to be used with MIMO - IBFD relays. In this approach, we estimate channel across each OFDM symbol. These estimates are then used to carry out beamforming in subsequent OFDM symbol. The analysis developed in this thesis are applicable to both AF and DF relaying schemes. The following lists the main contribution of this thesis:

1. We employ adaptive processing at the relay transmit side to mitigate the full-duplex node self-interference. The optimal beam forming vectors to employ
adaptive processing are derived.

2. We propose a novel approach of simultaneous signaling and channel estimation of the self-interference channel for adaptive transmit processing in MIMO in-band relays.

3. We evaluate the performance of simultaneous signaling and estimation approach by comparing the channel estimate error variance with the Cramer-Rao bound.

4. We evaluate the relay receive eigenvalue distribution to determine how successful the adaptive transmit protection is in the presence of channel estimation errors.

There are numerous issues associated with hardware nonidealities, such as those discussed in [25], that can potentially limit the performance gains discussed in this paper; however, those issues are beyond the scope of these results.

1.4 Thesis Outline

The rest of the thesis is organised as follows. Chapter 2 provides the reader with various terminologies used throughout this thesis. We also discuss some of the self-interference mitigation techniques for full-duplex relays proposed in literature. Description of our system set-up and mathematical equations representing the system model are presented in Chapter 3. In Chapter 4, we derive optimal vectors for performing transmit beamforming. We also introduce the novel technique of simultaneous communications for channel estimation and compare its performance against Cramer-Rao bound. Chapter 5 presents the simulation results evaluating the effectiveness of our simultaneous approach in interference mitigation by comparing against no simultaneous approach. Conclusions are drawn in Chapter 6.
In this chapter, firstly we provide a brief overview of various common terminologies to familiarize the reader and explain some popular relaying protocols. Later, we discuss the problem of self-interference associated with full-duplex relays and summarize techniques to mitigate the self-interference. Also, we discuss how our proposed technique differ from these existing techniques.

2.1 Multiple-Input Multiple-Output System

Diversity combining is one of the techniques to mitigate the deleterious effects of multipath fading. Diversity provides the receiver with multiple copies of the same signal, such that each transmitted signal experiences independent fading. Diversity combining exploits the fact that independently faded signals have low probability of all simultaneously being in a deep fade. Thus, diversity increases reliability of transmission over severely faded wireless channels by providing diversity gain. Diversity gain can be achieved in many ways:

1. Frequency Diversity: Frequency diversity is achieved by transmitting redundant information over different frequencies, such that the frequencies are separated by coherence bandwidth of the channel.
2. Time Diversity: Time diversity is achieved by transmitting redundant information over different time slots, such that the consecutive time slots are separated by coherence time of the channel.
3. Spatial Diversity: Spatial diversity is achieved by transmitting redundant in-
formation over multiple antenna or receiving the signal by multiple antenna or both. The antenna array should be spaced apart such that fading amplitudes corresponding to each antenna are almost independent.

Spatial diversity is widely popular among these techniques as it does not consume additional bandwidth. The system with multiple antenna at the transmitter and receiver side is referred to as MIMO system. The multiple antenna can be used to increase the data rates through spatial multiplexing or performance through diversity [32]. The advantages of MIMO to communications systems are widely studied in [33], [34]. In fact, the huge success of MIMO is a key to the innovation of relay-assisted communication. Relays with multiple antennas are considered in [16], [27], [31]. Diversity techniques can be used in full-duplex relays to combat the self-interference.

In this work, we consider MIMO techniques that exploit CSI at the transmit side. The capacity of the MIMO system can be increased, if the channel is known at the transmit side. Channel state information may include complex attenuation between transmit and receive antenna, knowledge of the statistical properties of the noise and interference (interference plus noise co-variance matrix) [35]. Access to the channel state information at the transmit side is problematic. One way is to estimate the channel at the transmit side and fed back the estimates to the transmit side through separate communication link. If the communication is bi-directional, on the same frequency and the same antenna, then reciprocity can be invoked to estimate the channel in the receive mode and exploit during the transmit mode.

2.2 Orthogonal-Frequency Division-Multiplexing

Orthogonal frequency division multiplexing (OFDM) is a multi-carrier modulation technique that is suitable for high data rate transmission. It converts the high rate data stream into a number of low rate streams that are transmitted over orthogonal
sub-channels and thus simplifies the receiver circuitry [32], [36]. OFDM like frequency
division multiple access, divides the available wideband into a number of sub-channels.
However in FDM, guard bands are used between sub-channels to avoid any sub-
carrier interference which is bandwidth inefficient. In OFDM, sub-carries are chosen
at orthogonal frequencies avoiding the use of guard bands.

![Figure 2.1: Power Spectral Density of OFDM.](image)

Figure 2.1 shows the spectrum of an OFDM signal. The channel is divided into
8 orthogonal sub-channels. The spectra of different modulated carriers overlap but
each carrier is in the spectral nulls of all other carriers. Therefore, as long as the
receiver does the good job, the data streams of any two sub-carriers will not interfere.

The number of sub-carriers is chosen such that the coherence bandwidth of each
of the sub channels is larger than the bandwidth of the signal and so each of the
sub-carriers experiences flat fading. This is to assure that each sub-carrier channel
response can be characterized by a single complex term. OFDM is implemented
by taking the inverse discrete Fourier transform (IDFT) of the transmit sequence
and then transmitting it through the channel. In the receiving end, discrete Fourier
transform (DFT) is applied to recover the original signals. With advances in digital
signal processing, IDFT and DFT can be effectively implemented by fast Fourier
transform (FFT) and its counterpart inverse fast Fourier transform (IFFT) as shown
in Figure 2.2.

The time-dispersion nature of wireless channels results in inter-symbol interference
(ISI) between adjacent OFDM symbols. ISI can be avoided by inserting cyclic prefix
Figure 2.2: OFDM Transmitter and Receiver Block Diagram.

either at the end or start of OFDM frame. In general, the last \( n_{ch} \) bits (equal to length of channel taps) of an OFDM symbol are used as cyclic prefix. The insertion of cyclic prefix converts the linear convolution of channel response and transmitted signal into circular convolution. At the receiver, cyclic prefix i.e., first \( n_{ch} \) bits of an OFDM frame are discarded and processed as usual. The length of cyclic prefix should be at least equal to the maximum delay spread of the channel.

OFDM is widely popular due to its effectiveness against multipath delay spread. In conventional single carrier systems, equalization techniques are employed to combat the ISI. The complexity of the equalizers increases with increase in number of delay lines. However, the complexity of OFDM is not dependent on the number of delay lines. However, there are some drawbacks associated with OFDM; peak-to-average power ratio, frequency offset, timing offset. The reader can refer to [36] for further discussion regarding them.
OFDM is widely used in applications like digital audio broadcasting (DAB), digital video broadcasting terrestrial (DVB-T), wireless local area networks (WLAN), 4G cellular networks and WiMax. OFDM is proposed as the modulation technique in IEEE 802.11a and IEEE 802.11g and it supports very high data rates. Relays employing OFDM have been extensively studied in [37], [18].

2.3 Channel Estimation in OFDM

The information transmitted over a wireless channel undergo distortion in amplitude and phase due to time varying characteristics of the channel. The coherent detection of transmitted information at the receive side requires an estimate of the channel state information (CSI). In wide-band OFDM system, channel is generally estimated in frequency domain. There exists some co-relation between adjacent sub-carriers in frequency domain and each sub-carrier behaves as a flat-fading channel as they have narrow bandwidth. OFDM channel estimation techniques can be categorized as blind and pilot based techniques. Blind channel estimation techniques are based on the statistics of the received signal. No special training sequences are required to estimate the channel. Since the receiver has no knowledge on transmitted signals, the receiver has to analyse large amount of data to get accurate CSI.

Pilot based channel estimation uses special training sequences called pilot signals to estimate the channel. Pilots have no useful information, they are merely used for the purpose of channel estimation. They are assumed to be known both at the transmitter and receiver. Based on the arrangement of pilot symbols in OFDM block, they are of two types [38]: block-type, comb-type as shown in Figure 2.3.

In block type, pilots are transmitted across all the sub-carriers of OFDM symbol. Pilot inserted OFDM symbols are transmitted periodically (for every coherence time interval) for any channel variations. Block-type channel estimation is used in slowly
Figure 2.3: Block-Type and Comb-Type Pilot Arrangement.

varying channels i.e., channel characteristics remain almost the same for one OFDM data block. In general, the wireless channel changes rapidly over time even within one OFDM data block. Hence, block type estimation is not effective for fast fading channels. In comb-based estimation, pilots are sent in each of the OFDM symbols but not across all the sub-carriers. Pilots are sent only across some of the sub-carriers and interpolation is done to estimate the channel across remaining data sub-carriers. The accuracy of the channel estimates increases with increase in pilot density but it decreases the goodput.

The channel can be estimated by either least squares (LS) or minimum mean square estimator (MMSE) in both block-type and comb-type pilot arrangements. The least squares estimated for arbitrary random variable from another random variable is the same as the MMSE estimate of a Gaussian random variable from another joint Gaussian random variable with same mean and covariance [39]. In this work, we implemented least-squares approach for channel estimation.

2.3.1 Least-Squares Method

Least square technique is based on minimizing the error signal between received signal and the estimated signal. Consider the wideband channel, which is divided
into N narrowband sub-carriers. Let \( \mathbf{X}_i \) represent the training sequence transmitted across each sub-carrier through a fading channel \( \mathbf{H}_i \). The signal received across each sub-carrier is given by

\[
\mathbf{Y}_i = \mathbf{H}_i \mathbf{X}_i + \mathbf{N}_i, \quad i = 1, 2, 3, \ldots, N
\]  

(2.1)

where \( \mathbf{N}_i \) indicates the additive white Gaussian noise across each sub-carrier and \( N \) represents number of sub-carriers.

Suppressing explicit sub-carrier indexing, the channel across each sub-carrier is estimated by minimising the square of the error signal and is given by

\[
\epsilon^2 = (\mathbf{Y} - \hat{\mathbf{H}} \mathbf{X})^\dagger (\mathbf{Y} - \hat{\mathbf{H}} \mathbf{X})
\]

\[
= \mathbf{Y}^\dagger \mathbf{Y} - \mathbf{X}^\dagger \hat{\mathbf{H}}^\dagger \mathbf{Y} - \mathbf{Y}^\dagger \hat{\mathbf{H}} \mathbf{X} + \mathbf{X}^\dagger \hat{\mathbf{H}}^\dagger \hat{\mathbf{H}} \mathbf{X}
\]

(2.2)

where \( \hat{\mathbf{H}} \) represents the estimate of channel \( \mathbf{H} \). The goal of least-squares algorithm is to minimize the cost function \( \epsilon^2 \). The value of \( \hat{\mathbf{H}} \) that minimizes the error square can be found by taking the derivation of \( \epsilon^2 \) with respect to \( \alpha \) (where \( \mathbf{H} \) is function of \( \alpha \)) and equating it to zero, i.e.,

\[
\frac{\partial \epsilon^2}{\partial \alpha} = tr\{ \hat{\mathbf{H}}^\dagger \mathbf{X}^\dagger \mathbf{X} - \mathbf{X}^\dagger \mathbf{Y} \} = 0
\]

\[
\hat{\mathbf{H}}^\dagger \mathbf{X}^\dagger \mathbf{X} = \mathbf{X}^\dagger \mathbf{Y}
\]

\[
\hat{\mathbf{H}} = (\mathbf{X}^\dagger \mathbf{X})^{-1} \mathbf{X}^\dagger \mathbf{Y}
\]

(2.3)

The advantages of least-squares method is its simplicity because it does not consider noise into account. Least-squares method provides the best linear based estimate of channel matrix and is given by

\[
\hat{\mathbf{H}} = (\mathbf{X} \mathbf{X}^\dagger)^{-1} \mathbf{X}^\dagger \mathbf{Y}
\]

(2.4)

2.4 Relaying Protocols

Relaying protocols are distinguished based on the way the relay processes the received signal. Each protocol has its own limitations in terms of complexity, delay
encountered, achievable signal-noise ratio (SNR). In here, we present some relaying protocols:

1. Amplify and Forward relays (AF): Relays employing AF protocol receive the noisy signal from the source, amplify it by certain factor and forward it to the destination. The signal does not undergo any decoding and encoding process. The relay acts as analog repeater, amplifying the noise signal along with the desired signal [8]. AF protocol puts minimal burden on relays and involves minimal delay among the relaying protocols.

2. Decode and Forward relays (DF): Relays with underlying DF protocol do some processing on the received signal. They fully regenerate the received signal, by decoding and recoding before transmitting it to the destination node [8]. Thus a very high delay is associated with these type of relays. These relays are also referred to as Digital repeaters, bridges or routers. DF relays are most commonly used relays as they increase the SNR at the destination.

3. Compress and Forward (CF): CF is similar to AF protocol. That the received signal is not decoded at the relay, but it is forwarded. But unlike AF, CF does quantization and compression on the signal before forwarding it similar to source encoding [40]. At the destination, the signal coming from the relay is used along with the direct signal from source to estimate the original transmitted signal. This is more difficult to implement when compared to AF and DF protocols and is not considered in our work.

2.5 Eigen Analysis

In this thesis, we evaluated the performance of proposed approach in mitigating self-interference in the context of eigenvalue spread of relay-receive self-interference signal. Eigenvalues physically represent the power of the received signal’s principal
component i.e., the modulus direction in the signal subspace in which maximum variance lies, then the second direction in which next maximum variance lies and so on.

2.6 Self-Interference Mitigation Techniques

In this section, we discuss various existing techniques for reduction of self interference in IBFD relay. The techniques can be divided into three classes for our discussion purpose: propagation-domain, analog circuit-domain and spatial-domain.

2.6.1 Propagation-Domain Interference Suppression

Propagation-domain interference suppression techniques are based on isolating the relay receiver from the transmitter i.e., the interference is suppressed even before the signal has reached the receiver circuitry. This greatly simplifies the relay receive processing, since the relay is no longer need to operate on signals with large dynamic range. Here in, the interference is reduced by a combination of pathloss, polarization and antenna directivity.

The deleterious effects of pathloss on signal amplitude can be used to reduce the self-interference. This is achieved by physically separating the relay transmit antenna from receive antenna [12]. The presence of any obstacle in the transmitter-receiver path, also shields the receiver against the self-interference as in [13], [14]. The degree of isolation depends upon the physical separation between the transmit and receive antenna. Greater the separation between them, greater the path loss and lesser the interference. Antenna isolation can provide interference suppression up to 30 dB [39]. To achieve perfect isolation, the signal transmit power should be equal to or less than the amount of pathloss encountered by the signal in the loop. Using different polarizations for transmit and receive antenna, also reduces the self interference [15]. One
can also reduce the self interference by using directional antenna [12] on either side of the relay and steering them in opposite directions. The location in which the relay is installed, heavily influences the design. For example, the amount of isolation achieved outdoors is high compared to indoors due to large reflections indoors. The isolation achieved in small form factor devices is not enough for reliable communication, hence we use propagation domain interference cancellation techniques in combination with other cancellation techniques.

2.6.2 Circuit-Domain Cancellation of Self-interference

Circuit-domain cancellation techniques can be further divided into a) Analog-domain cancellation techniques, b) Digital-domain cancellation techniques based on where the interference cancellation occurs. Analog domain cancellation techniques cancel out self-interference in analog domain i.e., before the signal reaches the Analog-Digital converter (ADC) of relay receive chain. This is achieved by tapping the signal at the transmit antenna feed, processing it through a analog filter to include the effects of self-interference channel and subtracting it from the receive antenna feed [26], [41], [42]. Hence, additional RF path from transmitter to receive chain is required for the cancelling signal. Cancelling the interference before ADC, relaxes the requirements on ADC dynamic range. Analog domain cancellation of self-interference includes all the effects of non-linearities induced by the transmitter like phase noise, third order non-linearities. The accuracy of the technique depends on the accuracy of the channel estimate. It is shown in [17], the amount of cancellation achieved increase with self-interference signal power. This is because the channel estimate accuracy increase with signal power. Since the channel varies over time, adaptive analog filter is needed to capture the time varying characteristics of the channel. For narrowband signals, the channel can be modelled by a complex gain and a single delay element.
for every antenna pair. For wideband signals, the channel is frequency selective in nature. Adaptive analog filter becomes quite complex, as it requires multiple taps to model the frequency selective channel.

One alternative solution is to cancel out the self-interference in digital domain. Digital domain interference cancellation techniques cancel the interference after ADC in receive chain. This is achieved by tapping the digital signal i.e., before DAC in transmit chain, process it by digital FIR filter and cancel out the interference in digital domain as proposed in [27], [43], [23] for wideband system. Adaptive digital filters are quite easy to model as the complexity is shifted from hardware to software. On the down side, these techniques do not capture the effects of transmitter non-linearities on transmitted signal. Hence this results in imperfect cancellation of self-interference. For perfect cancellation, a model that captures everything between transmitter DAC to receiver ADC is proposed in [44], [45]. Also, the receiver ADC is required to operate at huge dynamic range since the cancellation occurs after ADC.

2.6.3 Spatial-Domain Suppression

The isolation offered by time-domain techniques in single-input single-output (SISO) full-duplex relays is not good enough to establish reliable communication [22]. The relays can be equipped with the multiple antennas on either side and antenna arrays can be exploited to cancel the self-interference. In particular, the increased spatial diversity offered by MIMO opens up a new class of techniques to mitigate the interference called spatial suppression. All the interference mitigation techniques applicable to SISO relays are applicable to MIMO in-band relays. Spatial suppression also belongs to the class of propagation domain interference suppression, but the former requires the additional knowledge of the self-interference channel at the relay transmitter and receiver.
Taneli et al. [22], proposed null space projection as a technique to eliminate the self-interference completely. The scheme uses a set of linear filters; transmit and receive MIMO filters. The signal goes through transmit filter, self-interference channel and then through receive filter before received by the receive antenna. Given accurate channel estimate, the self-interference can be forced to zero by properly choosing filter pairs. The filter designs are based on the singular value decomposition (SVD) of the self-interference channel. The filters can be designed jointly or individually. In individual design, first a transmit filter is designed and then number of filters that minimize the interference are obtained. Among these designs, the design that produces the least interference is selected for receive filter. It is shown that the joint design of filter improves performance as it allows more spatial input and output streams.

In [22] rather than concentrating on a local problem i.e., interference mitigation, the authors aim to optimize the overall system by minimizing the mean square error at the relay receiver. This ultimately leads to reduced self-interference. This technique involves designing the relay transmit filter as described in null space projection method and receive filter is designed such that it minimizes the mean square error. Minimum mean square spatial estimate filtering and null space projection are based on ideal CSI. When there is some error in channel estimation which usually exists, it results in imperfect cancellation leaving behind some residual self-interference. The problem of residual self-interference is analysed in [24].

Beamforming may be employed at the relay transmitter side to mitigate the self-interference. The optimal beamforming vectors that mitigate the interference are based on SVD of self-interference channel. The self-interference channel can be represented in matrix form and can be decomposed into left singular matrix, right singular matrix and a diagonal matrix containing singular values along diagonals. By choosing
the eigen vectors that correspond to least singular value of self-interference channel, the power sent from relay transmit side to relay receiver is minimised [23].

The effectiveness of all the spatial domain suppression techniques discussed above is dependent on the estimate of CSI accuracy. In all the work discussed above, they have used conventional pilot based channel estimation techniques. This method has the drawback of additional overhead involved in transmitting pilot signals and for rapidly varying channels frequent estimation reduces the data rates significantly. To overcome these drawbacks, simultaneous signalling approach is discussed in [21], in case of SISO on frequency repeaters. The simultaneous signalling approach is used to continuously estimate the channel and update the filter co-coefficients adaptively. So far, to our knowledge no such technique is proposed in literature for MIMO IBFD relays.

In this work, we employed spatial adaptive techniques proposed in [25], [27] to mitigate the self-interference in FD relays. What we did different from the earlier works, is that we proposed a novel approach of simultaneous signaling to be used in conjunction with spatial techniques in MIMO IBFD relays. Also, in this work we considered the self-interference mitigation in the context of eigenvalue spread of relay receive co-variance matrix.
In this chapter, we introduce the system model which forms the basis for the entire discussion of this thesis. Followed by, we introduce various channel models and the model we adopted in our work. Finally, we provide mathematical model of the system.

3.1 System Model

A typical network consists of arbitrary number of nodes involved in communication at any time. For the sake of discussion, we consider a simple scenario in which there are only three nodes: source, relay and destination. However, analysis developed in this paper can be easily extended to multiple nodes. Consider Figure 3.1, where the source node (S) is communicating with the destination node (D) with the help of a relay (R). The relay is assumed to be operated in the full-duplex mode. The relay is equipped with multiple antenna on both transmit and receive side. Although we use MIMO, multiple antennas of the relay are used to steer the same information i.e., the transmit co-variance matrix is of rank-1. It is assumed that, the relay has atleast one transmit antenna higher than the number of receive antenna. The relay receives a strong interfering signal from its own transmitter while receiving signal from the source node.

Although we consider the problem of two-hop full-duplex communications explicitly, the idea can be easily extended to bi-directional communications. Figure 3.2 shows the bidirectional communication between two nodes. Here the two nodes A and B acts as IBFD relay simultaneously receiving and transmitting signals. The
Figure 3.1: Two-Hop Communications Using Full-Duplex MIMO Relay.

blue line indicates the desired communication, whereas red curve indicates the self-interference signal. In bi-directional scenario, two nodes communicate through the same channel. It is sufficient to estimate the single channel in bi-directional case, whereas in full-duplex communications we need to estimate both source-relay channel and relay-destination channel in addition to the self-interference channel. Since both the nodes in bi-directional scenario act as IBFD relays, the self-interference mitigation techniques developed for full-duplex have to be applied at the transmit side of both the nodes A and B.

Figure 3.2: Bi-Directional Communication Between Two Nodes.

3.2 Channel Model

The data stream transmitted over a wireless channel suffers from multipath delay spread which cause time dispersion [32], [40]. Due to dispersion, the transmit signal may undergo either flat-fading or frequency selective fading depending on the nature of the transmit signal with respect to channel characteristics.
3.2.1 Flat Fading

If the transmitted signal bandwidth is less than the coherence bandwidth of the channel, then the signal experiences flat-fading. In this type of fading, signal changes only in amplitude but its characteristics are preserved by the channel. Flat fading channels are also called narrow-band channels. The criteria for signals to undergo flat-fading is given by [32]

\[ B_s \ll B_c \]  

(3.1)

where \( B_s \) represents signal bandwidth and \( B_c \) represents coherence bandwidth of the channel.

Flat-fading channel can be modeled effectively by single tap delay line representing amplitude and phase distortion. According to central limit theorem, the envelope of the flat-fading channel can be statistically modeled by Rayleigh distribution and phase by uniform distribution on \([-\pi, \pi]\). The probability density function of Rayleigh model is given by [32],

\[ p_R(r) = \frac{2r}{\sigma^2} \exp \left(-\frac{r^2}{\sigma^2}\right) \quad r \geq 0 \]  

(3.2)

where \( \sigma^2 \) is the variance of complex Gaussian variable \( r \).

3.2.2 Frequency Selective Fading

If the transmitted signal bandwidth is greater than the coherence bandwidth of the channel, then the signal experiences frequency selective fading. The coherence bandwidth is defined as the bandwidth over which the wireless channel has constant gain in amplitude and has linear phase response. Hence frequencies which are separated by coherence bandwidth experience different amounts of fading levels in amplitude.
and delay. In time domain, this corresponds to reception of multiple versions of transmitted signal. Hence frequency selective fading introduces ISI. Frequency selective channels are also called wide-band channels. To summarize, frequency selective fading occurs if

\[ B_s \gg B_c \] (3.3)

Frequency selective fading is very difficult to model compared to flat-fading channels, since each multi-path component must be considered.

### 3.3 Mathematical Modeling

In this contribution, we consider wide-band frequency selective channels. However, we employ OFDM scheme which divides the wideband into several narrow-band channels. Because we employ OFDM waveforms, we will process the channel within each OFDM subcarrier as if it were constructed from a flat-fading channel; For the flat-fading model, the source and the destination nodes are equipped with \( n_{S,t} \) and \( n_{D,r} \) antennas respectively, and the full-duplex relay is equipped with \( n_{R,t} \) transmit antennas and \( n_{R,r} \) receive antennas. The matrices \( H_{S,D} \in \mathbb{C}^{n_{D} \times n_{S,t}} \), \( H_{S,R} \in \mathbb{C}^{n_{R} \times n_{S,t}} \), \( H_{R,R} \in \mathbb{C}^{n_{R} \times n_{R,t}} \), \( H_{R,D} \in \mathbb{C}^{n_{D} \times n_{R,t}} \) represent the channel matrices between source to destination, source to relay, relay to relay (self-interference channel) and relay to destination respectively. The elements of channel matrix are drawn from a complex, circularly symmetric Gaussian distribution with variance \( \sigma^2 \).

Let \( S_S \in \mathbb{C}^{n_{S,t} \times n_s} \) and \( S_R \in \mathbb{C}^{n_{R,t} \times n_s} \) represent the signal transmitted by the source and relay node, respectively. The number of samples in a block of data is \( n_s \). To include the effects of the frequency-selective channel, we employ a channel as a function of delay. The transmitted symbol repeated at various delays \( \delta_n \) is given by

\[ \tilde{S}_S \equiv (S_{S,\delta_1}^\dagger, S_{S,\delta_2}^\dagger, S_{S,\delta_3}^\dagger, \ldots) \in \mathbb{C}^{(n_{S,t} \cdot n_{\delta}) \times n_s} \] (3.4)
\[
\tilde{S}_R \equiv (S_{R, \delta_1}^\dagger, S_{R, \delta_2}^\dagger, S_{R, \delta_3}^\dagger, \ldots ) \in \mathbb{C}^{(n_{R,r}, t \cdot n_\delta) \times n_s},
\]

The received signal \( Z_R \in \mathbb{C}^{n_{R,r} \times n_s} \) at the relay is given by

\[
Z_R = \tilde{H}_{S,R} \tilde{S}_S + \tilde{H}_{R,R} \tilde{S}_R + N_R,
\]

where \( N_R \in \mathbb{C}^{n_{R,r} \times n_s} \) indicates the additive white Gaussian receiver noise. Here, we explicitly removed any temporal dependence on the received signal. The channel response incorporating the set of delays \( \delta_n \) is given by

\[
\tilde{H}_{R,R} \equiv (H_{R,R, \delta_1}, H_{R,R, \delta_2}, \ldots ) \in \mathbb{C}^{n_{R,r} \times (n_{R,r} \cdot n_\delta)};
\]

\[
\tilde{H}_{S,R} \equiv (H_{S,R, \delta_1}, H_{S,R, \delta_2}, \ldots ) \in \mathbb{C}^{n_{R,r} \times (n_{S,r} \cdot n_\delta)}.
\]

The term \( \tilde{H}_{R,R} \tilde{S}_R \) represents the self interference. The signal received by the destination node \( Z_D \in \mathbb{C}^{n_{R,r} \times n_b} \) is given by

\[
Z_D = \tilde{H}_{R,D} \tilde{S}_R + \tilde{H}_{S,D} \tilde{S}_S + N_D,
\]

where \( N_D \in \mathbb{C}^{n_{R,r} \times n_b} \) indicates the additive white Gaussian receiver noise. We can construct a similar form for the source-to-relay channel \( \tilde{H}_{R,D} \). Here, we assume that the direct link between source to destination link is negligible. i.e. \( \tilde{H}_{S,D} = 0 \). Hence the received signal at the destination node is given by

\[
Z_D = \tilde{H}_{R,D} \tilde{S}_R + N_D.
\]

### 3.4 Twisted SINR at Relay Receiver

Instead of solving for maximising the capacity of the relay-system as whole, we solve for sub-optimal solution. We try to optimize the twisted signal-interference noise ratio (SINR) at the relay receiver i.e., we try to minimize the power sent from relay
transmitter in the direction of relay receiver and maximize in the direction towards
destination. In [27], it is shown that this sub-optimal approach leads to good stem
performance. The twisted SINR at relay receiver is given by:
\[
P_{R,D} \frac{||H_{R,D}S_R||^2}{P_{R,R} + \sigma^2_{R,r}} = \frac{\langle||H_{R,D}S_R||^2\rangle}{\langle||H_{R,R}S_R + n_R||^2\rangle},
\tag{3.11}
\]
where \(\cdot\) is the norm or absolute value, and \(\langle\cdot\rangle\) indicates expectation. The optimal
solution for beamformer weight vectors for maximising the twisted SINR is nothing but
the eigenvector associated with the least eigenvalue of the self-interference channel
and the proof is provided in following chapter.
Chapter 4

OPTIMIZATION PROBLEM

This chapter is organized as follows: In Section 4.1, we discuss transmit beamforming and derive the optimal beamforming vectors for nulling self-interference at relay receiver. In Section 4.2, we introduce the novel simultaneous signaling approach for channel estimation and compare it with traditional pilot-based approach. In Section 4.3, we evaluate the performance of simultaneous approach against the Cramer-Rao bound.

4.1 Derivation of Optimal Transmit Beamforming Vectors

Transmit beamforming is an advanced spatial-signal processing technique used for directional signal transmission. The directionality of an antenna array is adjusted by adaptively controlling the gain and phase of each antenna in an array in such a way that signals combine in constructive manner for some directions and destructive manner in certain directions. The weights for transmit beamforming are usually derived from the estimate of the channel between the transmitter and receiver. Transmit beamforming is relatively a new area in the field of wireless communications and can be exploited to increase the efficiency of full-duplex communications. Transmit beamforming for IBFD relays has been studied in [19]-[22].

In this work, we perform transmit beamforming to minimize the relay transmit signal power in the direction of relay receiver which is otherwise known as null-steering. Consider the system model shown in Section 3.1, the power of the received
signal \((P_R)\) at the relay receiver due to signal from relay’s own transmission is

\[
P_R = (H_{R,R}v_RS_R)^\dagger(H_{R,R}v_RS_R)
\]

\[
= v_R^\dagger H_{R,R}^\dagger s_R^\dagger s_R H_{R,R} v_R
\]

\[
= P_s v_R^\dagger H_{R,R}^\dagger H_{R,R} v_R , \quad (4.1)
\]

where \(v_R\) represents the transmit beamformer, \(s_R\) represents the actual signal to be transmitted and \(P_s = s_R^\dagger s_R\) indicates the transmit signal power. \(S_R = v_R s_R\) indicates the beamformed transmit signals.

The simplest method to nullify power sent in the direction of relay receiver is to choose all zero vectors as transmit beamforming vector. However, it should be noted that for transmit beamforming, the total power of the transmitted signal should remain constant i.e., beamformer should be a normalised unit vector. Hence, all zero vector cannot be used as transmit beamformer. This can be mathematically represented as

\[
v_R^\dagger v_R = 1 . \quad (4.2)
\]

We need to solve for optimum beamforming vectors that minimize the received signal power at the relay receiver due to relay transmitter, provided that the beamformers satisfy the constraint given by (4.2). Lagrange method of multipliers is a useful tool to find the maximum or minimum of a function subject to some equality constraint. Let us consider

\[
f = v_R^\dagger H_{R,R}^\dagger H_{R,R} v_R
\]

\[
g = v_R^\dagger v_R
\]

\[
h = 1 .
\]
then we specify the Lagrange function $\Lambda$ as

$$
\Lambda = f + \lambda(g - h) = v_R^\dagger H_{R,R}^H H_{R,R} v_R + \lambda(v_R^\dagger v_R - 1),
$$

where $\lambda$ is a Lagrangian multiplier. We first evaluate the critical points of Lagrange function $\Lambda$. Critical points are the points where the gradient of Lagrange function evaluates to zero. Taking partial derivative of Lagrange function $\Lambda$ with respect to $\alpha$ and $\lambda$ gives

$$
\frac{\partial \Lambda}{\partial \lambda} = v_R^\dagger v_R - 1 = 0
\quad v_R^\dagger v_R = 1,
$$

(4.4)

$$
\frac{\partial \Lambda}{\partial \alpha} = v_R^\dagger H_{R,R}^H H_{R,R} \frac{\partial v_R}{\partial \alpha} + \lambda v_R^\dagger \frac{\partial v_R}{\partial \alpha} + \lambda v_R \frac{\partial v_R^\dagger}{\partial \alpha} = 0
\quad (v_R H_{R,R}^H H_{R,R} v_R + \lambda v_R \frac{\partial v_R^\dagger}{\partial \alpha}) = 0,
$$

(4.5)

where $v_R$ is a function of $\alpha$. The equation (4.4) is nothing but the original constraint. From equation (4.5), we can solve for either $v_R$ or $v_R^\dagger$, both these solutions provides the same result. In here, we solve for $v_R$ and is given by

$$
v_R^\dagger H_{R,R}^H H_{R,R} \frac{\partial v_R}{\partial \alpha} + \lambda v_R^\dagger \frac{\partial v_R}{\partial \alpha} = 0
\quad v_R^\dagger H_{R,R}^H H_{R,R} = -\lambda v_R^\dagger
\quad H_{R,R}^H H_{R,R} v_R = \lambda v_R.
$$

(4.6)

From the above equation, it is clear that critical points of Lagrange function $\Lambda$ are eigenvector of self-interference channel co-variance matrix $H_{R,R}^H H_{R,R}$ and $\lambda$ is eigenvalue of $H_{R,R}^H H_{R,R}$. Thus Lagrange method provided us with a set of critical points
however, we have to choose optimum set for \( v_R \) that minimizes the received signal power. The power can be determined as follows:

\[
H_{R,R}^\dagger H_{R,R} v_R = \lambda v_R
\]

\[
v_R^\dagger H_{R,R}^\dagger H_{R,R} v_R = v_R^\dagger \lambda v_R
\]

\[
= v_R^\dagger v_R \lambda
\]

\[ P_R = \lambda. \quad (4.8) \]

From equation (4.8), we can infer that the power of received signal at critical points is nothing but the eigenvalue of relay receive covariance matrix. Hence the optimal transmit beamformer \( v_R \) should be eigenvector associated with minimum eigenvalue \( \lambda_{\text{min}} \).

### 4.2 Simultaneous Signaling and Channel Estimation

Transmit beamforming approach requires the knowledge of the CSI at the transmitter. We estimate the channel at relay receiver side and feed back the estimates to the relay transmitter. The conventional methods of channel estimation described in Section (2.3), with separate training and communications periods, reduces the goodput due to the additional overhead involved in the transmission of pilot signals. In this Section, we propose simultaneous communications technique to estimate the self-interference channel. The basic idea behind simultaneous communications and channel estimation is to transmit relatively low power pilot signals simultaneously with data signals without reducing the goodput. The simultaneous signaling and channel estimation approach, used to estimate the channel, offers significant advantages over conventional time-interleaved channel estimation approaches by providing more flexibility in working with legacy waveforms, increasing data rate, and improving estimation performance in the case of dynamic channels.
The proposed signal flow diagram from relay transmitter-relay receiver is shown in Figure 4.2. The relay transmitter beamforms the signals in a way that the power sent to the relay receiver is reduced while maximizing the power along the direction to the destination node. The beamforming is carried out in the frequency domain. For accurate cancellation of self-interference, the transmitter needs accurate relay-relay channel state information. By transmitting the high power communications signal only within a spatial subspace that protects the receivers and simultaneously transmitting a full-rank transmit covariance at a lower power, we can achieve a full-duplex radio system dynamic range that is greater than the receiver’s dynamic range.

We now consider the channel within each OFDM subcarrier. We suppress any explicit index to the subcarrier, but the following operations must be repeated for each subcarrier. To estimate the relay-relay channel $H_{R,R}$, the transmitter sends the pilot signals simultaneously with the beamformed signals. The signal model at the relay transmitter, for each frequency subcarrier, is given by

$$T_R = v_R s_R + \sqrt{\eta} Q$$

$$S_R = v_R s_R,$$  \hspace{1cm} (4.9)
where the pilot signal used for channel estimation is represented by $Q \in \mathbb{C}^{m_{\mathcal{R},t} \times n_s}$, the ratio of channel-estimation power to communications power by $\eta$, the transmit beamformer by $v_R$, and the beamformed time-domain signal over a number of OFDM symbols by $s_R \in \mathbb{C}^{1 \times n_s}$. The pilot signals be selected such that they are orthogonal and are of relatively low in power compared to transmit beam former,

$$
\|v_R s_R\|_F^2 \gg \|\sqrt{\eta} Q\|_F^2.
$$

(4.10)

This is to ensure that pilot signals are both within the dynamic range of the full-duplex receiver and well below the noise floor of the destination node. The relay receiver upon receiving the signal makes an estimate of the interference channel corresponding to each frequency bin. The receiver then mitigates the pilot signals by projecting onto a temporal space orthogonal to the sequence [35], and decodes the resulting signal. The channel estimates are fed back to the transmitter for performing the beamforming during subsequent instants. The channel is estimated using the least square estimator,

$$
\hat{H}_{R,R} = Z_R T_R (T_R T_R^\dagger)^{-1}.
$$

(4.11)
Consider the singular value decomposition (SVD) of the estimated self-interference channel of one of the OFDM subcarriers

\[
\hat{H}_{R,R} = U_{R,R} \Sigma_{R,R} V_{R,R}^\dagger,
\]

(4.12)

where $\Sigma$ contains the singular values and the columns of $U_{R,R}$ and $V_{R,R}$ contain the left and right singular vectors, respectively. We protect the relay receive antenna by selecting transmit vectors that are contained with in the null space (associated with the zeros of $\Sigma_{R,R}$).

Although, simultaneous signaling approach is advantageous over conventional approach in the context of goodput, it has one draw-back. Due to the transmission of pilot embedded data signal, the rank of transmit co-variance matrix has increased from rank 1 to full rank. The increase in transmit rank is good in terms of channel estimation which avoids singular matrix inversion, but on the other hand

### 4.3 Performance Analysis of Simultaneous Signaling Approach

The fundamental measure of the performance is the level and spatial structure of the self-interference observed at the receive array; however, as an intermediate measure of performance, we consider the variance of the self-interference channel estimation error. If the spatial self protection is performing well, then the channel estimation will be essentially equivalent to estimating the channel without the communications signal, because little of that signal will be observed at the receiver. We evaluate the channel estimation performance of the simultaneous signaling and channel estimation by comparing against the Cramer-Rao bound under the assumption of training signal exclusively.
4.3.1 Cramer-Rao Bound

Cramer-Rao bound is a useful tool to estimate the lower bounds on the variance of an unbiased estimator. Cramer-Rao bound is a local bound and it states that the variance of an unbiased estimator is lower bounded by the inverse of fisher information. By far it is the simplest bound to derive. In this section, we reproduce the derivation for Cramer-Rao bound for variance of channel estimate error in the presence of additive noise provided in [35].

The Cramer-Rao bound for the self-interference channel estimation error variance of the $m^{th}$, $n^{th}$ element of the channel is given by the inverse of fisher information matrix

$$\text{var} \left\{ \left( \hat{H}_{\mathcal{R},\mathcal{R}} \right)_{m,n} \right\} = \left\{ J^{-1} \right\}_{(m,m),(j,k)} \quad (4.13)$$

here $\{m,n\}, \{j,k\}$ is used to specify an element of a matrix at row $\{m,n\}$ and column $\{j,k\}$.

Let us assume $X$ be the sequences transmitted by relay transmitter and the source node is not transmitting any signal. The signal received at the relay receiver $Y$ is given by

$$Y = H_{\mathcal{R},\mathcal{R}}X + N \quad (4.14)$$

where $N$ represents the additive white Gaussian noise with zero-mean and unit variance. The mean of the received signal $Y$ is given by

$$\langle Y \rangle = \langle H_{\mathcal{R},\mathcal{R}}X + N \rangle \quad (4.15)$$

$$= H_{\mathcal{R},\mathcal{R}}X$$

The co-variance matrix $R$ does not contain the channel matrix. Hence

$$\frac{\partial R}{\partial (H_{\mathcal{R},\mathcal{R}})_{m,n}} = 0, \quad (4.16)$$
and the derivative of one conjugation with respect to the other is zero,

$$\frac{\partial H_{\mathcal{R},\mathcal{R}}}{\partial (H_{\mathcal{R},\mathcal{R}})^*} = 0, \quad (4.17)$$

The fisher information matrix $J_{m,n}$ element associated with $\{m, n\}, \{j, k\}$ for the observation vector $Y$ with conditional probability density function $p(Y|H_{\mathcal{R},\mathcal{R}}, X)$ is given by

$$\{J\}_{\{m,n\},\{j,k\}} = -\frac{\partial^2}{\partial (H_{\mathcal{R},\mathcal{R}})^* \partial (H_{\mathcal{R},\mathcal{R}})} \log p(Y|H_{\mathcal{R},\mathcal{R}}, X)$$

$$= -\frac{\partial^2}{\partial (H_{\mathcal{R},\mathcal{R}})^* \partial (H_{\mathcal{R},\mathcal{R}})} \log(\exp^{-tr\{(Z-H_{\mathcal{R},\mathcal{R}}X)^\dagger R^{-1}(Z-H_{\mathcal{R},\mathcal{R}}X)\}})$$

$$= tr\{\frac{\partial^2}{\partial (H_{\mathcal{R},\mathcal{R}})^* \partial (H_{\mathcal{R},\mathcal{R}})} (H_{\mathcal{R},\mathcal{R}}X)^\dagger R^{-1}(H_{\mathcal{R},\mathcal{R}}X)\}$$

$$= tr\{\frac{\partial (H_{\mathcal{R},\mathcal{R}}X)^\dagger}{\partial (H_{\mathcal{R},\mathcal{R}})^*} R^{-1} \frac{\partial (H_{\mathcal{R},\mathcal{R}}X)}{\partial (H_{\mathcal{R},\mathcal{R}})}\}$$

$$= tr\{X^\dagger \frac{\partial (H_{\mathcal{R},\mathcal{R}})}{\partial (H_{\mathcal{R},\mathcal{R}})^*} R^{-1} \frac{\partial (H_{\mathcal{R},\mathcal{R}})}{\partial (H_{\mathcal{R},\mathcal{R}})} X\}, \quad (4.18)$$

The derivatives of the channel are given by

$$\frac{\partial (H_{\mathcal{R},\mathcal{R}})}{\partial (H_{\mathcal{R},\mathcal{R}})^*}_{j,k} = e^T_j e_k$$

$$\frac{\partial (H_{\mathcal{R},\mathcal{R}})^\dagger}{\partial (H_{\mathcal{R},\mathcal{R}})^*}_{j,k} = e^T_n e_m, \quad (4.19)$$

where $e_n$ indicates the vector of zeros with a one at the $n^{th}$ row,

$$e_n = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \quad (4.20)$$

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For the sake of evaluating the bound, consider that the source node is not transmitting and hence the interference-plus-noise co-variance matrix is given by

\[ R = I_n, \]  

(4.21)

where the noise per channel is normalised to unity.

The fisher information matrix is then modified as

\[
\{ J \}_{m,n},\{j,k\} = tr\{ X^\dagger \frac{\partial (H_{RR}^*)^T}{\partial (H_{RR})_{j,k}} R^{-1} \frac{\partial (H_{RR})}{\partial (H_{RR})_{j,k}} X \} \\
= tr\{ (e_m e_n^\dagger X)^\dagger \ I^{-1} (e_j e_k^\dagger X) \} \\
= x_k x_n^\dagger \delta_{m,j},
\]

(4.22)

where \( x_k \) indicates a row vector containing the \( k^{th} \) row of the training sequence \( X \) and \( \delta \) represents the Kronecker delta function. The training samples \( X \) is normalised and for sufficiently long sequences it is reasonable that

\[ x_k x_n^\dagger = n_s \delta_{k,n} \]  

(4.23)

Then the Fisher information matrix can be written as

\[
\{ J \}_{m,n},\{j,k\} = n_s \delta_{k,n} \delta_{m,j}
\]

(4.24)

The Fisher information matrix is a diagonal matrix with same elements along the diagonal. From Cramer-Rao bound theorem, the variance of the channel estimate with out the presence of interference signal is given by

\[
\text{var} \left\{ \left( \hat{H}_{RR} \right)_{m,n} \right\} = \{ J^{-1} \}_{m,m},\{j,k\} = \frac{1}{n_s}.
\]

(4.25)

Translating the above result, the Cramer-Rao bound for the self-interference channel estimation error variance of the \( j^{th},k^{th} \) element of the channel in case of simulta-
neous signaling is given by
\[
\text{var}\left\{ (H_{\mathcal{R},\mathcal{R}})_{j,k} \right\} = \frac{1}{n_s \eta},
\]}

(4.26)

where \( \eta \) indicates the relative power ratio between the pilot sequence and the communication signal.

In Figure 4.3, we show the comparison between the error variance and the Cramer-Rao bound. The communication signal power is set to 15 dB. The self-interference channel is estimated for various pilot signal powers according to the Equation (4.11). The estimation was performed over 100 OFDM symbols. From Figure 4.3, we infer that the simultaneous communications and channel estimation gives a good estimate of the channel and it closely follows the Cramer-Rao bound. The accuracy of the estimate increases with increase in pilot signal power.

![Figure 4.3](image)

**Figure 4.3:** Comparison Between Estimate Error Variance Due to Simultaneous Signaling and Channel Estimate Versus Cramer-Rao Bound As a Function of The Relative Pilot to Communications Signal Power. Estimation Was Performed Over 100 OFDM Symbols. The Communications Signal Power is 15 dB.
This chapter is organized as follows: in Section 5.1, we discuss the parameters considered in our simulation and in Section 5.2, we provide the simulation results. First, we compare the simultaneous signaling approach with that of no-simultaneous signaling approach. Then we compare the simultaneous signaling approach for two different number of samples.

5.1 Simulation Parameters

Table 5.1 summarises the simulation parameters. We employ an OFDM waveform using a 128-point FFT (in accordance with IEEE 802.11n standard). The source-relay, relay-relay and relay-destination channel is assumed to be frequency selective. The frequency selective self-interference channel is drawn from an independent circular

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of antenna at source node</td>
<td>1</td>
</tr>
<tr>
<td>Number of antenna at destination node</td>
<td>1</td>
</tr>
<tr>
<td>Number of antenna at relay transmitter</td>
<td>7</td>
</tr>
<tr>
<td>Number of antenna at relay receiver</td>
<td>3</td>
</tr>
<tr>
<td>Relay receive noise floor</td>
<td>0 dB</td>
</tr>
<tr>
<td>Number of sub-carriers</td>
<td>128</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
<td>10</td>
</tr>
<tr>
<td>Number of symbols</td>
<td>100</td>
</tr>
</tbody>
</table>
complex Gaussian distribution with a full-bandwidth 10 delay tap channel with an amplitude exponential delay weighting, $e^{-\delta/4}$.

5.2 Simulation Results

In Figure 5.1, we display the eigenvalue distribution of the relay self-interference signal along each sub-carrier, with and without using the simultaneous signaling and channel estimation. The estimation was performed over 100 OFDM symbols. The eigenvalues indicate the self-interference power plus noise observed at the receiver in each eigenmode. There were three eigenmodes which is the $\min(\text{number of relay transmit antenna, number of relay receive antenna})$. If the self-interference signal is perfectly mitigated then all the eigenvalues will be along the noise floor. On the other hand if there is any self-interference signal then some of the eigenvalues will be non-zero and lie above the receiver noise floor of 0 dB. The set of blue lines (lying close to the noise floor of 0 dB), indicate the eigenvalue distribution for a system that is not employing simultaneous channel estimation and signaling and is estimating

\[\begin{array}{c}
\text{Simultaneous signaling} \\
\text{Without Simultaneous Signaling}
\end{array}\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.1}
\caption{Eigenvalue of Relay Receive Self-Interference Covariance Matrix With and Without Simultaneous Signaling and Channel Estimation. In Both Cases, Estimation was Performed Over 100 OFDM Symbols.}
\end{figure}
the channel at full power (which might saturate the receiver in practice). This is because of accurate channel estimates which results in perfect null-space projection and leads to perfect cancellation of self-interference to the noise floor. The eigenvalue distribution in case of the simultaneous approach (shown in red lines) spreads around 15 dB from the noise floor. The spreading is due to the inaccurate channel estimates, however the spreading is well within the dynamic range of most receivers and can be overcome by using either spatial or temporal approaches.

If the channel is relatively static, the performance of simultaneous channel estimation and communications approach can be improved by considering the use of a larger number of samples in the channel estimation. In Figure 5.2, we show the comparison between the eigenvalue spread of the self-interference signal with 100 samples (shown in blue) and 1000 samples (shown in red) used for channel estimation. It is observed that, the eigenvalue spread in case of 1000 samples is 10 dB lower compared to that of 100 samples.

![Figure 5.2: Comparison of Receive Eigenvalue Spread With Number of Samples.](image)

The self-interference channel estimate accuracy can be improved by increasing the relative pilot to communications signal power. However, increasing the pilot signal
power increases the amount of self-interference at the relay receiver. In Figure 5.3, we plot the dominant eigenvalue of the relay receive self-interference signal for different pilot to communications signal power ($\eta$). The increase in dominant eigenvalue indicates the increase in self-interference signal power.

![Graph showing the relationship between dominant eigenvalue and relative pilot signal power](image)

**Figure 5.3:** Dominant Eigenvalue of Relay Receive Self-Interference Covariance Matrix As a Function of The Relative Pilot to Communications Signal Power. Estimation was Performed Over 100 OFDM Symbols.

In Figure 5.4, we compare the dominant eigenvalue of the relay receive self-interference signal for different combinations of relay transmit and receive antenna. The number of antenna at relay transmit side is varied to be 4, 5, 6, 7, 8, 9 and at relay receive side to be 3, 4, 5, 6. The lower the eigenvalue difference, the greater the amount of suppression of self-interference. It is observed from the figure that if the number of antenna at the relay transmit side is more than the number of relay receive antenna, then the difference is low and lies around 15 dB. This is because of the availability of the additional degrees of freedom at the transmit side to suppress the self-interference.

When the number of transmit antenna is equal or less than the number of relay receive antenna, then there is a 50dB difference between the eigenvalues indicating
large self-interference due to the reduced degrees of freedom at the transmit side to project the self-interference signal in null-space.

Figure 5.4: Comparison of Dominant Eigenvalue of Relay Receive Self-Interference Covariance Matrix for Different Number of Relay Transmit and Receive Antenna. In All the Cases, Estimation was Performed Over 100 OFDM Symbols.
CONCLUSIONS AND FUTURE DIRECTIONS

6.1 Main Conclusion

In this thesis, we addressed the spatial suppression of full-duplex relay self-interference to improve the relay performance. We proposed simultaneous signaling and channel estimation, a new approach for MIMO full-duplex radios. Our simulation results show that, the proposed technique provides a good estimate of the channel without decreasing the goodput or saturating the receiver. Also it is demonstrated that, simultaneous signaling and channel estimation approach in combination with adaptive transmit spatial techniques results in significant cancellation of relay self-interference.

6.2 Summary of Research

This chapter concludes our research work by highlighting the key contributions of our work. The primary objective of this thesis is to introduce effective channel estimation techniques to be used with spatial interference mitigation techniques without reducing the goodput. In this thesis, we addressed the problem of self-interference in IBFD relays with the aim of optimising the relay twisted-SINR.

Full-duplex relays suffers from heavy self-interference due to the coupling of relay’s own transmission with its reception. If somehow the problem of self-interference is solved, full-duplex operation provides increased throughput compared to half-duplex operation. In Chapter 1, We provided the leverages to address the problem of self-interference in full-duplex relays and summarized the important contributions in the field of relaying.
In Chapter 2, we provided a brief description about MIMO, OFDM, block and pilot based channel estimation techniques in OFDM systems, cooperative relaying protocols and various interference mitigation techniques in full-duplex relays.

In this contribution, we addressed the spatial suppression of full-duplex relay self-interference to improve the relay performance. In particular, we employed adaptive beamforming at the relay transmit side such that the power sent in the direction of relay receiver is minimized and maximized along the direction of destination node. The optimal beamforming vectors that minimizes the self-interference are derived in Chapter 4 using Lagrange theorem of multipliers. The performance of this approach in mitigating interference depends on the accuracy of channel estimation. The conventional methods of channel estimation, with separate training and communication periods, reduce the goodput due to additional overhead involved in transmission of pilot signals. We proposed simultaneous signaling and channel estimation, a new approach for full-duplex radios. Simultaneous signaling is based on burying low power pilot signal in the data signal. In chapter 4, we evaluated the performance of simultaneous approach by comparing against the Cramer-Rao bound. Our simulation results show that, the proposed technique provides a good estimate of the channel without decreasing the goodput or saturating the receiver.

In Chapter 5, we analysed the performance of simultaneous signaling and channel estimation approach in combination with adaptive transmit spatial techniques in the context of relay receive eigenvalue spread. It is observed from the simulations, that the eigenvalue distribution in case of the simultaneous approach spreads around 15 dB from the noise floor and the spread can be decreased further by considering more number of OFDM symbols. It is demonstrated that increasing pilot signal power, although increases the channel estimate accuracy it also contributes to the increased self-interference signal power at the relay receiver. The difference between
the dominant and next dominant eigenvalues of the relay receive co-variance matrix is observed for various combinations of relay transmit and receive antenna. It is observed that if the relay transmitter has at least one degree of freedom (i.e., number of antenna) higher than the receiver then the interference is significantly reduced.

6.3 Future Work

Future extension of the research could be a detailed study of the impact of various hardware non-linearities on the simultaneous communications approach developed in this thesis. Further extension could be to develop models that model these non-linearities and methods to compensate for these non-linearities to make simultaneous communications practically viable in MIMO relays. Another possible extension could be to study the problems of peak-to-average power ratio (PAPR) associated with OFDM on simultaneous communications approach.
REFERENCES


