Optimal Substation Ground Grid Design Based on
Genetic Algorithm and Pattern Search

by

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ABSTRACT

Substation ground system insures safety of personnel, which deserves considerable attentions. Basic substation safety requirement quantities include ground grid resistance, mesh touch potential and step potential, moreover, optimal design of a substation ground system should include both safety concerns and ground grid construction cost. In the purpose of optimal designing the ground grid in the accurate and efficient way, an application package coded in MATLAB is developed and its core algorithm and main features are introduced in this work.

To ensure accuracy and personnel safety, a two-layer soil model is applied instead of the uniform soil model in this research. Some soil model parameters are needed for the two-layer soil model, namely upper-layer resistivity, lower-layer resistivity and upper-layer thickness. Since the ground grid safety requirement is considered under the earth fault, the value of fault current and fault duration time are also needed.

After all these parameters are obtained, a Resistance Matrix method is applied to calculate the mutual and self resistance between conductor segments on both the horizontal and vertical direction. By using a matrix equation of the relationship of mutual and self resistance and unit current of the conductor segments, the ground grid rise can be calculated. Green's functions are applied to calculate the earth potential at a certain point produced by horizontal or vertical line of current. Furthermore, the three basic ground grid safety requirement quantities: the mesh touch potential in the worst case point can be obtained from
the earth potential and ground grid rise; the step potential can be obtained from two points' earth potential difference; the grid resistance can be obtained from ground grid rise and fault current.

Finally, in order to achieve ground grid optimization problem more accurate and efficient, which includes the number of meshes in the horizontal grid and the number of vertical rods, a novel two-step hybrid genetic algorithm-pattern search (GA-PS) optimization method is developed. The Genetic Algorithm (GA) is used first to search for an approximate starting point, which is used by the Pattern Search (PS) algorithm to find the final optimal result. This developed application provides an optimal grid design meeting all safety constraints. In the cause of the accuracy of the application, the touch potential, step potential, ground potential rise and grid resistance are compared with these produced by the industry standard application WinIGS and some theoretical ground grid model from [19] and [27].

In summary, the developed application can solve the ground grid optimization problem with the accurate ground grid modeling method and a hybrid two-step optimization method.
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NOMENCLATURE

\(a\)
Diameter of Conductor

\(a_1\)
Length of L Shape Ground Grid

\(a_2\)
Length of L Shape Ground Grid

\(a_3\)
Width of L Shape Ground Grid

\(a_4\)
Width of L Shape Ground Grid

\(b\)
Metallic Disc Radius

\(d\)
Grid Burial Depth

\(dx_s\)
Length of Infinitesimal Segment

\(h_0\)
Reference Depth

\(i\)
Segment Current Distribution Factor

\(i_k\)
Leakage Current of the \(k\)th Segment

\(k_{21}\)
Reflection Factor between Upper Layer and Lower Layer

\(k_{32}\)
Reflection Factor between Upper Layer and the Air

\(l\)
Length of Single Conductor or Rod

\(l_{rod}\)
Rod Length

\(l_{dr}\)
Rod Driven Depth

\(n\)
Geometric Factor

\(n_{a,b,c,d}\)
Geometric Factor

\(r\)
Cylindrical Coordinate

\(r_{jk}\)
Mutual Resistance between Segment \(j\) and Segment \(k\)

\(r_{jj}\)
Self-Resistance of Segment \(j\)

\(t_f\)
Fault Current Duration
z  
Cylindrical Coordinate

$(x_r, y_r)$  
Search Range in Pattern Search

$(x, y, z)$  
Coordinate of the Midpoint of A Field Conductor in A Rectangular-Coordinate System

$C_{cond}$  
Cost of Conductor and Rod Per Ft

$C_{exoth}$  
Cost of Exothermic

$C_{trench}$  
Labor Cost to Trench, Install and Backfill Conductor

$C_{drive}$  
Labor Cost to Trench, Install and Backfill Rod

$C_{connect}$  
Labor Cost to Connection

$D$  
Depth of Upper Layer

$D_m$  
Square Mesh Spacing

$D_r$  
Search Resolution in Pattern Search

$D_X$  
Mesh Size on Horizontal X Direction

$D_Y$  
Mesh Size on Horizontal Y Direction

$D_f$  
Current Division Factor

$E_{earth}$  
Earth Potential of A Grounding System at Certain Point

$E_{mesh}$  
Worst Touch Potential of A Grounding System
$E_{touch}$  Touch Potential

$E_{step}$  Worst Step Potential of A Grounding System

$E_{touch\_allowable}$  Allowable Touch Voltage

$E_{step\_allowable}$  Allowable Step Voltage

$G_{23}$  Green’s function for the Field Point in the Air

$G_{22}$  Green’s function for the Field Point in the Upper-layer Soil

$G_{21}$  Green’s function for the Field Point in the Lower-layer Soil

$HLA$  Length of Segment A

$HLB$  Length of Segment B

$I_p$  Current of Infinitesimal Segment

$I_F$  Fault Current

$I_g$  Maximum Current Flowing Through A Grounding System

$I$  Current Injecting into the Point Source

$I_B$  Electric Body Current

$K_m$  Geometrical Factor

$K_i$  Irregularity Factor

$K_{II}$  Correction Factor for Grid Geometry

$K_s$  Spacing Factor for Step Voltage

$K_{12}$  Reflection Coefficient between Region 1 and Region 2

$K_{32}$  Reflection Coefficient between Region 3 and Region 2

$L_1$  Length of Half Segment

$L_x$  Length of Square or Rectangular Grid
$L_y$ Width of Square or Rectangular Grid

$L_s$ Effective Length for Step Voltage

$L_p$ Ground Grid Perimeter

$L_T$ Total Effective Length of Grounding System Conductor

$N_X$ Number of Meshes on Horizontal X Direction

$N_Y$ Number of Meshes on Horizontal Y Direction

$N_{rod}$ Number of Rods

$PARL$ Parallel Parameter Used in Matrix Method

$PERP$ Perpendicular Parameter used in Matrix Method

$PERP_X$ Perpendicular Parameter used in Matrix Method on X Direction

$PERP_Y$ Perpendicular Parameter used in Matrix Method on Y Direction

$PERP_Z$ Perpendicular Parameter used in Matrix Method on Z Direction

$R_B$ Body Resistance

$R_f$ Foot Resistance

$R_g$ Grounding Resistance to Remote Earth

$R$ $V/A$

$R_{g,1}$ Single Rod and Conductor Resistance Calculated by MATLAB

$R_{g,2}$ Single Rod and Conductor Resistance Calculated by WinIGS

$R_{g,3}$ Single Rod and Conductor Resistance Calculated by Theoretical Model

SRP Salt River Project
\( V \):
Voltage Measurement (volts) from the Voltmeter

\( VDF \):
Voltage-distribution-factor Matrix including Self and Mutual Resistances

\( VX \):
Potential at a Point Caused by a X-directed Line Current Source

\( VY \):
Potential at a Point Caused by a X-directed Line Current Source

\( VZ \):
Potential at a Point Caused by a Z-directed Line Current Source

\( V_{22}G \):
Potential at Upper Layer Point Due to a Horizontal Current Line in Upper Layer

\( V_{21}G \):
Potential at Upper Layer Point Due to a Horizontal Current Line in Lower Layer

\( V_{22}ZG \):
Potential at Upper Layer Point Due to a Vertical Current Line in Upper Layer

\( V_{21}ZG \):
Potential at Upper Layer Point Due to a Vertical Current Line in Lower Layer

\((X_{ps1}, Y_{ps1})\):
Starting Point in Pattern Search

\((X_A, Y_A, Z_A)\):
Coordinate of the Midpoint of Segment A

\((X_B, Y_B, Z_B)\):
Coordinate of the Midpoint of Segment B

\((X_{\text{mesh}}, Y_{\text{mesh}}, Z_{\text{mesh}})\):
Coordinate of the Touch Potential Worst Case Point

\((X_l, Y_l, Z_l)\):
Coordinate of the Midpoint of a Line Current Source in a Rectangular-coordinate System

\(Z_{Th}\):
Thevenin Equivalent of Body Resistance

\(\rho_1\):
Upper Layer Soil Resistivity

\(\rho_2\):
Lower Layer Resistivity

\(\rho\):
Homogeneous Earth of Resistivity

\(\sigma_{Ri}\):
Expected Standard Deviation of \(i\)th Measurement
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<td>$\sigma_1$</td>
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CHAPTER 1.
INTRODUCTION

1.1 Background Introduction

With the continuously increasing capacity of the power system, ground fault current has also continued to grow. Grounding system analysis and design to ensure reliable and safe operation of power substation is essential. The objective of this research is to ensure personnel safety through the design of a ground grid which meets all safety standards and with minimal cost.

The aim of this project and research is to develop an applications which has the capacity of modeling and optimizing regular shape (rectangular, square or L shape) ground grid under a two-layer soil model assumption. Generally, there are two modeling methods to design a grid model. One is IEEE standard equations which is an approximate modeling method. In this work, the second one, a more accurate modeling and calculation method is used, which includes the Resistance Matrix method and Green's function.

For the optimization model, it includes the objective function and constraints. The costs in the objective function include: material cost of horizontal conductor and vertical rod, material cost of exothermic welds, cost of labor to trench and install conductors and rods, cost of labor to make exothermic connections of conductor to conductor or conductor to rod. The safety requirements include: 1) the step potential and touch potential of designed ground grid must be lower than the max allowable values, 2) ground grid resistance must be lower than the acceptable value 0.5 ohms. Some other grid design requirements are also provided
by Salt River Project (SRP), such as max and min mesh size and max and min number of ground rods. From the above information, the objective function and constraints can be obtained.

The most popular optimization methods are not appropriate for this application since nonlinear and mixed integer problems need to be solved. Some heuristic and artificial intelligence methods were applied.

1.2 Literature Review

The substation grounding system, which serves to insure the safety of personnel during earth faults, has a long history in the research literature [1]-[18]. As defined in IEEE Std. 80 –2000 [19], basic substation safety requirement quantities including ground grid resistance, mesh touch potential, step potential and ground potential rise (GPR), should be limited as constraints in design for the sake of safety. Hence, the researchers have focused on how to get these quantities. Optimal ground-grid design only used a “trial-and-error” approach [10] and [13] earlier, which could not provide a real optimal result for our problem. Subsequently, we can see other optimization methods were applied in [14], [18]. But in these papers, in general, a fast ground grid modeling method with some approximate functions has been used, which can't meet the Salt River Project precision requirement. Several commercial software products have been introduced in [31], [32], and self-developed application has been developed [23]. However, in [31], WinIGS can only simulate the ground grid without any optimal functions. However, in this work, some application functions such as touch
potential 2D and 3D plots are motivated by WinIGS. In [23], this simple application can only make the optimal design for rectangular shape ground grid. While our application can handle more grid shapes based on the project requirements.

The literature is categorized in the following seven main topics:

1) **The soil model**: In [17], the uniform soil model has been used to analyze the grounding system. In [10], a non-uniform two-layer soil model was used for the ground grid modeling, the accuracy of the ground grid calculation has been improved accordingly.

Some ground grid modeling methods with approximate expressions have been adopted by the IEEE standard and have been used for years. However, these traditional methods cannot fulfill the accuracy requirement. Some numerical computing methods, such as ground-grid segment strategy and Green's function methods, have been used since the 1970s.

2) **The ground grid segment method**: In [9] and [10] F. Dawalibi introduced a method which segments the grounding conductors and rods in the ground grid. Moreover, two ways to compute potentials contributed by each segment have been proposed. The first method for computing potentials models a single segment as a point current source leading to a series expression for the potential. The second method models a single segment as a line current source leading to an integral expressions. In this work, both methods are used as discussed later, the selection of these two methods is depending on the distance between two segments. Dawalibi
was the first one to show that multi-step analysis of interconnected grounding electrodes can be used to handle unbalanced current distribution.

3) **The Resistance Matrix method:** This method gets its name from the observation that the voltage distribution factors and mutual-self resistance of the conductors and rods segments is calculated using a resistance matrix. In [30], Y. L. Chow proposed using image conductors to model ground electrodes in layered soils. Chow modeled a toroidal electrode in a four-layer soil by using one real image and four complex images.

4) **The Green's functions:** In this approach, the earth potentials of a certain point in the ground grid is calculated using Green’s functions. In [4], Robert J. Heppe proposed a method to calculate the effects of the variation of conductors with leakage current, cross conductors, angled conductors, and end effects. Heppe also derived functions to calculate the surface voltage near ground conductors, which are used to obtain the earth potential at arbitrary points on the earth surface. After obtaining the earth potential, the touch potential and step potential can be calculated.

5) **The shape and mesh size of ground grid:** This topic of research refers to the grounding grid shape, and the grid with uniform mesh size [14] and/or non-uniform mesh size [15]. In [14] and [15], the influence of uniform/non-uniform mesh size to touch potential and step potential values has been discussed. In this work, all grids are assumed to have uniform mesh size. In the future work, the un-equal mesh size can be
considered.

Since the number of equations used to represent a ground grid varies based on the mesh resolution, the traditional optimization methods, such as Newton’s method which required the number of equations to be fixed during the optimization process are no applicable in this work. Therefore, we use heuristic methods and direct search methods, such as a genetic algorithms and pattern search.

6) **Optimization model:** For the objective function in the optimization model, reference [23] considered the material cost and labor cost which is the same as this work. The constraints that guarantee personnel safety are found in IEEE Std.80-2000 [19]. Of these constraints it is found that the step potential is not the binding constraint. Thus, in some papers, such as [26] and [28], step potential has not been studied. However, in this work, the step potential is still considered as a grid safety constraint.

7) **Optimization method:** Mixed-integer linear programming was used in [14] and [15] which applies only when the objective function is linear, a condition that is not obeyed in most cases. In this work, the optimization problem is nonlinear, since some of the constraint functions are nonlinear, which include the constraints on the touch potential, step potential and grid resistance. In order to solve the nonlinear optimization problem, some heuristic methods have been used, such as the genetic algorithm (GA), which was applied in [16] and [17]. Alternatively, the pattern
search method (PS) was investigated in [10] and shown to be complementary to GA. In [10] the author presented not only the theory of GA and PS, but also the rationale that should be used when selecting among the optimization solvers available in MATLAB and the Global Optimization Toolbox. The selection strategy provides recommendations on how to select different optimization methods to solve linear or nonlinear optimization problems. The particle swarm optimization (PSO) was introduced in [18] and compared with GA yielding optimal solutions, which were close (the result difference was less than 5% ) for same optimization problem while requiring less computation time.
1.3 Report Organization

There are three main chapters in this work covering, analysis of the ground grid system and its safety requirements, the optimization model and the hybrid GA and PS optimization method, and instructions for using the ground grid optimization application developed in this work, respectively. In the last chapter, the conclusions and future work are stated.

In Chapter 2, a mathematical model is described to compute the key safety requirement quantities (step potential, mesh touch potential and ground grid resistance) of substation ground system. Some ground grid analysis and safety requirement calculation cases are shown.

In Chapter 3, a two-step hybrid GA-PS algorithm is proposed. Some ground grid optimization design cases are shown.

In Chapter 4, instructions for using the application developed for ground-grid optimization is shown. The application was developed using MATLAB and with a GUI (graphical user interface) which satisfies Salt River Project’s requirements.

In Chapter 5, the conclusion and the recommendations for future research are provided.
CHAPTER 2.

GROUND GRID MODELING METHOD

This chapter presents the modeling method used for a ground grid system in a two-layer soil model. In Fig. 2.1, the flowchart of the ground grid safety requirements calculation steps are presented. Subsequent sections will mainly discuss the calculation method.

Fig. 2.1 Flowchart of Ground Grid Safety Requirement Calculation
In the above flowchart, the first step is segmenting conductors to divide the ground conductors and rods into sufficiently small segments so that an accurate answer is achieved. Then the Resistance Matrix method is utilized to calculate the self and mutual resistance among each segment. In the third step, a simple matrix method is used to calculate current distribution factors in ground conductor and rod segments, ground grid resistance and ground potential rise. In the fourth step, the Green’s functions are used to calculate the earth potential at desired points on the earth’s surface produced by fault current flowing in the horizontal and vertical conductor segments. To validate the program, some simple cases of a single horizontal conductor and vertical rod are compared to WinIGS simulation results and to theoretical results [27]. Additionally, some safety ground grid design results for grids of rectangular shape, square shape and L shape are compared to WinIGS.

2.1 The Equivalent Circuit of Body Shock

The influence of an electric current passing through the vital parts of a human body depend on the duration, magnitude and frequency of the current. Humans are very vulnerable to the effects of electric current. For purposes of calculating body current due to ac voltages, with direct current or alternating current in 60 Hz, the human body can be modeled as a resistance. Currents with magnitude as little as 0.1 Amp can be lethal at these frequencies.
As shown in Fig. 2.2, when there is a fault or induced voltage in the substation equipment, a fault current or induced current can flow into the soil and the substation grounding system. The flow of this current will lead to the touch potentials and step potentials which can create a flow of body current as shown in Fig. 2.2. The current paths associated with touch potential and step potential are from hand to both feet and from one foot to the other one, respectively.

According to [19], the touch potential body current circuit and step potential body current circuit are shown in following Fig. 2.3 and Fig. 2.4.
In the above two figures, the $R_B$ is the body resistance. The variable $R_f$, represents the ground resistance of one foot. The variable $Z_{Th}$ represents the Thevenin impedance of the body circuit as seen from the two terminals. The two terminals are shown in the above Fig. 2.3 and Fig. 2.4. The value of the body resistance depends on many unpredictable factors, for example, skin condition, touch condition and magnitude and duration of shock current. According to [19], the value of body resistance is typically chosen as 1000 ohms. And the human foot is modeled as a conducting metallic disc and the contact resistance of shoes.
From [19], the ground resistance in ohms for a metallic disc with radius $b$ on the surface of the homogeneous earth of resistivity $\rho$ is given as following (2.1) from [19].

$$R_f = \frac{\rho}{4b} \quad (2.1)$$

where the metallic disc radius $b$ is traditionally taken as 0.08 m. Therefore, to a close approximation, the equation of $Z_{Th}$ can be obtained as shown in (2.2) and (2.3) for the touch voltage and step voltage conditions, respectively.

For touch voltage accidental circuit:

$$Z_{Th} = \frac{R_f}{2} = \frac{\rho}{4 \times 0.08} \times \frac{1}{2} \approx 1.5 \rho \quad (2.2)$$

And for the step voltage accidental circuit:

$$Z_{Th} = 2R_f = \frac{\rho}{4 \times 0.08} \times 2 \approx 6 \rho \quad (2.3)$$

2.2 Grounding System Safety requirement

Generally, a safe ground grid design has to meet the following two requirements [19]:

- To provide a means to carry electric currents into earth under normal and fault conditions without exceeding any operating and equipment limits or adversely affecting continuity of service.
- To ensure that a person in the vicinity of grounded facilities is not exposed to the danger of critical electric shock.
The three critical values required by the IEEE standard to guarantee safety are: \( R_g \) (ground grid resistance), \( E_{\text{touch}} \) (touch potential) and \( E_{\text{step}} \) (step potential).

A good grounding system should provide a low resistance to remote earth to minimize the \( GPR \) (ground potential rise). For most large substations, the ground resistance is usually about 1 ohms [19]. However, based on the design requirements of Salt River Project, a more critical and conservative resistance value 0.5 ohms is required.

In [29], the step and touch voltage are defined as follows:

- **Step Voltage**: When current is flowing through a conductor to the earth, a high voltage gradient will occur based on the resistivity of the soil, resulting in a voltage difference, also known as a potential difference, between two points on the ground. This is called a step potential as it can cause voltage difference between a person's feet.

- **Touch Voltage**: Touch potential is the voltage between any two points on a person's body, hand to hand, shoulder to back, elbow to hip, hand to foot and so on.

From the ANSI/IEEE Std 80-2000 [19], the allowable touch and step potential values can be calculated using (2.4) and (2.5), respectively.

\[
E_{\text{touch\_allowable}} = (R_B + 1.5\rho)I_B \tag{2.4}
\]

\[
E_{\text{step\_allowable}} = (R_B + 6\rho)I_B \tag{2.5}
\]
where the \( I_B \) is the current value through the body, and, for a 50 kg body weight, the allowable maximum \( I_B \) value is given by (2.6).

\[
I_B = \frac{0.116}{\sqrt{t_f}} \quad (2.6)
\]

where \( t_f \) is the shock duration which can be obtained from the below Table 2.1 from [19].

Based on the ground grid design requirements from SRP, the standard body weight for calculating step and touch potential should be 50 kg (110 lbs). And the separation distance between feet for the step potential calculation should be 1 meter.

**Table 2.1**

**RELATIONSHIP BETWEEN VOLTAGE CLASS AND FAULT DURATION TIME**

<table>
<thead>
<tr>
<th>Voltage class (kV)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;250</td>
<td>0.25</td>
</tr>
<tr>
<td>200 ~ 250</td>
<td>0.50</td>
</tr>
<tr>
<td>22~ 200</td>
<td>0.58</td>
</tr>
<tr>
<td>&lt;22</td>
<td>1.10</td>
</tr>
</tbody>
</table>

According to Section 2.1, the body resistance \( R_B \) is usually taken as 1000 ohms. Therefore, by combining The above equations, (2.4), (2.5) and (2.6), the final equations for allowable touch potential and step potential values can be simplified to (2.7) and (2.8).

\[
E_{\text{touch, allowable}} = (1000 + 1.5\rho)^{0.116} \frac{0.116}{\sqrt{t_f}} \quad (2.7)
\]

\[
E_{\text{step, allowable}} = (1000 + 6\rho)^{0.116} \frac{0.116}{\sqrt{t_f}} \quad (2.8)
\]
Since in this work, the two-layer soil model is used. The $\rho$ in above equations equals the upper layer resistivity, based on the IEEE standard [19].

2.3 Methods of Ground Grid Modeling

Corresponding to Fig. 2.1, the steps needed for calculating the ground grid safety requirements parameters are shown in Section 2.3.1 to Section 2.3.4, with each step discussed in detail.

2.3.1. Electromagnetic Analysis and Segmentation Method

A different number of segments will influence the accuracy of the results. Due to the differing geometries, the segmentation rules used in the symmetrical shape grid (rectangular and square) and unsymmetrical shape grid (L-shape) are different.

First, some guidelines on ground grid segmentation from literature are discussed to provide context for the decisions made in the algorithm development. This discussion will explain why conductor/rod segmentation is needed and will describe the single segment model.

From [4], in order to find the leakage current distribution, one conductor or one rod is divided into many segments, each segment consisting of a single piece of straight conductor. Within each segment, leakage current density is assumed to be constant, but will be allowed to vary between segments. If the electrode is in the form of a rectangular grid, a natural choice for the segments would be the
pieces of conductor between the crossings. If greater accuracy is needed, one can further subdivide these pieces of conductor into smaller segments. Consistent with [21], a horizontal conductor or a vertical rod segment is assumed to be a cylindrical homogeneous conductor.

There are no guidelines in the literature relating segmentation to accuracy. Thus, after testing many cases, a segmentation strategy yielding acceptable accuracy (difference between proposed method and WinIGS simulation should less than 2%) was obtained empirically. The objective of this strategy is to not only guarantee calculation accuracy but also to keep the execution time reasonable. In a symmetrically shaped grid, namely rectangular and square shape, the segmentation strategy is given in following Table 2.2.

<table>
<thead>
<tr>
<th>$N_x + N_y \leq 10$</th>
<th>$Seg_x = L_x / 10$</th>
<th>$Seg_y = L_y / 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 \leq N_x + N_y \leq 20$</td>
<td>$Seg_x = L_x / 15$</td>
<td>$Seg_y = L_y / 15$</td>
</tr>
<tr>
<td>$N_x + N_y \geq 20$</td>
<td>$Seg_x = L_x / 20$</td>
<td>$Seg_y = L_y / 20$</td>
</tr>
</tbody>
</table>

In above Table 2.2, $N_x$ and $N_y$ are the number of divisions on ground mat. And $L_x$ and $L_y$ are the length and width of the substation dimensions, respectively. The $Seg_x$ and $Seg_y$ are segment length values on ground mat side length and width directions, respectively.

If the rod does not penetrate to the lower layer, the rod will be subdivided into five segments with equal length. If the rod does penetrate to the lower layer, the rod will be subdivided into five segments with three equal length segments in the
upper layer and two equal length segments in the lower layer.

However, in an un-symmetrically shaped grid (in our case an L shape), the segmentation method is different as discussed in Section 2.4.4.

2.3.2. Resistance Matrix Method Used to Calculate Mutual and Self Resistance of Each Conductor and Rod Segments

Based on the segmentation strategy in Section 2.3.1, assume that the horizontal mesh and vertical rods on the ground grid are divided into \( n \) segments, where \( k \) is the \( k^{th} \) segment in the total number of segments. Each segment of conductor is modeled as a lumped resistance in the matrix \( r \). If \( j \) is equal to \( k \), it is self-resistance \( r_{jj} \); if \( j \) is not equal to \( k \), it is mutual resistance \( r_{jk} \). Thus, the mutual and self resistance matrix of each segment can be presented using (2.9).

\[
  r = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1n} \\
    r_{21} & \cdots & \cdots & \cdots \\
    \cdots & \cdots & r_{jk} & \cdots \\
    r_{n1} & r_{n2} & \cdots & r_{nn}
  \end{bmatrix}_{(n)^2}
\]  

(2.9)

In order to obtain each entry in the matrix \( r \), the Resistance Matrix method is used. In Fig. 2.5, the two layer soil model plot and reflection coefficients \( K_{32} \) and \( K_{12} \) are shown. The variables \( \sigma_3, \sigma_2, \sigma_3 \) and \( \rho_3, \rho_2 \) and \( \rho_3 \) are the conductivity and resistivity values of region 3 (air), region 2 (upper layer) and region 1 (lower layer). The reflection coefficients \( K_{32} \) and \( K_{12} \) are given by (2.11) and (2.12), respectively.
In equation (2.11), the conductivity value $\sigma_3$ of region 3 (air) is 0. Thus, the $K_{32}$ is equal to -1.

In this work, each conductor or rod segment is modeled as a cylindrical metallic conductor or rod. For each segment, the segment location area (upper layer or lower layer), segment unit diameter ($2a$) and length ($2L$), the coordinate of segment center ($X_i$, $Y_i$, $Z_i$) should be specified. In Fig. 2.6, an example of a horizontal conductor segment is shown.
Fig. 2.6 An Example of Horizontal Conductor Segment

In order to satisfy the competing goals of accuracy and acceptable execution time, two different calculation methods are used here. In the first method, two conductor or rod segments are modeled as lines of uniform current density (line-line modeling) imbedded in the layered media (upper layer or lower layer). In the second method, two conductor or rod segments are modeled as two point sources (point-point modeling) imbedded in the layered media. The criterion used for selecting which of these two methods to use is: the distance between the two segments for which the mutual resistances are needed.

2.3.3. Resistance Matrix Method Line-Line Modeling

The line-line modeling method is introduced first. In the following discussion, all the possible mutual line orientation combinations will be discussed in detail.

- Line-Line modeling: This method is used when the distance between two segments is less than the total length of two segments. Depending on the
locations (upper layer or lower layer) and the geometrical relationship (parallel or perpendicular) of the two lines, there will be eight distinct cases requiring different modeling equation which are needed to calculate the resistance matrix $r$.

Before calculating, the notation to be used should be explained. The central point coordinates of segment A and B are $(X_A, Y_A, Z_A)$ and $(X_B, Y_B, Z_B)$, respectively. The lengths of segment A and B are $HL_A$ and $HL_B$ respectively.

In addition, since segment A and segment B may be parallel to the X axis, Y axis or Z axis direction, the coordinates of the segments should be changed (described below) for the further calculation.

In deriving Cases 1 to 4 of the equations needed to calculate the mutual resistance values, it is assumed that segment A is parallel to segment B, in other words, they have same orientation.

When both segment A and segment B are parallel to X axis, the segment center coordinates should be changed as shown in (2.13), (2.14) and (2.15) from [33]:

$$X = X_B - X_A$$ \hspace{1cm} (2.13)  

$$Y = Y_B - Y_A$$ \hspace{1cm} (2.14)  

$$Z = Z_B - Z_A$$ \hspace{1cm} (2.15)  

When both segment A and segment B are parallel to Y axis, the segment center coordinates should be changed as shown in (2.16), (2.17) and (2.18) from [33]:

$$X = Y_B - Y_A$$ \hspace{1cm} (2.16)
\[ Y = X_B - X_A \]  \hspace{1cm} (2.17)  \\
\[ Z = Z_B - Z_A \]  \hspace{1cm} (2.18)

When both segment A and segment B are parallel to Z axis, the segment center coordinates should be changed as shown in (2.19), (2.20) and (2.21) from [33]:

\[ X = Z_B - Z_A \]  \hspace{1cm} (2.19)  \\
\[ Y = Y_B - Y_A \]  \hspace{1cm} (2.20)  \\
\[ Z = X_B - X_A \]  \hspace{1cm} (2.21)

After rearranging the coordinates of segments as above steps, it still needs to take segment length into account as following equations (2.22), (2.23), (2.24) and (2.25) from [33]. The \( X_1, X_2, X_3 \) and \( X_4 \) are the values which are used in the next steps.

\[ X_1 = X + HLB - HLA \]  \hspace{1cm} (2.22)  \\
\[ X_2 = X + HLB + HLA \]  \hspace{1cm} (2.23)  \\
\[ X_3 = X - HLB - HLA \]  \hspace{1cm} (2.24)  \\
\[ X_4 = X - HLB + HLA \]  \hspace{1cm} (2.25)

Next the variable P ARL is introduced which will be used in derivation of the functions which specify the mutual and self resistance values for parallel segments. Depending on the distance between segment A and segment B, P ARL can be calculated in two different functions.

If the projective distance between segment A with coordinate \((X_A, Y_A, Z_A)\) and segment B with coordinate \((X_B, Y_B, Z_B)\) in Y-Z plane \( (\sqrt{(Y_B - Y_A)^2 + (Z_B - Z_A)^2}) \) is greater than 1.0e-6 meter, the value of the variable P ARL is calculated using (2.26) from [33].
\[ PARL = \sqrt{X_1^2 + Y^2 + Z^2} - \sqrt{X_2^2 + Y^2 + Z^2} \]
\[ + \sqrt{X_3^2 + Y^2 + Z^2} - \sqrt{X_4^2 + Y^2 + Z^2} \]
\[ - X_1 \ln[X_1 - X_1 \cdot (Y^2 + Z^2)] + X_2 \ln[X_2 + X_2 \cdot (Y^2 + Z^2)] \]
\[ + X_3 \ln[X_3 + X_3 \cdot (Y^2 + Z^2)] + X_4 \ln[X_4 - X_4 \cdot (Y^2 + Z^2)] \] (2.26)

If the projective distance between segment A and segment B in Y-Z plane is less than or equal to 1.0e-6 meter, the value of the variable \( PARL \) is calculated using (2.27) from [33].

\[ PARL = \sqrt{X_1^2 + Y^2 + Z^2} - \sqrt{X_2^2 + Y^2 + Z^2} \]
\[ + \sqrt{X_3^2 + Y^2 + Z^2} - \sqrt{X_4^2 + Y^2 + Z^2} \]
\[ - X_1 \ln[X_1] + X_2 \ln[X_2] \]
\[ + X_3 \ln[X_3] + X_4 \ln[X_4] \] (2.27)

In deriving Cases 5 to Case 8 of the equations needed to calculate the mutual resistance values, it is assumed that segment A is perpendicular to segment B, in other words, they have different orientations.

Just like above steps, the coordinate of segments with perpendicular relationship should also be rearranged first.

When segment A and segment B are parallel to the X axis and Y axis respectively, the segment center coordinates should be changed as (2.28), (2.29), (2.30), (2.31) and (2.32) from [33]:

\[ X_1 = |X_B - X_A| + HLA \] (2.28)
\[ X_2 = |X_B - X_A| - HLA \] (2.29)
\[ Y_1 = |Y_B - Y_A| + HLB \] (2.30)
\[ Y_2 = |Y_B - Y_A| - HLB \] (2.31)
\[ Z = X_B - X_A \] (2.32)
When segment A and segment B are parallel to the Y axis and X axis respectively, the segment center coordinates should be changed as (2.33), (2.34), (2.35), (2.36) and (2.37) from [33]:

\[ X_1 = \left| X_B - X_A \right| + HLB \]  \hspace{1cm} (2.33)

\[ X_2 = \left| X_B - X_A \right| - HLB \]  \hspace{1cm} (2.34)

\[ Y_1 = \left| Y_B - Y_A \right| + HLA \]  \hspace{1cm} (2.35)

\[ Y_2 = \left| Y_B - Y_A \right| - HLA \]  \hspace{1cm} (2.36)

\[ Z = X_B - X_A \]  \hspace{1cm} (2.37)

When segment A and segment B are parallel to the X axis and Z axis respectively, the segment center coordinates should be changed as (2.38), (2.39), (2.40), (2.41) and (2.42) from [33]:

\[ X_1 = \left| X_B - X_A \right| + HLA \]  \hspace{1cm} (2.38)

\[ X_2 = \left| X_B - X_A \right| - HLA \]  \hspace{1cm} (2.39)

\[ Y_1 = \left| Z_B - Z_A \right| + HLB \]  \hspace{1cm} (2.40)

\[ Y_2 = \left| Z_B - Z_A \right| - HLB \]  \hspace{1cm} (2.41)

\[ Z = Y_B - Y_A \]  \hspace{1cm} (2.42)

When segment A and segment B are parallel to the Z axis and X axis respectively, the segment center coordinates should be changed as (2.43), (2.44), (2.45), (2.46) and (2.47) from [33]:

\[ X_1 = \left| X_B - X_A \right| + HLB \]  \hspace{1cm} (2.43)

\[ X_2 = \left| X_B - X_A \right| - HLB \]  \hspace{1cm} (2.44)

\[ Y_1 = \left| Z_B - Z_A \right| + HLA \]  \hspace{1cm} (2.45)

\[ Y_2 = \left| Z_B - Z_A \right| - HLA \]  \hspace{1cm} (2.46)

\[ Z = Y_B - Y_A \]  \hspace{1cm} (2.47)

When segment A and segment B are parallel to the Y axis and Z axis respectively, the segment center coordinates should be changed as (2.48), (2.49), (2.50), (2.51) and (2.52) from [33]:

\[ X_1 = \left| X_B - X_A \right| + HLB \]  \hspace{1cm} (2.48)

\[ X_2 = \left| X_B - X_A \right| - HLB \]  \hspace{1cm} (2.49)

\[ Y_1 = \left| Z_B - Z_A \right| + HLA \]  \hspace{1cm} (2.50)

\[ Y_2 = \left| Z_B - Z_A \right| - HLA \]  \hspace{1cm} (2.51)

\[ Z = Y_B - Y_A \]  \hspace{1cm} (2.52)
respectively, the segment center coordinates should be changed as (2.48), (2.49), (2.50), (2.51) and (2.52) from [33]:

\[ X_1 = |Y_B - Y_A| + HLA \]  
\[ X_2 = |Y_B - Y_A| - HLA \]  
\[ Y_1 = |Z_B - Z_A| + HLA \]  
\[ Y_2 = |Z_B - Z_A| - HLA \]  
\[ Z = X_B - X_A \]

When segment A and segment B are parallel to the Z axis and Y axis respectively, the segment center coordinates should be changed as (2.53), (2.54), (2.55), (2.56) and (2.57) from [33]:

\[ X_1 = |Y_B - Y_A| + HLB \]  
\[ X_2 = |Y_B - Y_A| - HLB \]  
\[ Y_1 = |Z_B - Z_A| + HLA \]  
\[ Y_2 = |Z_B - Z_A| - HLA \]  
\[ Z = X_B - X_A \]

Similar to PARL, another variable PERP is introduced here which will be used in derivation of the functions which specify the of mutual and self resistance values for perpendicular segments. Since the calculation of the function of PERP is complex, some intermediate variables should be calculated first. They are \( PERP_X, PERP_Y, PERP_Z_1, PERP_Z_2, PERP_Z_3 \) and \( PERP_Z_4 \) and are given by (2.58) to (2.64) from [33].

\[
PERP_X = X_1 \ln \left[ \frac{Y_1 + \sqrt{X_1^2 + Y_1^2 + Z^2}}{Y_2 + \sqrt{X_1^2 + Y_2^2 + Z^2}} \right]
+ X_2 \ln \left[ \frac{Y_2 + \sqrt{X_2^2 + Y_2^2 + Z^2}}{Y_1 + \sqrt{X_2^2 + Y_1^2 + Z^2}} \right]
\]
\[
\text{PERP}_Y = Y_1 \ln \left[ \frac{X_1 + \sqrt{X_1^2 + Y_1^2 + Z^2}}{X_2 + \sqrt{X_2^2 + Y_2^2 + Z^2}} \right] \\
+ Y_2 \ln \left[ \frac{X_2 + \sqrt{X_2^2 + Y_2^2 + Z^2}}{X_1 + \sqrt{X_1^2 + Y_1^2 + Z^2}} \right]
\]  
(2.59)

\[
\text{PERP}_Z_1 = \sin^{-1} \left\{ \text{SIGN} \left[ \frac{X_1 Y_2 + Z^2 \sqrt{X_1^2 + Y_2^2 + Z^2}}{(Y_2^2 + Z^2) \sqrt{X_1^2 + Z^2}} \right] \right\}
\]  
(2.60)

\[
\text{PERP}_Z_2 = \sin^{-1} \left\{ \text{SIGN} \left[ \frac{X_2 Y_1 + Z^2 \sqrt{X_2^2 + Y_1^2 + Z^2}}{(Y_1^2 + Z^2) \sqrt{X_2^2 + Z^2}} \right] \right\}
\]  
(2.61)

\[
\text{PERP}_Z_3 = \sin^{-1} \left\{ \text{SIGN} \left[ \frac{X_2 Y_1 + Z^2 \sqrt{X_2^2 + Y_1^2 + Z^2}}{(Y_1^2 + Z^2) \sqrt{X_2^2 + Z^2}} \right] \right\}
\]  
(2.62)

\[
\text{PERP}_Z_4 = \sin^{-1} \left\{ \text{SIGN} \left[ \frac{X_2 Y_1 + Z^2 \sqrt{X_2^2 + Y_1^2 + Z^2}}{(Y_1^2 + Z^2) \sqrt{X_2^2 + Z^2}} \right] \right\}
\]  
(2.63)

\[
\text{PERP}_Z = |Z_B - Z_A| \left[ \frac{\text{PERP}_Z_1 + \text{PERP}_Z_3}{\text{SIGN}(Y_2) \left( \text{PERP}_Z_2 + \text{PERP}_Z_4 \right)} \right]
\]  
(2.64)

\[
\text{PERP} = \text{PERP}_X + \text{PERP}_Y + \text{PERP}_Z
\]  
(2.65)

Note that \text{PERP} is simply the sum of \text{PERP}_X, \text{PERP}_Y and \text{PERP}_Z.

Moreover, in (2.60)-(2.64), the operator \text{SIGN} returns the following values: the 1 if the argument is greater than zero and the 0 if the argument is equal to zero and the -1 if the argument is less than zero. Thus, after obtaining all the values of \text{PERP} and \text{PARL}, the final functions of the resistance factors for Case 1 to Case 8 can be calculated.
Case 1: When segment A and segment B are both placed in lower layer and segment A is parallel to segment B, the resistance factor \( r_{jk} \) is given by (2.66) from [33].

\[
r_{jk}(A,B) = \frac{I}{4\pi \cdot HLA \cdot HLB \cdot \sigma_1} \left[ \frac{\text{PARL}(A,B)}{\sigma_1 + \sigma_2} \right] + K_{12} \cdot \text{PARL}(A,B) \right] - K_{32} \cdot \text{PARL}(A,B) \right] \]

Case 2: When segment A and segment B are placed in the lower layer and upper layer, respectively and segment A is parallel to segment B, the resistance factor \( r_{jk} \) is given by (2.67) from [33].

\[
r_{jk}(A,B) = \frac{I}{2\pi \cdot HLA \cdot HLB \cdot (\sigma_1 + \sigma_2)} \left[ \frac{\text{PARL}(A,B)}{\sigma_1 + \sigma_2} \right] - K_{32} \cdot \text{PARL}(A,B) \right] \]

Case 3: When segment A and segment B are placed in upper layer and lower layer, respectively, and segment A is parallel to segment B, the resistance factor \( r_{jk} \) is given by (2.68) from [33].

\[
r_{jk}(A,B) = \frac{I}{2\pi \cdot HLA \cdot HLB \cdot (\sigma_1 + \sigma_2)} \left[ \frac{\text{PARL}(A,B)}{\sigma_1 + \sigma_2} \right] - K_{32} \cdot \text{PARL}(A,B) \right] \]

Case 4: When segment A and segment B are both placed in upper layer and segment A is parallel to segment B, the resistance factor \( r_{jk} \) is given by (2.69) from [33].

\[
r_{jk}(A,B) = \frac{I}{16\pi \cdot HLA \cdot HLB \cdot \sigma_2} \left[ \frac{\text{PARL}(A,B)}{\sigma_2} \right] + K_{12} \cdot \text{PARL}(A,B) \right] - K_{32} \cdot \text{PARL}(A,B) \right] \]
Case 5: When segment A and segment B are both placed in lower layer and segment A is perpendicular to segment B, the resistance factor $r_{jk}$ is given by (2.70) from [33].

$$r_{jk}(A, B) = \frac{I}{4\pi * HLA * HLB * \sigma_1} \begin{bmatrix} \text{PERP}(A, B) \\ + K_{12} * \text{PERP}(A, B) \\ - K_{32} * \text{PERP}(A, B) \end{bmatrix}$$ (2.70)

Case 6: When segment A and segment B are placed in lower layer and upper layer, respectively, and segment A is perpendicular to segment B, the resistance factor $r_{jk}$ is given by (2.71) from [33].

$$r_{jk}(A, B) = \frac{I}{2\pi * HLA * HLB * (\sigma_1 + \sigma_2)} \begin{bmatrix} \text{PERP}(A, B) \\ - K_{32} * \text{PERP}(A, B) \end{bmatrix}$$ (2.71)

Case 7: When segment A and segment B are placed in upper layer and lower layer, respectively, and segment A is perpendicular to segment B, the resistance factor $r_{jk}$ is given by (2.72) from [33].

$$r_{jk}(A, B) = \frac{I}{2\pi * HLA * HLB * (\sigma_1 + \sigma_2)} \begin{bmatrix} \text{PERP}(A, B) \\ - K_{32} * \text{PERP}(A, B) \end{bmatrix}$$ (2.72)

Case 8: When segment A and segment B are both placed in upper layer and segment A is perpendicular to segment B, the resistance factor $r_{jk}$ is given by (2.73) from [33].

$$r_{jk}(A, B) = \frac{I}{16\pi * HLA * HLB * \sigma_2} \begin{bmatrix} \text{PERP}(A, B) \\ + K_{12} * \text{PERP}(A, B) \\ - K_{32} * \text{PERP}(A, B) \end{bmatrix}$$ (2.73)
2.3.4. Resistance Matrix Method Point-Point Modeling

Then, the point-point modeling method is introduced.

Point-Point modeling: This method is used in [33], when the distance between two segments is greater than or equal to the total length of two segments. Depending on the two points’ locations (upper layer or lower layer), there will be four distinct cases requiring different modeling equation which are needed to calculate the entries in the resistance matrix \( r \), Case 1 to Case 4.

Case 1: When the first point and second point are both placed in upper layer, the resistance factor \( r_{jk} \) is given by (2.74) from [33].

\[
 r_{jk}(A, B) = \frac{1}{4\pi \sigma_z^2} \left[ \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B - Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (Z_B + Z_A)^2} - 1} \right. \\
+ \left. \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B + Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B - Z_A)^2} - 1} \right] \\
+ \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D - Z_B + Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D - Z_B - Z_A)^2} - 1} \\
+ \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B - Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D - Z_B + Z_A)^2} - 1} \right)
\]  

Case 2: When the first point and second point are placed in upper layer and lower layer, respectively the resistance factor \( r_{jk} \) is given by (2.75) from [33].

\[
 r_{jk}(A, B) = \frac{1}{2\pi (\sigma_2 + \sigma_1)} \left[ \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B + Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B - Z_A)^2} - 1} \right. \\
+ \left. \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D - Z_B + Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D - Z_B - Z_A)^2} - 1} \right] \\
+ \frac{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B - Z_A)^2} - 1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D - Z_B + Z_A)^2} - 1} \right)
\]
Case 3: When the first point and second point are placed in lower layer and upper layer, respectively, the resistance factor \( r_{jk} \) is given by (2.76) from [33].

\[
r_{jk}(A, B) = \frac{1}{2\pi*(\sigma_2 + \sigma_1)} \left\{ \frac{1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B + Z_A)^2}} \right\}^{-1}
\]

Case 4: When the first point and second point are both placed in lower layer, the resistance factor \( r_{jk} \) is given by (2.77) from [33].

\[
r_{jk}(A, B) = \frac{1}{4\pi*\sigma_1} \left\{ \frac{1}{\sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2 + (2D + Z_B - Z_A)^2}} \right\}^{-1}
\]

In general, by combining all the above functions and equations, the mutual and self resistance entries in the resistance matrix, \( r \), can be calculated.

2.3.5. Simple Matrix Method Used to Calculate Current Distribution Factors, Ground Grid Resistance and G.P.R.

After evaluating the entries in the resistance matrix, \( r \), using the equations of Section 2.3.2, a simple matrix method can be used to calculate segment current distribution factor \( i \), grounding potential rise (GPR) and ground grid resistance \( R_g \).

The voltage \( v_j \) of each segment \( j \) should equal to the product of each source current \( i_k \) on segment \( k \) and the corresponding mutual resistance between segment \( j \) and segment \( k \) (when \( j=k \), it should be the self resistance) The sum of source
currents from all segments should be equal to the fault current returning to the remote sources through the earth. In the following equations, the $r_{jk}$ is the mutual resistance ($j \neq k$) between segment $j$ and segment $k$, the $r_{jj}$ is the self resistance ($j = k$) of segment $j$. The variable $v_j$ is the voltage of each segment $j$. Therefore, the following must be obeyed:

$$\sum_{k=1}^{n} r_{jk} i_k = v_j \text{, } j = 1, 2, \ldots, n$$  \hspace{1cm} (2.78)

$$\sum_{k=1}^{n} i_k = I_F$$  \hspace{1cm} (2.79)

In equation (2.79), $I_F$ is the fault current. Then, by combining the two above equations, the following matrix equation, (2.80), is formed:

$$AX = b$$  \hspace{1cm} (2.80)

where,

$$A = \begin{bmatrix}
  r_{11} & \cdots & r_{1n} & -1 \\
  \vdots & \ddots & \vdots & \vdots \\
  r_{n1} & \cdots & r_{nn} & -1 \\
  1 & \cdots & 1 & 0
\end{bmatrix}^{(n+1) \times (n+1)}$$  \hspace{1cm} (2.81)

$$b = \begin{bmatrix}
  0 \\
  \vdots \\
  0 \\
  I_F
\end{bmatrix}^{(n+1) \times 1}$$  \hspace{1cm} (2.82)

$$X = \begin{bmatrix}
  i_1 \\
  \vdots \\
  i_n \\
  \frac{V}{I_F}
\end{bmatrix}^{(n+1) \times 1}$$  \hspace{1cm} (2.83)

$$R_g = \frac{V}{I_F}$$  \hspace{1cm} (2.84)
In above equations, $V$ is the value of GPR. From [19], the definition of GPR is shown as following:

**GPR**: GPR (ground potential rise) is the maximum electrical potential that a substation grounding grid may attain relative to a distant grounding point assumed to be at the potential of remote earth. This voltage is equal to the maximum grid current times the grid resistance.

In matrix $A$, expect for the last row and column, the values of self-resistance and mutual resistance can be obtained by using Resistance Matrix method from above Section 2.3.2 and [33]. Since the value of fault earth current $I_f$ is known, the unknown values include current density in every segment. Note that GPR can then be found by solving (2.80). Finally the resistance of ground grid is solved by following equation (2.84).

After using the simple matrix method, the values of current distribution factors in ground conductor and rod segments, grid resistance and GPR can be obtained. In the following sections, the grid resistance should be considered as a constraint value in the ground grid optimal design model. The GPR along with the earth potential can be used to calculate the touch potential and step potential values at interested point.

2.3.6. Green’s Functions Used to Calculate Earth Potential at Certain Point

In order to calculate the grounding system safety requirements (touch potential, step potential and ground grid resistance) more accurately and
efficiently, the horizontal grounding conductors and vertical rods are divided into $N$ horizontal and vertical segments, where $N$ is the total number of segment. These underground segments may be located in either the upper layer soil or lower layer soil. The location of the conductor and rod segments will lead to different Green's Functions. Since a two-layer soil model is applied here, based on Fig. 2.5 in Section 2.3.2, to keep the consistency of the terms in the following Green's functions and Laplace's equations, the subscript of 2 and 1 represent region 2 (upper layer) and region 1 (lower layer) respectively.

In the following functions, the calculated voltage in three regions (air, upper layer and lower layer soil) is generated by a point of current located in the upper layer or lower layer.

In addition, each segment of conductor or rod is modeled as a cylindrical metallic conductor or rod, which has already been discussed in above Section 2.3.3.

Laplace's equation solution in cylindrical coordinates can be represented by Green's functions. While this theory is well know, it is included here for the sake of documentation required by the sponsor of this research. The solution for the voltage can be separable into function $Z(z)$ and $R(r)$ as below (2.85) from [34]:

$$V(r, z) = R(r)Z(z) \quad (2.85)$$

where $V$ is the voltage at the cylindrical coordinates $r$ and $z$ and $R$ and $Z$ are functions needed to be determined. Hence, Laplace's equations for this case reduces to (2.86) from [34]:

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\[ \nabla^2 V(r, z) = \frac{r}{R(r)} \frac{\partial}{\partial r} \left( r \frac{\partial R(r)}{\partial r} \right) + \frac{r^2}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} = 0 \quad (2.86) \]

In general, the solution for the voltage at an upper layer point produced by a line current of uniform density in the upper layer and the solution for the voltage at an upper layer point produced by a line current of uniform density in the lower layer are given by (2.87) and (2.88) from [34], respectively:

\[
V_2(r, z) = \frac{I_p}{4\pi \sigma_2} \left[ \int_0^\infty J_0(kr)e^{-zk} dk + \int_0^\infty \phi(k) J_0(kr)e^{-zk} dk + \int_0^\infty \theta(k) J_0(kr)e^{zk} dk \right] \quad (2.87)
\]

\[
V_1(r, z) = \frac{I_p}{4\pi \sigma_2} \int_0^\infty \phi(k) J_0(kr)e^{-zk} dk \quad (2.88)
\]

The above two equations have three unknown functions \( \theta(k), \phi(k) \) and \( \psi(k) \) which must be determined before the voltage at a point can be evaluated. Therefore, by applying the appropriate boundary conditions to the solutions at two boundaries \( z=0 \) and \( z=D \) (where \( D \) is the upper layer depth shown in above Fig. 2.6), these functions can be obtained. At each planar boundary the voltage on each side of the boundary must be continuous at the boundary.

After solving for the three unknown functions in (2.87) and (2.88), and then after expanding each integral in a power series and integrating term by term, the two voltage solutions yield the (2.89) and (2.90) from [34] as infinite series expressions for the voltage in the upper layer and lower layer soil, respectively:
In above equations (2.89) and (2.90), the \( I_p \) is the current associated with each segment. \( I_p \) can be obtained by the below equation (2.92). The reflection coefficient \( K_{12} \) and \( K_{32} \) are obtained by (2.10) and (2.12), respectively. In Fig. 2.6 from Section 2.3.2, the segment for which this voltage is to be calculated is assumed to be parallel to x axis or y axis with the length \( 2L_i \) and the center located at the cylindrical line coordinate \((X_i, Y_i, Z_i)\). It is important to notice that, the total current, \( I \), will be uniformly distributed along the length of the conductor line, so that it leads to a current density \( \rho \) with the unit (amps/meter) as given by (2.91) from [34]:

\[
I = \rho \ast (2L_i)
\]  

Then let the line be divided into infinitesimal segments with length \( dx \). The
injected current associated with each segment is given by (2.92) from [34]:

\[ I_p = \rho dx_s = \left( \frac{I}{2L_i} \right) dx_s \quad \text{(2.92)} \]

Then the contribution to the voltage at a point \((x, y, z)\) in upper layer due to the segment with x-coordinate \(x_s\) can be obtained by using (2.92) with \(I_p\) replaced by (2.91) and \(r\) replaced with (2.93) given below from [34].

\[ r = \sqrt{(x - X_i)^2 + (y - Y_i)^2} \quad \text{(2.93)} \]

The form of the voltage solution equation needed will depend the orientation of current filament and the layer in which the current filament lies.

1) When the voltage is produced by a horizontal current filament parallel with the x axis or y axis and located in the upper layer and the point at which the voltage is desired is also located in the upper soil layer as shown in Fig. 2.7:
Fig. 2.7 Condition of Point of Interest and Horizontal Current Filament are both located at Upper Layer Soil

The $G$ function can be obtained as (2.94) from [34]. Let $V_{22}(x, y, z)$ be the voltage at a point $(x, y, z)$ in upper layer due to a horizontal current filament (model segment) in the upper layer, is given by (2.95) from [34]:

$$G(x, y, z) = F(x, y, z, L_1)$$
$$= \ln \left( \frac{\sqrt{(x + L_1)^2 + y^2 + z^2 + x + L_1}}{\sqrt{(x - L_1)^2 + y^2 + z^2 + x - L_1}} \right)$$

(2.94)
\[
V_{22}G(x, y, z) = \frac{I}{8\pi L_1 \sigma_2} \begin{bmatrix}
G(x - X_1, y - Y_1, z - Z_1) \\
- K_{12} \sum_{i=0}^{\infty} G(x - X_1, y - Y_1, z + Z_1 - 2iD) \\
- K_{12} \sum_{i=0}^{\infty} (K_{12} K_{32})^i \\
- K_{32} \sum_{i=0}^{\infty} G(x - X_1, y - Y_1, z + Z_1 + 2iD) \\
+ \sum_{i=1}^{\infty} G(x - X_1, y - Y_1, z - Z_1 - 2iD)
\end{bmatrix}
\]

where \( L_1 \) is half of the segment length, the coordinates \((x, y, z)\) are the coordinates of the point of interest, and the \((X_i, Y_i, Z_i)\) is the central point of segment.

2) When the voltage is produced by a horizontal current filament parallel with the x axis or y axis and located in the lower layer and the point at which the voltage is desired is located in the upper soil layer as shown in Fig. 2.8:
Fig. 2.8 Condition of Point of Interest and Horizontal Current Filament are part located at Upper Layer Soil and Lower Layer Soil

Let $V_{21}G(x, y, z)$ be the voltage at a point $(x, y, z)$ in upper layer due to a horizontal current filament (model segment) in lower layer, the function of $V_{21}G(x, y, z)$ is given by (2.96) from [34]:

$$
V_{21}G(x, y, z) = \frac{I}{4\pi L_1(\sigma_1 + \sigma_2)} \left[ \sum_{i=1}^{\infty} \frac{(K_{12}K_{32})^i ...}{z - Z_1 - 2iD} - K_{32} \sum_{i=0}^{\infty} \frac{(K_{12}K_{32})^i ...}{z + Z_1 + 2iD} \right]
$$

(2.96)

When the vertical rod segments are placed in the ground grid, Green’s functions are also used to determine the voltage produced in the three regions due to a vertical line of current located in either the upper layer or lower layer. Since when the length of vertical rod $2L_j$ is greater than the depth of upper layer soil $D$,
the rod will be divided into two pieces in upper layer and lower layer respectively, rod line segments are located in upper layer and lower layer soil, respectively.

The derivation proceeds in the same sequence used in the derivation of the voltage due to horizontal lines of current presented above. However, the order of some terms may change.

3) When the voltage is produced by a vertical current filament parallel with the z axis and located in the upper layer and the point at which the voltage is desired is also located in the upper soil layer as shown in Fig. 2.9:

![Diagram](https://example.com/diagram.png)

Fig. 2.9 Condition of Point of Interest and Vertical Current Filament are both located at Upper Layer Soil

The voltage due to a vertical current filament (model segment) located in the upper layer for the point \((x, y, z)\) also located in the upper layer can be presented as (2.97) from [34]. Note the change in the order of the arguments on opposite sides of the equation. This ordering is important.
\[ V_{22ZG}(x, y, z) = V_{22G}(z, x, y) \] (2.97)

The voltage function of this condition is similar to (2.95). However, due to the change of ordering from \((x, y, z)\) to \((z, x, y)\), the \(G\) function and \(V_{22ZG}(x, y, z)\) will be changed as shown in (2.98) and (2.99) from [34], respectively.

\[
G(z, x, y) = F(z, x, y, L_1) = \ln \left( \frac{\sqrt{(z + L_1)^2 + y^2 + x^2 + z + L_1}}{\sqrt{(z - L_1)^2 + y^2 + x^2 + z - L_1}} \right) \] (2.98)

\[
V_{22ZG}(x, y, z) = \frac{I}{8\pi L_4 \sigma_2} \left[ G(z - Z_1, y - Y_1, x - X_1) - K_{12} \sum_{i=0}^{\infty} (K_{12}K_{32})^i \cdots G(z + Z_1 - 2iD, y - Y_1, x - X_1) - K_{32} \sum_{i=0}^{\infty} (K_{12}K_{32})^i \cdots G(z + Z_1 + 2iD, y - Y_1, x - X_1) + \sum_{i=1}^{\infty} G(z - Z_1 - 2iD, y - Y_1, x - X_1) \right] \] (2.99)

4) When the voltage is produced by a vertical current filament parallel with the \(z\) axis and located in the lower layer and the point at which the voltage is desired is located in the upper soil layer as shown in Fig. 2.10:
The voltage due to the vertical current filament (model segment) located in the lower layer for the point \((x, y, z)\) located in the upper layer is presented as following (2.100) from [34]:

\[
V_{21}ZG(x, y, z) = V_{21}G(z, x, y)
\]  

(2.100)

Similarly, for the voltage function the order of the argument variables is changed from \((x, y, z)\) to \((z, x, y)\), in going from the LHS to RHS. Thus, the \(V_{21}ZG\), though similar to (2.96), is given by (2.101) from [34]:

\[
V_{21}ZG(z, x, y) = \frac{I}{4\pi L_1 (\sigma_1 + \sigma_2)} \left[ \sum_{i=1}^{\infty} (K_{12}K_{32})^i G(z - Z_1 - 2iD, y - Y_1, x - X_1) 
- K_{32} \sum_{i=0}^{\infty} (K_{12}K_{32})^i G(z + Z_1 + 2iD, y - Y_1, x - X_1) \right]
\]  

(2.101)

Thus, by combining (2.95), (2.96), (2.99) and (2.101), a function used to...
calculate the earth potential at an arbitrary point can be obtained. When a ground rod is to be modeled there are two scenarios with two different functions for earth potential calculation: 1) when the ground rod exists only in the top layer; 2) when the ground rod penetrates both layers. Therefore, evaluating the voltage at the point of interest may require using two voltage functions, namely, equation (2.99) and (2.101).

Consider first the scenario where the length of vertical rod is less than or equal to the upper layer depth, which means both the horizontal grid mesh conductor (aka ground conductors) and vertical rods are restricted to the upper layer. The earth potential calculation function is given by (2.102) from [34].

\[
E_{\text{earth}} = V_{22} G(x, y, z) + V_{22} ZG(z, x, y) \tag{2.102}
\]

In second scenario, the length of vertical rod is greater than the upper layer depth; in other words, the horizontal grid mesh and part of the vertical rod segments are installed in the upper layer, and a portion of the vertical rod segments exist in the lower layer. The earth potential calculation function for this second scenario is given by (2.103) from [34].

\[
E_{\text{earth}} = V_{22} G(x, y, z) + V_{22} ZG(z, x, y) + V_{21} ZG(z, x, y) \tag{2.103}
\]

After obtaining the earth potential function at the point of interest, the worst case mesh (touch) potential and step potential can be calculated.

According to the [2] and [19], the worst touch potential point is typically located at the central point of corner mesh. The examples, shown in Fig. 2.11, are
corner meshes of square and rectangular grids.

Fig. 2.11 Mesh Touch Potential Worst Case Point Examples for Rectangular and Square Shape Corner Meshes

In Fig. 2.11, it has been assumed the central point of a corner mesh is \((X_{\text{mesh}}, Y_{\text{mesh}}, Z_{\text{mesh}})\). Based on the definition of touch potential from [19] and Section 2.2, touch potential is the difference between the GPR and the surface earth potential at the point, in this worst case, the coordinate point is \((X_{\text{mesh}}, Y_{\text{mesh}}, Z_{\text{mesh}})\). Thus, the touch potential in the worst case can be calculated using (2.104).

\[
E_{\text{touch}} = GPR - E_{\text{earth}}\left(X_{\text{mesh}}, Y_{\text{mesh}}, Z_{\text{mesh}}\right)
\] (2.104)

According to [19], the worst two step potential points are typically line at a 45 degree angle from the horizontal as shown in Fig. 2.12. The examples, shown in Fig. 2.12, show the corner meshes of square and rectangular grids.
In Fig. 2.12, it has been assumed that the coordinates of first and second points are \((X_1, Y_1, Z_1)\) and \((X_2, Y_2, Z_2)\) respectively. The distance between first and second point \(L\) is 1 meter. Based on the definition of step potential from [2], [19] and Section 2.2, the step potential in the worst case can be calculated using (2.105).

\[
E_{step} = E_{earth}(X_1, Y_1, Z_1) - E_{earth}(X_2, Y_2, Z_2)
\]  
(2.105)

2.4 Results Validation

In order to guarantee the accuracy of the grounding system modeling application developed in this work, some results are validated in the following Sections 2.4.1 to 2.4.4. The grounding system models used for validation are chose to include: a single vertical rod model in Section 2.4.1, a single horizontal conductor model in Section 2.4.1, a rectangular (square) shape ground grid model in Section 2.4.3, an L-shape ground grid model in Section 2.4.4.

The grounding system modeling methods compared in the validation sections include results from: (1) the MATLAB code developed in this work, (2) a simple
closed-form solution for a single conductor and a rod from [27], (3) a simple ground grid case using the IEEE Std [19] equations, (4) WinIGS simulations.

2.4.1. Model of Single Vertical Rod

The ground resistance, $R_g$, value of a single vertical rod in uniform soil with 0 ft burial depth is given by [27]:

$$R_g = \frac{\rho}{2\pi l} \left( \ln \frac{4l}{a} - 1 \right)$$  \hspace{1cm} (2.106)

where, $\rho$ is the resistivity of the uniform soil model in Ohm\text{*}meters, $l$ is the length of rod in ft, $a$ is the conductor radius in feet. In all of the following examples, $\rho$ is taken as 100 Ohm\text{*}meters, and $a$ is 0.628/2 inch. The range of ground rod lengths is 10 ft to 30 ft in SRP’s design requirement. Thus, in the following tests, two cases are selected, 10 ft and 30 ft. Also tested are two cases that are not in the requirement range, 8 ft and 60 ft.

In the following tables, $R_g$ values are obtained by three different methods:

- $R_{g\_1}$: Proposed Method (based on all equations presented in this work)
- $R_{g\_2}$: WinIGS Results
- $R_{g\_3}$: Closed Form Solution (Sunde's Equation [27])

The most significant difference in the above three conductor-and-rod modeling methods is: in the proposed method and the WinIGS simulations, the conductor and rod segmentation method is applied (though the segmentation methods of the proposed method and WinIGS are not the same); however, the closed form solution [27] does not use segmentation. In the following tables, the segment number is only applicable to the proposed method.
When \( l \) is 8 ft, the function \( R_g \) is shown in (2.107). The results for different segment lengths are shown in Table 2.3:

\[
R_g = \frac{\rho}{2\pi l} \left( \ln \frac{4l}{a} - 1 \right)
\]

\[
= \frac{100}{2\pi \times 8 \times 0.3048} \left( \ln \frac{4 \times 8}{0.628/(2 \times 12)} - 1 \right)
\]

\[
= 39.8736 \text{ Ohms}
\]

**Table 2.3**

**RESULTS COMPARISON OF MATLAB CODE, WINIGS SIMULATION AND SUNDE'S THEORETICAL MODEL OF 8 FT SINGLE VERTICAL ROD**

<table>
<thead>
<tr>
<th>Seg. Length (ft)</th>
<th>Length_Rod (ft)</th>
<th>1. Proposed Method</th>
<th>2. WinIGS</th>
<th>3. Closed Form</th>
<th>Diff.1&amp;2 (%)</th>
<th>Diff.1&amp;3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.67</td>
<td>12 Segs</td>
<td>39.8320</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.08%</td>
<td>0.10%</td>
</tr>
<tr>
<td>0.80</td>
<td>10 Segs</td>
<td>39.8025</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.16%</td>
<td>0.18%</td>
</tr>
<tr>
<td>1.00</td>
<td>8 Segs</td>
<td>39.7769</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.22%</td>
<td>0.24%</td>
</tr>
<tr>
<td>1.14</td>
<td>7 Segs</td>
<td>39.7665</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.24%</td>
<td>0.27%</td>
</tr>
<tr>
<td>1.33</td>
<td>6 Segs</td>
<td>39.7584</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.26%</td>
<td>0.29%</td>
</tr>
<tr>
<td>1.60</td>
<td>5 Segs</td>
<td>39.7539</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.28%</td>
<td>0.30%</td>
</tr>
<tr>
<td>2.00</td>
<td>4 Segs</td>
<td>39.7549</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.27%</td>
<td>0.30%</td>
</tr>
<tr>
<td>2.67</td>
<td>3 Segs</td>
<td>39.7657</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.25%</td>
<td>0.27%</td>
</tr>
<tr>
<td>4.00</td>
<td>2 Segs</td>
<td>39.7965</td>
<td>40.6797</td>
<td>39.8736</td>
<td>2.17%</td>
<td>0.19%</td>
</tr>
<tr>
<td>8.00</td>
<td>1 Seg</td>
<td>39.8797</td>
<td>40.6797</td>
<td>39.8736</td>
<td>1.97%</td>
<td>0.02%</td>
</tr>
</tbody>
</table>

When \( l \) is 10 ft, the function \( R_g \) is shown in (2.108). The results for different segment lengths are shown in Table 2.4:

\[
R_g = \frac{\rho}{2\pi l} \left( \ln \frac{4l}{a} - 1 \right)
\]

\[
= \frac{100}{2\pi \times 10 \times 0.3048} \left( \ln \frac{4 \times 10}{0.628/(2 \times 12)} - 1 \right)
\]

\[
= 33.7179 \text{ Ohms}
\]
When $l$ is $30$ ft, the function $R_g$ is shown in (2.109). The results for different segment lengths are shown in Table 2.5:

$$R_g = \frac{\rho}{2\pi l} \left( \ln \frac{4l}{a} - 1 \right)$$

$$= \frac{100}{2\pi \times 30 \times 0.3048} \left( \ln \frac{4 \times 30}{0.628/(2 \times 12)} - 1 \right)$$

$$= 12.9335 \text{ Ohms}$$

### Table 2.4

**RESULTS COMPARISON OF MATLAB CODE, WINIGS SIMULATION AND SUNDE’S THEORETICAL MODEL OF 10 FT SINGLE VERTICAL ROD**

<table>
<thead>
<tr>
<th>Seg. Length (ft)</th>
<th>Length_Rod (ft)=10</th>
<th>1. Proposed Method</th>
<th>2. WinIGS</th>
<th>3. Closed Form</th>
<th>Diff. 1&amp;2 (%)</th>
<th>Diff. 1&amp;3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.83</td>
<td>12 Segs</td>
<td>33.0058</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.11%</td>
<td>0.18%</td>
</tr>
<tr>
<td>1.00</td>
<td>10 Segs</td>
<td>32.988</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.16%</td>
<td>0.23%</td>
</tr>
<tr>
<td>1.25</td>
<td>8 Segs</td>
<td>32.9731</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.21%</td>
<td>0.28%</td>
</tr>
<tr>
<td>1.43</td>
<td>7 Segs</td>
<td>32.9675</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.23%</td>
<td>0.29%</td>
</tr>
<tr>
<td>1.67</td>
<td>6 Segs</td>
<td>32.9636</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.24%</td>
<td>0.30%</td>
</tr>
<tr>
<td>2.00</td>
<td>5 Segs</td>
<td>32.9625</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.24%</td>
<td>0.31%</td>
</tr>
<tr>
<td>2.50</td>
<td>4 Segs</td>
<td>32.9655</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.23%</td>
<td>0.30%</td>
</tr>
<tr>
<td>3.33</td>
<td>3 Segs</td>
<td>32.976</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.20%</td>
<td>0.27%</td>
</tr>
<tr>
<td>5.00</td>
<td>2 Segs</td>
<td>33.0018</td>
<td>33.7179</td>
<td>33.0641</td>
<td>2.12%</td>
<td>0.19%</td>
</tr>
<tr>
<td>10.00</td>
<td>1 Seg</td>
<td>33.0678</td>
<td>33.7179</td>
<td>33.0641</td>
<td>1.93%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>
TABLE 2.5
RESULTS COMPARISON OF MATLAB CODE, WINIGS SIMULATION AND SUNEDE'S THEORETICAL MODEL OF 30 FT SINGLE VERTICAL ROD

<table>
<thead>
<tr>
<th>Seg. Length (ft)</th>
<th>Length_Rod (ft)=30</th>
<th>1. Proposed Method</th>
<th>2. WinIGS</th>
<th>3. Closed Form</th>
<th>Diff.1&amp;2 (%)</th>
<th>Diff.1&amp;3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.50</td>
<td>12 Segs</td>
<td>12.8959</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.06%</td>
<td>0.29%</td>
</tr>
<tr>
<td>3.00</td>
<td>10 Segs</td>
<td>12.895</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.07%</td>
<td>0.30%</td>
</tr>
<tr>
<td>3.75</td>
<td>8 Segs</td>
<td>12.8948</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.07%</td>
<td>0.30%</td>
</tr>
<tr>
<td>4.29</td>
<td>7 Segs</td>
<td>12.8952</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.07%</td>
<td>0.30%</td>
</tr>
<tr>
<td>5.00</td>
<td>6 Segs</td>
<td>12.8961</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.06%</td>
<td>0.29%</td>
</tr>
<tr>
<td>6.00</td>
<td>5 Segs</td>
<td>12.8977</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.05%</td>
<td>0.28%</td>
</tr>
<tr>
<td>7.50</td>
<td>4 Segs</td>
<td>12.9004</td>
<td>13.1674</td>
<td>12.9335</td>
<td>2.03%</td>
<td>0.26%</td>
</tr>
<tr>
<td>10.00</td>
<td>3 Segs</td>
<td>12.9051</td>
<td>13.1674</td>
<td>12.9335</td>
<td>1.99%</td>
<td>0.22%</td>
</tr>
<tr>
<td>15.00</td>
<td>2 Segs</td>
<td>12.9139</td>
<td>13.1674</td>
<td>12.9335</td>
<td>1.93%</td>
<td>0.15%</td>
</tr>
<tr>
<td>30.00</td>
<td>1 Seg</td>
<td>12.9339</td>
<td>13.1674</td>
<td>12.9335</td>
<td>1.77%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

When \( l \) is 60 ft, the function \( R_g \) is shown in (2.110). The results for different segment lengths are shown in Table 2.6:

\[
R_g = \frac{\rho}{2\pi l} \left( \ln \frac{4l}{a} - 1 \right)
= \frac{100}{2\pi * 60 * 0.3048} \left( \ln \frac{4 * 60}{0.628/(2 * 12)} - 1 \right)
= 7.06999 \text{ Ohms}
\]
TABLE 2.6
RESULTS COMPARISON OF MATLAB CODE, WINIGS SIMULATION AND SUNDE’S THEORETICAL MODEL OF 60 FT SINGLE VERTICAL ROD

<table>
<thead>
<tr>
<th>Seg. Length (ft)</th>
<th>Length_Rod (ft)=60</th>
<th>1. Proposed Method</th>
<th>2. WinIGS</th>
<th>3. Closed Form</th>
<th>Diff. 1&amp;2 (%)</th>
<th>Diff. 1&amp;3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>12 Segs</td>
<td>7.0503</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.94%</td>
<td>0.28%</td>
</tr>
<tr>
<td>6.00</td>
<td>10 Segs</td>
<td>7.0504</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.94%</td>
<td>0.28%</td>
</tr>
<tr>
<td>7.50</td>
<td>8 Segs</td>
<td>7.0509</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.93%</td>
<td>0.27%</td>
</tr>
<tr>
<td>8.57</td>
<td>7 Segs</td>
<td>7.0513</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.93%</td>
<td>0.26%</td>
</tr>
<tr>
<td>10.00</td>
<td>6 Segs</td>
<td>7.0519</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.92%</td>
<td>0.26%</td>
</tr>
<tr>
<td>12.00</td>
<td>5 Segs</td>
<td>7.0529</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.91%</td>
<td>0.24%</td>
</tr>
<tr>
<td>15.00</td>
<td>4 Segs</td>
<td>7.0543</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.89%</td>
<td>0.22%</td>
</tr>
<tr>
<td>20.00</td>
<td>3 Segs</td>
<td>7.0567</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.85%</td>
<td>0.19%</td>
</tr>
<tr>
<td>30.00</td>
<td>2 Segs</td>
<td>7.0609</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.80%</td>
<td>0.13%</td>
</tr>
<tr>
<td>60.00</td>
<td>1 Seg</td>
<td>7.0901</td>
<td>7.19</td>
<td>7.06999</td>
<td>1.39%</td>
<td>0.28%</td>
</tr>
</tbody>
</table>

In general, from above tables in Section 2.4.1, some conclusions can be drawn. For the single rod model, the proposed method agrees marginally better with the closed form solution [27] than the WinIGS simulation and the maximum difference with the closed form solution is 0.30%. If the segmentation rules present in Table 2.3 are used, then the maximum error with the WinIGS simulation is 2.28%.

2.4.2. Model of Single Horizontal Conductor

The ground resistance value, $R_g$, of a single horizontal conductor in uniform soil with 1.5 ft burial depth is given by [27]:

$$R_g = \frac{\rho}{\pi d} \left( \ln \frac{2l}{\sqrt{2ad}} - 1 \right)$$  \hspace{1cm} (2.111)

where, $\rho$ is the resistivity of the uniform soil model in Ohm*metres, $l$ is the length of rod in ft, $a$ is the conductor radius in feet, $d$ is the burial depth in ft. In all the
following examples, the \( \rho \) is taken as 100 Ohm* meters, and \( a \) is 0.628/24 ft. The
two values of conductor length selected are 100 ft and 50 ft respectively.

When \( l \) is 100 ft, the function \( R_g \) is shown in (2.112). The results for different
segment lengths are shown in Table 2.7:

\[
R_g = \frac{\rho}{\pi l} \left( \ln \frac{2l}{\sqrt{2ad}} - 1 \right)
\]

\[
= \frac{100}{\pi \times 100 \times 0.3048} \left( \ln \frac{2 \times 100}{\sqrt{2 \times 1.5 \times 0.628 / 24}} - 1 \right)
\]

\[
= 5.9081 \text{ Ohms}
\]

### Table 2.7
**RESULTS COMPARISON OF MATLAB CODE, WINIGS SIMULATION AND SUNDRE'S THEORETICAL MODEL OF 100 FT SINGLE HORIZONTAL CONDUCTOR**

<table>
<thead>
<tr>
<th>Seg. Length (ft)</th>
<th>Length_Conductor(ft)=100</th>
<th>1. Proposed Method</th>
<th>2. WinIGS</th>
<th>3. Closed Form</th>
<th>Diff.1&amp;2 (%)</th>
<th>Diff.1&amp;3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00</td>
<td>10 Segs</td>
<td>5.8726</td>
<td>5.8776</td>
<td>5.9081</td>
<td>0.09%</td>
<td>0.60%</td>
</tr>
<tr>
<td>11.11</td>
<td>9 Segs</td>
<td>5.8749</td>
<td>5.8776</td>
<td>5.9081</td>
<td>0.05%</td>
<td>0.56%</td>
</tr>
<tr>
<td>12.50</td>
<td>8 Segs</td>
<td>5.8806</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.05%</td>
<td>0.47%</td>
</tr>
<tr>
<td>14.28</td>
<td>7 Segs</td>
<td>5.8787</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.02%</td>
<td>0.50%</td>
</tr>
<tr>
<td>16.67</td>
<td>6 Segs</td>
<td>5.8872</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.16%</td>
<td>0.35%</td>
</tr>
<tr>
<td>20.00</td>
<td>5 Segs</td>
<td>5.8932</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.27%</td>
<td>0.25%</td>
</tr>
<tr>
<td>25.00</td>
<td>4 Segs</td>
<td>5.9029</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.43%</td>
<td>0.09%</td>
</tr>
<tr>
<td>33.33</td>
<td>3 Segs</td>
<td>5.9086</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.53%</td>
<td>0.01%</td>
</tr>
<tr>
<td>50.00</td>
<td>2 Segs</td>
<td>5.9275</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.85%</td>
<td>0.33%</td>
</tr>
<tr>
<td>100.00</td>
<td>1 Seg</td>
<td>5.9275</td>
<td>5.8776</td>
<td>5.9081</td>
<td>-0.85%</td>
<td>0.33%</td>
</tr>
</tbody>
</table>

When \( l \) is 50 ft, the function \( R_g \) is shown in (2.113). The results for different
segment lengths are shown in Table 2.8:
\[ R_s = \frac{\rho}{\pi l} \left( \ln \frac{2l}{\sqrt{2ad}} - 1 \right) \]
\[ = \frac{100}{\pi \times 50 \times 0.3048} \left( \ln \frac{2 \times 50}{\sqrt{2 \times 1.5 \times 0.628 / 24}} - 1 \right) \]
\[ = 10.3685 \text{ Ohms} \]

**TABLE 2.8**

**RESULTS COMPARISON OF MATLAB CODE, WINIGS SIMULATION AND SUNDE'S THEORETICAL MODEL OF 50 FT SINGLE HORIZONTAL CONDUCTOR**

<table>
<thead>
<tr>
<th>Seg. Length (ft)</th>
<th>Length_Conductor (ft)=50</th>
<th>1. Proposed Method</th>
<th>2. WinIGS</th>
<th>3. Closed Form</th>
<th>Diff.1&amp;2 (%)</th>
<th>Diff.1&amp;3 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>10 Segs</td>
<td>10.3300</td>
<td>10.3364</td>
<td>10.3685</td>
<td>0.06%</td>
<td>0.37%</td>
</tr>
<tr>
<td>5.56</td>
<td>9 Segs</td>
<td>10.3336</td>
<td>10.3364</td>
<td>10.3685</td>
<td>0.03%</td>
<td>0.34%</td>
</tr>
<tr>
<td>6.25</td>
<td>8 Segs</td>
<td>10.3437</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.07%</td>
<td>0.24%</td>
</tr>
<tr>
<td>7.14</td>
<td>7 Segs</td>
<td>10.3399</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.03%</td>
<td>0.28%</td>
</tr>
<tr>
<td>8.33</td>
<td>6 Segs</td>
<td>10.3559</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.19%</td>
<td>0.12%</td>
</tr>
<tr>
<td>10.00</td>
<td>5 Segs</td>
<td>10.3672</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.30%</td>
<td>0.01%</td>
</tr>
<tr>
<td>12.50</td>
<td>4 Segs</td>
<td>10.3866</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.49%</td>
<td>0.17%</td>
</tr>
<tr>
<td>16.67</td>
<td>3 Segs</td>
<td>10.3985</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.60%</td>
<td>0.29%</td>
</tr>
<tr>
<td>25.00</td>
<td>2 Segs</td>
<td>10.4380</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.98%</td>
<td>0.67%</td>
</tr>
<tr>
<td>50.00</td>
<td>1 Seg</td>
<td>10.4379</td>
<td>10.3364</td>
<td>10.3685</td>
<td>-0.98%</td>
<td>0.67%</td>
</tr>
</tbody>
</table>

In general, from the above tables in Section 2.4.2, some conclusions can be drawn. In the single horizontal conductor simulations, the proposed method agrees more closely with the WinIGS results than with the closed form solution. Since, the specific segmentation method used in the application WinIGS is proprietary, there is no way to know the number of segments used by WinIGS. If the segmentation rules state in Table 2.8 are used, then the maximum difference between the proposed method and WinIGS and the closed form solution are -0.98% and 0.67%, respectively. The general difference among three methods is in the acceptable range.
The reason why the proposed method may be more accurate than the closed form solution is the different assumptions of current density in these two methods. In the closed form solution, the current density is uniform along the conductor or rod. While in the proposed method, the current values are different in each conductor/rod segment, mimicking more closely the variation in current density experienced by a buried conductor.

For the single ground rod and horizontal conductor modeling methods, the accuracy of the proposed method is acceptable (under 2%). In the following sections, more complex ground grid models are discussed.

2.4.3. Model of Rectangular and Square Ground Grid With Rods

Based on the ground grid design requirements from SRP, the rectangular and square ground grids are of primary interest. Thus, in this work, these two geometries are discussed first. In Fig. 2.13, an example of rectangular ground grid is shown. In this example grid, there are four corner rods, ten horizontal divisions and six vertical divisions on the horizontal plane.
There are three different numerical methods to calculate its safety metric values. The first is the proposed method, which was developed in MATLAB and is based on the equations presented in this work. The second method is the WinIGS simulation. The third method is the IEEE Standard method[19]. This method is largely different from the above two methods, since it uses coarser approximations.

First, one simple designed ground grid case from [19] IEEE Std 80-2000 is presented:

- Case 1: Assume a 70 m by 70 m grid with equal size square meshes as shown in Fig. 2.14. The mesh size is 7 m and 7 m. Grid burial depth is 0.5 m, and no ground rods are used. Uniform soil is assumed and the soil resistivity is 400 Ohm*meters.
Fig. 2.14 An Example of Rectangular Shape Ground Grid from [19]

For the IEEE method used in the Case 1, the calculation steps and functions are present as following equations from (2.114) and (2.122).

The total length of buried conductor, \( L_T \), is \( 2 \times 11 \times 70 \text{ m} = 1540 \text{ m} \). The total area covered by the ground grid, \( A \), is \( 70 \text{ m} \times 70 \text{ m} = 4900 \text{ m}^2 \). The grid burial depth, \( d \), is 0.5 m. The conductor is AWG 2/0 with a diameter of 0.4180 inches.

Using the equation (2.114), the grid resistance \( R_g \) can be calculated.

\[
R_g = \rho \left( \frac{1}{L_T} + \frac{1}{\sqrt{20} \times A} \left( 1 + \frac{1}{1 + h \sqrt{20/A}} \right) \right)
\]

\[
= 400 \left( \frac{1}{1540} + \frac{1}{\sqrt{20} \times 4900} \left( 1 + \frac{1}{1 + 0.5 \sqrt{20/4900}} \right) \right)
\]

\[
= 2.78 \text{ Ohms}
\]
Then, the grounding potential rise, $GPR$, can be calculated using (2.15).

$$GPR = I_G * R_s = 1908 * 2.78 = 5304 V$$

(2.115)

The given system fault current, $I_G$, is 1908 A.

In order to calculate mesh voltage with IEEE method, a parameter $K_m$ called a geometrical factor needs to be calculated first using (2.116). In (2.116), $D_m$ is the 7 m mesh length, $d$ is the grid burial depth 0.5 m, $a$ is diameter of grid conductor 0.0053 m (0.418 inches) and $n$ is the geometry factor which is given by (2.120).

$$K_m = \frac{1}{2\pi} \left[ \ln \left( \frac{D_m^2}{16*d*a} + \frac{(D_m + 2 + d)^2}{8*D_m*d} - \frac{d}{4*a} \right) \right]$$

\begin{align*}
&+ \frac{K_{ii}}{K_h} \ln \left( \frac{8}{\pi(2*n-1)} \right) \\
\end{align*}

(2.116)

where,

$$K_{ii} = \frac{1}{(2n)^2} = \frac{1}{(2*11)^2} = 0.57$$

(2.117)

and

$$K_h = \sqrt{1 + \frac{d}{h_0}} = \sqrt{1 + \frac{0.5}{1.0}} = 1.225$$

(2.118)

Thus, $K_m$ is equal to 0.89. In above equations, the grid reference depth, $h_0$, is 1 meter. Another parameter $K_i$, the irregularity factor, also needs to be calculated using (2.119).

$$K_i = 0.644 + 0.148*n$$

(2.119)
where,
\[ n = n_a * n_b * n_c * n_d \tag{2.120} \]
\[ n_a = \frac{2 * L_T}{L_p} = \frac{2 * 1540}{280} = 11 \tag{2.121} \]

In above (2.120) and (2.121), the perimeter of the grid, \( L_p \), is 280 meter. And when the grid is square, \( n_b, n_c \) and \( n_d \) are all equal to 1. Thus the value of \( n \) is 11.

Finally \( K_i \) is equal to 2.272.

Using these values, the mesh touch potential can be calculated using (2.122).
\[ E_m = \frac{\rho * I_G * K_m * K_i}{L_c} \tag{2.122} \]
\[ = \frac{400 * 1908 * 0.89 * 2.272}{1540} = 1002.1 V \]

To compare the results, the application developed and WinIGS are used. It is shown in following Table 2.9. In this table, the \( E_{touch} \) is touch potential which equals to \( E_m \).

<table>
<thead>
<tr>
<th>Case</th>
<th>( R_g )</th>
<th>( GPR )</th>
<th>( E_{touch} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.IEEE</td>
<td>2.7800</td>
<td>5304.00</td>
<td>1002.1000</td>
</tr>
<tr>
<td>2.Proposed</td>
<td>2.6441</td>
<td>5045.20</td>
<td>925.9967</td>
</tr>
<tr>
<td>3.WinIGS</td>
<td>2.6490</td>
<td>5054.28</td>
<td>931.5100</td>
</tr>
<tr>
<td>Diff.1 &amp; 2</td>
<td>4.89%</td>
<td>4.88%</td>
<td>7.59%</td>
</tr>
<tr>
<td>Diff.1 &amp; 3</td>
<td>4.71%</td>
<td>4.71%</td>
<td>7.04%</td>
</tr>
<tr>
<td>Diff.2 &amp; 3</td>
<td>0.18%</td>
<td>0.18%</td>
<td>0.59%</td>
</tr>
</tbody>
</table>

From Table 2.9, it can be seen that the IEEE method is less conservative than
MATLAB application or the WinIGS simulation results. It is because that IEEE method is just an approximate method to evaluate the performance of ground grid. However, the difference between MATLAB application and WinIGS is small and acceptable (under 2%).

Further validation of the proposed method is presented in Cases 2 to 4 with different grid shapes, number of meshes and rods, rod lengths and soil resistivity values. In the following simulations, the horizontal conductor is AWG 4/0 with the diameter 0.528 inches and the vertical ground rod is AWG 5/8 with the diameter 0.628 inch. The fault current is 3.78 kA and the fault duration time is 0.53 s. Comparisons are made between the developed MATLAB application and WinIGS simulation results.

- Case 2: Assume a 600 ft by 400 ft grid with square meshes. The mesh size is 40 ft by 40 ft (with a total of 15*10=150 inner meshes). The grid burial depth is 1.5 ft, and no ground rods are installed. A two layer soil model is used, and the upper and lower layer soil resistivity values are 100 and 30 Ohm*meters, respectively. The depth of upper layer is 10 ft. The difference in the results is presented in Table 2.10. The difference is obtained by subtracting the results of the proposed method from the results of WinIGS simulation.
Table 2.10
Results Comparison of MATLAB Code and WinIGS Simulation Case 2

<table>
<thead>
<tr>
<th></th>
<th>$R_g$</th>
<th>$E_{step}$</th>
<th>$E_{touch}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Proposed</td>
<td>0.1226</td>
<td>69.1869</td>
<td>176.2958</td>
</tr>
<tr>
<td>2. WinIGS</td>
<td>0.1222</td>
<td>70.8000</td>
<td>174.4100</td>
</tr>
<tr>
<td>Diff.1 &amp; 2</td>
<td>-0.33%</td>
<td>2.28%</td>
<td>-1.08%</td>
</tr>
</tbody>
</table>

- Case 3: Assume a 400 ft by 400 ft grid with square meshes. Mesh size is 20 ft by 20 ft (with a total of 20*20=400 inner meshes). The grid burial depth is 1.5 ft, and 12 ground rods are placed around the grid perimeter, each with the length of 30 ft. A two layer soil model is used, and the upper and lower resistivity values are 100 and 30 Ohm*meters, respectively. The depth of upper layer is 10 ft.

A 2D plot of the touch potential along with the grid diagonal line can be plotted. It is shown in following Fig. 2.15. A 3D plot of the touch potential for points all over the grid is shown in Fig. 2.16. The difference in the results is presented in Table 2.11.
Fig. 2.15 An Example of Square Ground Grid Touch Potential 2D Plot

Fig. 2.16 An Example of Square Ground Grid Touch Potential 3D Plot
TABLE 2.11
RESULTS COMPARISON OF MATLAB CODE AND WINIGS SIMULATION CASE 3

<table>
<thead>
<tr>
<th></th>
<th>$R_g$</th>
<th>$E_{step}$</th>
<th>$E_{touch}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Proposed</td>
<td>0.1323</td>
<td>63.9728</td>
<td>108.2519</td>
</tr>
<tr>
<td>2. WinIGS</td>
<td>0.1324</td>
<td>63.2600</td>
<td>108.3200</td>
</tr>
<tr>
<td>Diff. 1 &amp; 2</td>
<td>0.07%</td>
<td>-1.13%</td>
<td>0.06%</td>
</tr>
</tbody>
</table>

- Case 4: Assume a 500 ft by 300 ft grid with rectangular meshes. The mesh size is 20 ft by 30 ft (25*10=250 inner meshes). The grid burial depth is 1.5 ft, and 12 ground rods are placed around the grid perimeter, each with the length of 30 ft. A two layer soil model is used, and the upper and lower layer resistivity values are 50 and 100 Ohm*meters, respectively. The depth of the upper layer is 40 ft. The differences in the relevant WinIGS and proposed method results is presented in Table 2.12. A 3D touch potential plot is shown in following Fig. 2.17. Unlike the 3D plot in Fig. 2.15, this plot is a flat view on X-Y plane.

TABLE 2.12
RESULTS COMPARISON OF MATLAB CODE AND WINIGS SIMULATION CASE 4

<table>
<thead>
<tr>
<th></th>
<th>$R_g$</th>
<th>$E_{step}$</th>
<th>$E_{touch}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Proposed</td>
<td>0.2955</td>
<td>81.1331</td>
<td>107.5056</td>
</tr>
<tr>
<td>2. WinIGS</td>
<td>0.2956</td>
<td>79.7200</td>
<td>107.2000</td>
</tr>
<tr>
<td>Diff. 1 &amp; 2</td>
<td>0.03%</td>
<td>-1.77%</td>
<td>-0.29%</td>
</tr>
</tbody>
</table>
In general, the difference between MATLAB application and WinIGS simulation results are less than 2.5%, which is considered as acceptable accuracy. Based on the SRP design requirement, equally spaced ground conductor (grid conductor is uniformly distributed) is applied in this work meshes in a ground grid is used by the developed applications. For uniform rectangular meshes, the touch potential will increase along meshes from the center to the corner of the mesh, it can be clearly observed from above figures Fig. 2.15 and Fig. 2.16. The rate of change will largely depend on the size of the mesh, number and location of the ground rods, spacing of parallel conductors, diameter and depth of the conductors, and the resistivity profile of the soil.

Fig. 2.17 An Example of Square Shape Ground Grid Touch Potential 3D Plot
2.4.4. Model of L Shape Ground Grid

The MATLAB application can also simulate an L shape ground grid. Unlike the rectangular or square shape ground grids, an L shape ground grid is unsymmetrical shape. An example of an L shape ground grid is shown in Fig. 2.18.

![Grid Diagram](image)

**Fig. 2.18 An Example of L Shape Ground Grid**

As seen in Fig. 2.18, the L shape substation, it has four geometrical parameters, $a_1$, $a_2$, $a_3$ and $a_4$. Ground rods are also installed at each corner and may be installed along the perimeter of grid, though non-corner rods are not present in

For the segmentation method used in L shape grid, since the segment length
values of horizontal ground mat must exactly be divided with six values, which are \( a_1, a_4 \) and \( a_1-a_4 \) on one length direction and \( a_2, a_3 \) and \( a_2+a_3 \) on width direction, respectively. Thus, the number of segments should be chosen the highest common factor of these length and width values. In Table 2.13, the segmentation method of L shape is shown, and the \( gcd \) function is used to calculate the greatest common divisor of given numbers.

<table>
<thead>
<tr>
<th>Number of segment on length direction</th>
<th>( gcd(a_1, a_4, a_1-a_4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of segment on width direction</td>
<td>( gcd(a_2, a_3, a_2+a_3) )</td>
</tr>
</tbody>
</table>

In Case 5 to Case 7 below, a comparison is made between important safety metrics using the MATLAB application and WinIGS. All the system parameters are the same as in the above rectangular and square grid cases including: fault current, fault duration time, conductor and rod size and composition, .

- Case 5: Assume an L shape grid and the size \((a_1, a_2, a_3, a_4)\) is \((100, 100, 100, 50)\) in units of ft. There are 3 divisions on x axis and 3 divisions on y axis. The grid burial depth is 1.5 ft. No ground rods are used as shown in Fig. 2.19. A uniform soil model is used and the resistivity values is 100 Ohm*meters. The results are shown in Table 2.14.
Case 6: Assume an L shape grid and the size \((a_1, a_2, a_3, a_4)\) is \((120, 80, 80, 60)\) in the units of ft. There are 2 divisions on grid x axis and 2 divisions on grid y axis. The grid burial depth is 1.5 ft and 6 ground rods are placed at each corner as shown in Fig. 2.20, each with a length of 30 ft. A two layer soil model is used and the upper and lower layer resistivity values are 30 and 100 Ohm*meters, respectively. The depth of upper layer is 10 ft. The results obtained using the MATLAB application and WinIGS are compared in Table 2.15.
Case 6: Assume an L shape grid and the size (a1, a2, a3, a4) is (120, 70, 80, 50) in the units of ft. There are 12 divisions on grid x axis and 10 divisions on grid y axis. The grid burial depth is 1.5 ft, and 12 30-ft. ground rods are placed along the perimeter, including the corners as shown in Fig. 2.21. A two layer soil model is used and the upper and lower layer resistivity values are 100 and 30 Ohm*metres, respectively. The depth of the upper layer is 10 ft. The results obtained using the MATLAB application and WinIGS are compared in Table 2.16.

![Fig. 2.20 Case 6 L Shape Grid](image)

**TABLE 2.15**

RESULTS COMPARISON OF MATLAB CODE AND WINIGS SIMULATION CASE 6

<table>
<thead>
<tr>
<th></th>
<th>$R_g$</th>
<th>$E_{step}$</th>
<th>$E_{touch}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Proposed</td>
<td>0.8462</td>
<td>297.5661</td>
<td>526.88</td>
</tr>
<tr>
<td>2. WinIGS</td>
<td>0.8480</td>
<td>301.7320</td>
<td>531.91</td>
</tr>
<tr>
<td>Diff.1 &amp; 2</td>
<td>0.22%</td>
<td>1.40%</td>
<td>0.95%</td>
</tr>
</tbody>
</table>

Case 7: Assume an L shape grid and the size (a1, a2, a3, a4) is (120, 70, 80, 50) in the units of ft. There are 12 divisions on grid x axis and 10 divisions on grid y axis. The grid burial depth is 1.5 ft, and 12 30-ft. ground rods are placed along the perimeter, including the corners as shown in Fig. 2.21. A two layer soil model is used and the upper and lower layer resistivity values are 100 and 30 Ohm*metres, respectively. The depth of the upper layer is 10 ft. The results obtained using the MATLAB application and WinIGS are compared in Table 2.16.
Fig. 2.21 Case 7 L Shape Grid

<table>
<thead>
<tr>
<th></th>
<th>$R_s$</th>
<th>$E_{step}$</th>
<th>$E_{touch}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Proposed</td>
<td>0.4178</td>
<td>243.8747</td>
<td>300.5048</td>
</tr>
<tr>
<td>2. WinIGS</td>
<td>0.4196</td>
<td>247.7635</td>
<td>306.4581</td>
</tr>
<tr>
<td>Diff. 1 &amp; 2</td>
<td>0.43%</td>
<td>1.55%</td>
<td>1.94%</td>
</tr>
</tbody>
</table>

From above tables, it can be observed that the grid resistance calculation with the proposed method agrees very close (less than 0.5%) with the corresponding WinIGS simulations. The worst touch potential and worst step potential calculation with the proposed method agrees are different from the WinIGS simulation by at most 1.94%, which is within the target difference band of +/- 2%. It is believed that the different segmentation strategies in these two methods may lead to these differences.
CHAPTER 3.

GROUNDING SYSTEM OPTIMIZATION

The purpose of the ground grid optimal design is to minimize the total ground grid construction cost while ensuring that the personnel are safe. The optimized ground grid design should be constructed and built using acceptable values of the IEEE safety metrics [19] along with design requirements specific to Salt River Project.

Fig. 3.1 The Flowchart of the Two-step Hybrid Optimization Method

In this work, a novel hybrid optimization method is used which combines a
genetic algorithm (GA) with a pattern search (PS) algorithm. The flowchart of the hybrid optimization method is shown in the Fig. 3.1. In the hybrid method, a GA is used first to find an approximate starting point. Once the initial estimate is obtained, then a PS algorithm is used for further optimization.

A GA can be classified as a stochastic method. In [25], it has been shown that deterministic methods often converge to one of the function's local minima. Therefore, if there is no or very little knowledge about the behavior of the objective function in the region of each local minima, or knowledge about the location of feasible and non-feasible regions in the multidimensional parameter space, it seems advisable to start the optimization process with a stochastic strategy.

Stochastic methods choose their path through the parameter space by using some random factors, which are discussed for this particular application in the Section 3.1. Stochastic methods are simple to implement, stable in convergence, and are able to find the desired region with a reasonable reliability. However, stochastic methods usually suffer from a high number of function evaluations and long execution time..

3.1 Use of Genetic Algorithm for Optimization

In the following Sections 3.1 and 3.2, the details about the chosen GA and PS method are introduced, respectively. In Section 3.3, the ground grid optimal design model is discussed. In Section 3.4, three cases of ground grid designs optimized using the application developed are shown.

A genetic algorithm is a search technique used in computing to find a true or
approximate solutions to an optimization problem. It is also a particular class of evolutionary algorithms which were inspired by evolutionary biology such as inheritance, mutation, selection and recombination. The following are the major steps needed by a GA algorithm.

- Creating initial population: At first, an individual is produced randomly. The process is repeated until the number of the individuals in the population equals the specified population size. The population size specifies how many individuals there are in each generation.

In this research, the initial population is defined as a binary string. The population size is set to 20 individuals. The individual size is set to 20 characteristics long. Thus, in this specific ground grid design problem, each grid design can be represented as an individual, which is in turn represented as a binary string with twenty 0’s or 1’s. This binary string will only be used in the GA optimization work. These twenty 0's and 1's will be transformed to the three variables \((N_x, N_y, N_{rod})\) in order to evaluate the objective function (grid construction cost) and constraint functions (touch potential, step potential and grid resistance) are calculated. However, since the MATLAB GA solver is applied in this work, the details of transforming binary string to variables cannot be obtained.

- Objective and fitness functions: A fitness function is determined by the objective function. According to the value of fitness function (which is the inverse of the objective function as discussed below), all the individuals will be ranked by using the "selection" (defined below.) The feasibility of fitness function is checked with problem specific constraints.
If the fitness function fails to be feasible, a penalty term (0.001) will be added to its fitness value.

In this research, the objective is to minimize the ground grid construction cost. However, the fitness function values for all the individuals are ranked by the MATLAB GA routine in descending order. Thus, the fitness function value of one individual should be taken as the inverse of its objective function value. In other words, when the objective function value is minimized, the fitness function value is maximized.

- Selection: In the process of choosing parent individuals with high fitness values, based on their scaled values from the ranked fitness function values from which the next generation will be produced.

In this research, selection is performed using the roulette wheel method. In other words, the individual with the high fitness values will have a high probability of being chosen as parents for the next step of "reproduction".

- Reproduction: In a process which defines how the GA creates children individuals at each generation. It contains two steps, crossover and mutation. Crossover will combine two parent individuals, and form a new individual for next generation. A crossover example is shown below. The $s_1$ and $s_2$ are two parent individuals, and $s_3$ and $s_4$ are two new individuals.

In the GA MATLAB solver, the crossover point is selected randomly.

Before crossover (parents):

$$s_1=10|11010001, s_2=11|10110101$$

After crossover (children):
The purpose of mutation is to simulate the effect of errors that happen with low probability during duplication. A mutation example is shown below. The $s_5$ and $s_6$ are individuals before mutation and after mutation, respectively. The number between the two symbols “|” (presented in the example) is selected to be mutated from “1” to “0” or from “0” to “1”. In this example, only one bit was selected, but each bit has the same probability of being selected.

Before mutation:

$s_1=10|1|1010001$

After mutation:

$s_3=10|0|1010001$

In the optimization solver, the crossover and mutation probability values are 0.8 and 0.2, respectively. The suitable crossover and mutation probabilities can preserve the diversity of GA and avoid local minima. If the probability is set too high, the search will turn into a primitive random search.

- Stopping criteria: This includes two stopping criteria. One is the maximum number of iterations. In the second one, when the best fitness value of an individual is less than or equal to the value of fitness function tolerance, the algorithm terminates.

In this research, the maximum number of iterations is defined as 20. In general, the range of objective function (grid construction cost) is $30,000 to
$200,000, thus the fitness function value will be $5.0e-6$ to $3.3e-5$. Therefore, the tolerance value is set as $1e-6$.

The reason for using a genetic algorithm as the first step to search for the PS starting point is:

1. By using the fast convergence speed of a GA, an approximate result can be obtained in only 15 or 20 iterations.

2. In general, the approximate start point from GA is close to the final optimal result, which saves much running time for the further optimization work.

The GA method flowchart is shown in Fig. 3.2.
3.2 Use of Pattern Search Method for Optimization

A pattern search (PS) algorithm is a direct search algorithm which searches a set of points in some neighborhood of the starting point, looking for an improved result where the value of the objective function is smaller than the value at the
current point. This kind of direct search algorithm can be used to optimize functions that are not continuous or differentiable, which is the case with the objective function and constraint functions in this work.

In the hybrid optimization method used in the optimal ground grid design application, the PS is used as the second step. In the first step, the GA is used obtain an approximate optimum. Generally, this approximate optimum is close to the final optimum. The PS is used to search in a certain region around the approximate optimum (starting point). The PS algorithm will test multiple points near the starting point. If one of these multiple points yields a smaller or larger value (depending on whether the objective function is to be minimized or maximized) of the objective function than the starting point, the new start point will be set as the starting point for the next iteration. This kind of search method is used in the MATLAB pattern search solver, which is called "Poll step". In this MATLAB function, the user can define the search range around the starting point. A simple pattern search example is stated.

- The coordinates of the starting point is \((X_{ps1}, Y_{ps1})\)

- Let the search range around the starting point be given by \((x_r,0), (-x_r,0), (0, y_r)\) and \((0, -y_r)\), respectively. Let the search resolution be \(D_r\). Thus the new vectors will be \((x_r*D_r, 0), (-x_r*D_r,0), (0, y_r*D_r)\) and \((0, y_r*D_r)\), respectively.

Therefore, the new four point are obtained as \((X_{ps1}+x_r*D_r, Y_{ps1}), (X_{ps1}+x_r*D_r, Y_{psl}), (X_{ps1}+y_r*D_r)\) and \((X_{ps1}-y_r*D_r)\), respectively. The objective function
value of each point will be tested, and the point with smaller or larger objective value will be set as the starting point for next iteration.

In addition, MATLAB’s implementation of the pattern search algorithm gives users the option of changing the search resolution scaling parameters, which is used to reduce the search resolution or expand the search resolution automatically.

In Fig. 3.3, the flowchart of PS is shown.

![Flowchart of Pattern Search Optimization Method](image)

The stopping criteria for the pattern search algorithm includes:

- Reaches the maximum number of iterations (200)
- Reaches the minimum searching range size (1e-6)
- Meets the minimum objective change (1e-6)
3.3. Optimization Modeling Based on SRP Design Rules

For the objective function of the grounding grid optimal design problem, there can be different functions with different aims. From [28], the objective function is minimizing the total length of underground conductor and rod.

However, in this work, based on the project sponsor’s, SRP’s, design requirements, the objective function is minimizing the grounding system total construction cost. Based on the data from the local utility, there are the (a) material cost of horizontal conductors and vertical rods, $C_{cond}=$$3.77/ft; (b) material cost of exothermic welds, $C_{exoth}=$$19.25 each; (c) cost of labor to trench, install and backfill conductors and drive ground rods, $C_{trench}=$$10.0/ft and $C_{drive},$ $32/ft$ respectively; (d) cost of labor to make the exothermic connection of conductor to conductor or conductor to rod, $C_{connect}=$$40 each. Thus, the objective function of square and rectangular girds is obtained using (3.1), (3.2), (3.3) and (3.4):

\[
\text{Obj.} \min \ Cost\left(N_x, N_y, N_{rod}\right) = (C_{cond} + C_{trench})*L_{cond} + (C_{rod} + C_{drive})*L_{rod} + (C_{connect} + C_{exoth})*(N_{rod} + N_{exoth})
\]

where,

\[
L_{cond} = (N_x + 1)*L_x + (N_y + 1)*L_y \quad (3.2)
\]

\[
L_{rod} = l_{rod} * N_{rod} \quad (3.3)
\]

\[
N_{exoth} = (N_x + 1)*(N_y + 1) \quad (3.4)
\]

In above equations, $L_x$ and $L_y$ are the given ground grid length and width in ft.
$L_{\text{cond}}$ is the total length of horizontal conductors, $L_{\text{rod}}$ is the total length of vertical rods, and $N_{\text{exph}}$ is the total number of exothermic connections.

In (3.3), $l_{\text{rod}}$ is the length of a single vertical rod. Based on design requirement, rod length is determine by SRP’s design rules, which are a function of the upper and lower layer soil resistivity, and upper layer depth. In the following, rules for determining ground rod length are shown.

- When the upper layer resistivity $\rho_1$ is equal to the lower layer resistivity $\rho_2$, or the depth of upper layer is greater than 30 ft, the length of rod $l_{\text{rod}}$ is 10 ft.
- When the upper layer resistivity $\rho_1$ is not equal to the lower layer resistivity $\rho_2$, and the depth of upper layer is lower than 10 ft, the length of rod $l_{\text{rod}}$ is 20 ft.
- When the upper layer resistivity $\rho_1$ is not equal to the lower layer resistivity $\rho_2$, and the depth of upper layer is smaller than 30 ft and greater than or equal to 10 ft, the length of rod $l_{\text{rod}}$ is 30 ft.

In the optimization model, there are three additional variables, $N_x$ is the number of meshes in grid length direction, $N_y$ is the number of meshes in grid width direction, and $N_{\text{rod}}$ is the number of rods in designed grid.

Based on the ground grid safety requirements and design requirements from SRP, the optimization constraints can be obtained as given by (3.5), (3.6), (3.7), (3.8) and (3.9).

\[
E_{\text{grid, touch}} \leq E_{\text{touch, allowable}} \tag{3.5}
\]

\[
E_{\text{grid, step}} \leq E_{\text{step, allowable}} \tag{3.6}
\]
\[ R_g \leq R_{\text{allowable}} \]  
\[ \frac{L_x}{50 \text{ ft}} \leq N_x \leq \frac{L_x}{8.5 \text{ ft}} \]  
\[ \frac{L_y}{50 \text{ ft}} \leq N_y \leq \frac{L_y}{8.5 \text{ ft}} \]

For the square grid and rectangular grid, the number of rods is restricted as following (3.10):

\[ 4 \leq N_{rod} \leq 12 \]  

For the L shape grid, the number of rods is restricted as following (3.11):

\[ 6 \leq N_{rod} \leq 18 \]

In the above constraint functions, the three variables \( N_x, N_y \) and \( N_{rod} \) are integer variables.

In constraints (3.5) and (3.6), the maximum allowable values for touch potential and step potential can be obtained from the (2.8) and (2.9) in Chapter 2 based on IEEE standard. In (3.7), the maximum allowable value for grid resistance is 0.5 ohms, which comes from the SRP design requirement.

Furthermore, in (3.8) and (3.9), the mesh size for ground grid is restricted in the range of 8.5ft and 50ft, in order to avoid the grid mesh is too small or too large.

In (3.10), for square and rectangular grids, the number of rods is restricted between 4 and 12. In (3.11), for L shape grid, the number of rods is restricted between 6 and 18. The origin of these restrictions come from SRP’s design rules.
First, vertical rods must be placed in every grid corner. Thus, the minimum rods number must be 4 for rectangular grids, or 6 for L-shape grids. In order to save the optimization running time and keep the high calculation efficiency, the maximum rods number is also set as 12 or 18 (depending on the grid shape) for simplicity.

Other ground grid design requirements from SRP are listed as following:

- Grid horizontal conductor size should be 4/0 AWG, 7 strand copper (0.528” diameter).
- Grid vertical rod size should be 5/8 AWG, 1 strand copper (0.628” diameter).
- The standard depth of the grounding system should be 1.5 ft. below finished grade. It does not include any surface material used to obtain a decreased touch and step potential.
- The ground grid should be designed for the maximum fault level expected for the life of the station.
- In this model, uniform potential distribution (no potential difference along grounding conductors) and uniform mesh size are assumed.
- The model ignores the influence of mutual inductance and capacitance.
- In this model, interior rods and surge arrestor loops are not considered to provide benefit for safety purposes.
- In order to apply the proposed hybrid two-step optimization method, the MATLAB genetic algorithm and pattern search solvers are utilized. In the GA solver, the inputs should include the variable range for \((N_x, N_y, N_{rod})\).
the objective and constraint functions, and all the required parameters which are used in these functions. For PS solver, in addition to the above inputs, the solution from the GA is inputted as the starting point.

3.4 Case Study of Ground Grid Optimization Design

In this section, three cases with different grid shapes are presented. For the following three cases, the fault current is 3.78 kA and fault duration time is 0.53 seconds.

Case 1: In this case, the optimal design of a square grid is desired. The soil model and other parameters are:

- Soil model parameters: upper layer resistivity $\rho_1$ is 100 Ohm*meters, lower layer resistivity $\rho_2$ is 30 Ohm*meters, the depth of upper layer is 10 ft.
- Grid size parameters: length and width are both 450 ft. Based on the design requirement, the length of the ground rods is 30 ft.

By using the hybrid optimization method introduced earlier, the result produced for this square grid is shown in Table 3.1. This table, also shows the touch potential $E_{touch}$ and its maximum allowable value, step potential $E_{step}$ and its maximum allowable value, grid resistance $R_g$ and its maximum allowable value. All these values are calculated based on the final optimal grid design.
Table 3.1
CASE 1 SQUARE GROUND Grid OPTIMIZATION DESIGN RESULT

<table>
<thead>
<tr>
<th>N_x</th>
<th>N_y</th>
<th>N_rod</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>7</td>
<td>4</td>
<td>$86,961.9</td>
</tr>
</tbody>
</table>

*E_{\text{touch}} (\text{Max. Allowed})* 178.69 V (183.24 V)  
*E_{\text{step}} (\text{Max. Allowed})* 92.39 V (254.94 V)  
*R_g (\text{Max. Allowed})* 0.16 Ohm (0.50 Ohm)

Case 2: In this case the optimal design of a rectangular grid is desired. The soil model and other parameters are listed as following:

- Soil model parameters: upper layer resistivity $\rho_1$ is 100 Ohm*meters, lower layer resistivity $\rho_2$ is 30 Ohm*meters, the depth of upper layer is 10 ft.
- Grid size parameters: length and width are 500 ft and 350 ft. The length of rod is 30 ft.

By using the hybrid optimization method introduced earlier, the result for this square grid is shown in Table 3.2. This table, also shows the touch potential $E_{\text{touch}}$ and its maximum allowable value, the step potential $E_{\text{step}}$ and its maximum allowable value, and the grid resistance $R_g$ and its maximum allowable value. All these values are calculated based on the final optimal grid design.

Table 3.2
CASE 2 RECTANGULAR GROUND Grid OPTIMIZATION DESIGN RESULT

<table>
<thead>
<tr>
<th>N_x</th>
<th>N_y</th>
<th>N_rod</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>7</td>
<td>4</td>
<td>$93,092.4</td>
</tr>
</tbody>
</table>

*E_{\text{touch}} (\text{Max. Allowed})* 180.74 V (183.24 V)  
*E_{\text{step}} (\text{Max. Allowed})* 81.02 V (254.94 V)  
*R_g (\text{Max. Allowed})* 0.14 Ohm (0.50 Ohm)

Case 3: This case is the optimal design of an L shape grid. The soil model and other parameters are listed as following:
- Soil model parameters: upper layer resistivity $\rho_1$ is 100 Ohm*meters, lower layer resistivity $\rho_2$ is 30 Ohm*meters, the depth of upper layer is 10 ft.

- Grid size parameters: The $(a1, a2, a3, a4)$ are (200, 300, 100, 180) ft. The length of ground rods are 30 ft.

By using the hybrid optimization method introduced earlier, the result for this square grid is shown in Table 3.3. This table, also shows the touch potential $E_{touch}$ and its maximum allowable value, the step potential $E_{step}$ and its maximum allowable value, and the grid resistance $R_g$ and its maximum allowable value. All of these values are calculated based on the final optimal grid design produced by the proposed method.

<table>
<thead>
<tr>
<th>$N_x$</th>
<th>$N_y$</th>
<th>$N_{rod}$</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>15</td>
<td>6</td>
<td>$29,038.2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$E_{touch}$ (Max. Allowed)</th>
<th>$E_{step}$ (Max. Allowed)</th>
<th>$R_g$ (Max. Allowed)</th>
<th>$\text{Cost}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>176.88 V(183.24 V)</td>
<td>107.28 V(254.94 V)</td>
<td>0.2 Ohm (0.5Ohm)</td>
<td>$29,038.2$</td>
</tr>
</tbody>
</table>

In general, from above three cases, it can be observed that the touch potential constraint is the binding constraint in the ground-grid design optimization problem. Among these cases, the step potential and grid resistance values of the optimal grid design are much less than the allowable values. However, the touch potential value is always just a little bit less the allowable value. Therefore, the touch potential is the most critical constraint in our problem.
A application interface has been designed using Graphical User Interface (GUI) functionality of MATLAB. Guidelines to run this application are stated in following sections. The flowchart of the application is shown in following Fig. 4.1.

4.1 Brief Introduction of the Application

The application requires the following input data.

- Soil model parameters: upper layer resistivity $\rho_1$ (Ohm* m), lower layer resistivity $\rho_2$ (Ohm* m) and the depth of upper layers $D$(ft).
- Fault system parameters: fault current $I_f$ (kA) and fault duration time $t_f$.
(second).

- Grid geometrical parameters: for example, when the rectangular ground grid is designed, the rectangular length (ft) and width (ft) are needed.

The screen capture of the input dialog box is shown in Fig. 4.2.

![Screen Capture of Input Parameter and Function Option]

Fig. 4.2 Application Interface Screen Capture of the Input Parameter and Function Option

Other options to be selected by the user are listed below

- Soil model parameters input methods: There are two parameter input
methods for soil model parameters.

1) The application can read-in a specifically formatted excel document and calculate all the soil model parameters. (The excel document shown in Appendix I should include all the data used in four-point Wenner method.)

2) User can define all the parameters by typing them into the GUI of the application.

- Shape of substation ground grid: In the current version, the application can handle square grid, rectangular grid and L shape grid.

- Ground grid touch potential 2D an 3D plotting: After the optimal result is obtained, the application can generate the touch potential 2D and 3D plots of the final ground grid design. This function is available for square grid and rectangular grid. However, since the plotting progress will take a long time, this is an optional step.

- Rod placements option: User has two options for the rod placement.

  1) The rods are only placed in the grid corners, in other words, the number of rods is no longer considered as a variable in the optimal grid design.

  2) The rods are placed in the grid corners and additional rods can be equally spaced around the perimeter of the grid. The number of rods is a integer variable in the optimal grid design calculated by the software.

After all of the above parameters and options have been entered into the
application, the user can click the "Optimize" button to start the calculations. During the grid optimal design work, there will be different progress bar windows to notify the users which step the application is processing. An example of the progress bar is shown as below Fig. 4.3.

![Fig. 4.3 A Screenshot of the Application Main Interface with the Progress Bar](image)

Once optimization problem has been solved by the MATLAB application, the results generated will be shown on the application interface. The results are listed as below:

- **Optimal model variables**: Number of meshes ($N_x$) on the X axis, number of meshes ($N_y$) on the Y axis, and number of rods ($N_{rod}$) on the Z axis.

- **Final grid design details**: Mesh size values (ft) on horizontal X and Y axis respectively, total construction cost of the final grid ($\$\$).

- **Safety requirements of the final grid design**: They are touch potential (Volts) in the worst case location, step potential (Volts) in worst case location and the grid resistance (Ohms). If the application cannot find an optimal
solution for the grid, which implies that the final solution is infeasible, the application will warn the user and notify which constraints have been violated. An example of this case is shown as below Fig. 4.4.

Fig. 4.4 An Example of Square Ground Grid Infeasible Design Result

In above Fig. 4.4, the grid design is infeasible, since the optimization method has failed to find a feasible solution. The violated constraint is the touch potential safety requirement; thus the system marks the touch potential box in red.

4.2 Case Study Presented by the Application

In this section, three application screen capture figures are shown. In the three figures discussed in this section, three ground grid optimization problems with square shape, rectangular shape and L shape are solved respectively.

In Fig. 4.5 and Fig. 4.6, one square and one rectangular ground grid optimization design cases are shown respectively. All the parameters and results
are shown in the following figures.

Fig. 4.5 An Example of Square Ground Grid Optimization Design Result

Fig. 4.6 An Example of Rectangular Ground Grid Optimization Design Result

In Fig. 4.7, an L shape ground grid optimization design case is shown. The touch potential 2D and 3D plots are not available in L shape. All the parameters and results are shown in Fig. 4.7.
Fig. 4.7 An Example of L Shape Ground Grid Optimization Design Result
5.1 Conclusions

In this work, there are three main parts, including ground grid physical model equations, it's the hybrid optimization algorithm, and the development of the application.

A summary of this work and major conclusions are drawn as follows:

- In the ground grid safety requirement, three values should be considered, touch potential at the worst case location, step potential at the worst case location and grid resistance. By changing the grid design, these three values will also be changed accordingly. Thus, in order to obtain the safety requirements, it is necessary to calculate these three values accurately for different grid shapes and designs. In this work, the modeling methods for square grid, rectangular grid and L shape grid are discussed and developed.

- For purpose of modeling the ground grid system in an accurate way, some specific methods are discussed. The first is the segmentation strategy to be applied to the grid conductors and rods. With different grid shapes and sizes, the segmentation methods are different. The second method discussed is the use of the Resistance Matrix method. This method can obtain the mutual-self resistance values for every segment in the ground grid. Third step in the solution process is a simple matrix equation solution, which yields the ground potential rise and, from which, the ground grid resistance and the current distribution factors can be calculated. The final step is using Green's
Functions to calculate the earth potential at the selected point. Thus, the grid resistance and the touch potential and step potential at worst case locations can be obtained.

- In order to test the modeling method results accuracy, the results generated by the optimal ground grid design application are compared to other methods, which include closed form solution for a single horizontal conductor and single vertical conductor (ground rod) [27], a simple ground grid modeling method from the IEEE standard [19] and an accurate ground grid modeling application known as WinIGS. In general, the difference in the results obtained by the most accurate methods is less than 2%, which is considered acceptable.

- The objective of this project is minimizing the ground grid construction cost while also guaranteeing personnel safe. To that end, a hybrid optimization method was applied in this work, which combines the genetic algorithm (to find an approximate optimum as starting point for further optimization) and a pattern search algorithm (which performs further optimization based on the GA-supplied starting point). By using the optimal solver box in MATLAB, the GA and PS methods are applied to the ground grid optimization problem. In the optimal model, the grid conductor and rod material cost and construction labor cost are considered in the objective function; the constraint functions include ground grid safety requirements (lower than the allowable safe values from the IEEE standard) and other grid design requirement (grid inner mesh size and number of ground rods).
• A MATLAB/GUI application package was developed based on all the methods introduced in this work. The application has been shown to successfully solve the ground grid optimization design problem for several cases: rectangular, square and L-shape substations.

• The work here built upon the work of a previous student. The author's contributions include:
  1) development of the rectangular, rather than square, mesh capability
  2) development of the rectangular and L shape ground grid models
  3) improvement of the ground rod model. (This has been accomplished by correcting errors in the Green's functions used by a previous student. Moreover, the complex image method has been abandoned, since this method was utilized incorrectly and led to errors in the results.)
  4) improvement of the accuracy of ground grid calculation
  5) development of a new hybrid two-step optimization method. (In the previous work, a simple nonlinear optimization solver with IEEE approximate ground grid modeling functions has been used as first step. The pattern search and genetic algorithm has been used as second step and third step. However, this method is not reasonable. Since pattern search is good at searching the optimal result within a small range. The genetic algorithm is good at searching the optimal result within a large range. The proposed hybrid two-step optimization method take full advantage of pattern search and genetic algorithm.)
6) development of a GUI interface for the proposed Optimal Ground Grid Design application

5.2 Future Work

In this work, the developed application can handle three grid shapes, square, rectangular and L shape. However, based on the project sponsor, Salt River Project, the triangular shape ground grid should also be considered. Moreover, based on the specific design requirements from SRP, equally spaced ground grid mesh is used in the application. According to some other references, the unequally spaced ground grid may be more economical.

In addition to the ground grid physical model, the optimization method is also very important. In the current version, the application uses a hybrid (genetic algorithm and pattern search algorithm) optimization method. The efficiency of this kind of stochastic method is not high. Hence, by changing to other optimization methods, the processing time may be reduced.

In general, the future work suggested are the following:

- Develop triangular shape ground grid physical model
- Develop unequally spaced ground grid mesh
- Find other optimization methods with high efficient
- Improve the current application version to one that executes more quickly by using another programming languages
REFERENCES


[33]. SOMIP: Legacy FORTRAN code taken from the industry standard open source program SOMIP

[34]. A.P. Meliopoulos, Graphical and Tabular Results of Computer Simulation of Faulted URD Cables, Electric Power Laboratory of the School of Electrical Engineering, Georgia Institute of Technology, 1981.
APPENDIX I

INPUT EXCEL FORMAT STANDARD
In this appendix, the required Excel format standard is stated. An Excel file may be used in the proposed application as a soil model parameter input method, rather than using the manual input. An example of this Excel format is given by below Fig. 1.

As shown above, the first row contains the name of site and the test date in separate columns. The second row is the probe length (ft) and diameter (inch). The measured data from the 4-point Wenner method starts in the fourth row. The

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<td>Test Date: 05/21/2008</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>Probe Length: 1 ft</td>
<td>Probe Diameter: 0.5 inches</td>
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<td>3</td>
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Fig. 1 Soil Model Formatted Excel Example

The measured data from the 4-point Wenner method starts in the fourth row. The
first column contains the distance between one voltage probe and the midline in feet as shown in Fig. 2, which is half the separation. The second column contains the distance between one current probe and the midline in feet as shown in Fig. 2, which is one-and-one-half of the separation. The third column includes the separation value in feet and the forth column includes the measured resistance value in Ohms. The $a$ is the separation distance used in Wenner method.