Cost-Effective and Privacy-Preserving Energy Management for Smart Meters
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Abstract—Smart meters, designed for information collection and system monitoring in smart grid, report fine-grained power consumption to utility providers. With these highly accurate profiles of energy usage, however, it is possible to identify consumers’ specific activities or behavior patterns, thereby giving rise to serious privacy concerns. This paper addresses this concern by designing a cost-effective and privacy-preserving energy management technique that uses a rechargeable battery. From a holistic perspective, a dynamic programming framework is designed for consumers to strike a tradeoff between smart meter data privacy and the cost of electricity. In general, a major challenge in solving dynamic programming problems lies in the need for the knowledge of future electricity consumption events. By exploring the underlying structure of the original problem, an equivalent problem is derived, which can be solved by using only the current observations. An online control algorithm is then developed to solve the equivalent problem based on the Lyapunov optimization technique. It is shown that without the knowledge of the statistics of the time-varying load requirements and the electricity price processes, the proposed online control algorithm, parametrized by a positive value \( V \), is within \( O(1/V) \) of the optimal solution to the original problem, where the maximum value of \( V \) is limited by the battery capacity. The efficacy of the proposed algorithm is demonstrated through extensive numerical analysis using real data.

Index Terms—Smart Meter, Smart Grid, Data Privacy, Load Monitor, Cost Saving, Battery

I. INTRODUCTION

Traditional power grids are being transformed into smart grids using advanced information control and communication technologies to offer higher reliability, security and efficiency in power systems [1]–[3]. To support both dynamic pricing and a two-way flow of electricity between homes (or micro grids) and power grids, smart meters are being widely deployed. Compared to conventional analog meters, smart meters measure power consumption at a much finer granularity. Such fine-grained information, however, may give rise to serious concerns of security regarding attacks [4] and data privacy [5], especially for residential consumers.

The privacy of smart meter data has recently garnered much attention. Recent work [5] provides an overview of the privacy implications of fine-grained power consumption monitoring. From the information collected by smart meters, complex usage patterns, such as residential occupancy and social activities, can be extracted without a priori knowledge of household activities [6]. Multiple methods [7]–[14] have been proposed to protect smart meter data privacy. One common approach is to introduce uncertainty in individual power consumption by perturbing the load measurements [7], [8]. However, this approach requires modification of the metering infrastructure, which may not be logistically and economically viable, with millions of smart meters already installed. Besides, the modification of usage data could result in inaccurate billing and grid controls, thereby undermining grid management.

In this paper, we address the threats to electricity consumer privacy by using energy storage devices (such as a rechargeable battery) [15]. In particular, a battery can be used to protect the usage patterns of electricity consumers. Ideally, all usage patterns can be perfectly masked by charging and discharging the battery to maintain a constant metered load, such that all load measurements equal to the average consumer’s load. Moreover, the rechargeable battery can also be used to reduce the cost of electricity, as the electricity price is time-varying. It is likely that consumers may tolerate some degree of information leakage to reduce their electricity costs by adapting their needs to the time-varying electricity price. However, it is challenging to control the battery charging/discharging to achieve this, without the knowledge of future electricity consumption events, not to mention the time-varying load requirements and electricity prices with possibly unknown statistics. To tackle this challenge, an online control algorithm with low computational complexity is developed to jointly protect smart meter data privacy and reduce the cost of electricity, while taking into account the cost of repeated charging and discharging on the battery’s lifetime.

A. Summary of Main Contributions

Our main contributions are summarized as follows:

- We develop a dynamic programming framework that can protect smart meter data privacy in a cost-effective manner, while taking into account the impact of repeated charging and discharging on the battery’s lifetime. One major challenge in solving this problem is the need for...
the knowledge of future electricity consumption events. Moreover, due to the finite battery capacity, the control actions at all slots are coupled. It turns out that had the charging/discharging constraints been relaxed, the average battery charging and discharging power would be evened out. By exploiting this structure, we recast the original problem as a decoupled optimization problem with a larger feasible set, such that the optimal control action at each slot can be solved by using only the current observations, and it is also optimal for the original problem, if it is feasible for the original problem.

- By carefully constructing the Lyapunov function, we develop an online control algorithm to solve the decoupled optimization problem based on the Lyapunov optimization technique [16], [17], such that the control action always lies in the feasible region of the original problem. The proposed online control algorithm requires solution of a mixed-integer nonlinear program, in order to consider the cost of repeated charging and discharging on the battery’s lifetime. By decomposing the problem into multiple cases, a closed-form solution to each case of the mixed-integer nonlinear program is derived.

- We show that the proposed online control algorithm, parametrized by a positive value $V$, is within $O(1/V)$ of the optimal solution to the original problem, where the maximum value of $V$ is limited by the battery capacity. In this way, we quantify the impact of the battery capacity on the performance of the proposed online control algorithm. Using real data, we demonstrate the efficacy of our online control algorithm through extensive numerical exploration. The results corroborate that our algorithm can protect smart meter data privacy in a cost-effective manner.

### B. Related Work

Smart meter data privacy protection has been studied in a number of papers. Heuristic algorithms are developed to tackle this challenge (see, e.g., [9], [10], [14] and [13]). Kalogridis et al. [9] proposes a “best-effort” algorithm against power load changes by charging/discharging the battery to maintain the current load equal to the previous load. McLaughlin et al. [10] proposes a non-intrusive load leveling (NILL) algorithm to mask appliance features that are used by non-intrusive load monitoring (NILM) algorithms (e.g., [18]–[20]) to detect appliance switch-on/off events. None of these works have considered optimal control algorithms to protect smart meter data privacy. Moreover, as pointed out in [14], the methods proposed in [9] and [10] suffer from precise load change recovery attacks, due to the leakage of load-change information. The information leakage is mainly due to the fact that the “best-effort” algorithm in [9] attempts to maintain the current load at all times, and that the NILL algorithm in [10] often encounters a period of peak loads. In contrast, the online control algorithm proposed here determines the observed load profile by solving a well-designed optimization problem with unobservable parameters, without which the original load profile cannot be recovered. Therefore, the proposed approach does not suffer from precise load change recovery attacks [14].

Recent work [13] proposes a Monte Carlo simulation based approach to jointly optimize the cost of electricity and privacy. Furthermore, [11] proposes wallet-friendly privacy protection for smart meters by using stochastic dynamic programming. Along a different line, recent work [12] protects smart meter data privacy by using energy harvesting and storage devices from an information theoretic perspective. All these works require the knowledge of the statistics of the load requirements or the electricity prices, which may not be readily available. Moreover, the solution to dynamic programming requires solution of a value function that can be computationally difficult when the state space of the system is large and hence suffers from the curse of dimensionality.

Unlike the prior works, this study proposes a low complexity online control algorithm that is within $O(1/V)$ of the optimal solution, without the knowledge of the statistics of the time-varying load requirements and the electricity price processes.

The rest of the paper is organized as follows. In Section II, we describe the system model. In Section III, we propose a cost-effective and privacy-preserving energy management framework. In Section IV, we propose an optimal online control algorithm. In Section V, we evaluate the performance of the proposed online control algorithm using real data. The paper is concluded in Section VI.

### II. System Model

We consider a discrete-time system, in which the length of each time slot matches the typical sampling and operation time scale of the smart meter. An overview of the system model is given in Fig. 1. We assume that a smart home contains an energy storage device (battery) and a power controller that can control the combination of power drawn from the utility and the battery to satisfy the load requirements. In the following, the model of each component in Fig. 1 is described in detail, aiming to optimize the load profile to mask individual consumption events in a cost-effective manner.

#### A. Battery Model

We denote by $B_{\text{max}}$ the battery capacity, by $B(t)$ the energy level of the battery at slot $t$, and by $P_B(t)$ the power charged to (when $P_B(t) > 0$) or discharged from (when $P_B(t) < 0$) the battery during slot $t$. Assume that the battery energy leakage is negligible, which is a reasonable assumption, since the time scale over which the loss takes place (e.g., about 3-20% a month for lead-acid batteries) is much larger than the minute-level time scale. Then, the dynamics of the battery energy level
can be expressed as
\[ B(t + 1) = B(t) + P_B(t). \] (1)

Since the charging and discharging rates of the battery are physically constrained, we denote by \( P_B^{\text{max}} \) the maximum charging rate and by \( P_B^{\text{min}} \) the maximum discharging rate. \( P_B^{\text{max}} \) and \( P_B^{\text{min}} \) are positive constants depending on the physical properties of the battery. Therefore, we have the following constraint on \( P_B(t) \):
\[ -P_B^{\text{min}} \leq P_B(t) \leq P_B^{\text{max}}. \] (2)

Since the battery energy level should always be nonnegative and cannot exceed the battery capacity, we need to ensure that in each slot \( t \),
\[ 0 \leq B(t) \leq B^{\text{max}}. \] (3)

Based on constraints (1), (2) and (3), we have the following equivalent constraints in each slot \( t \) for \( P_B(t) \):
\[ P_B(t) \geq -\min\{P_B^{\text{min}}, B(t)\}, \] (4)
\[ P_B(t) \leq \min\{P_B^{\text{max}}, B^{\text{max}} - B(t)\}. \] (5)

For simplicity, a basic battery model (cf. [21]) is used to model the cost of repeated charging and discharging. It is assumed that the number of charging/discharging cycles for each battery is limited. Given the battery’s cost and the number of charging/discharging cycles, the cost of repeated charging and discharging on the battery’s lifetime can be calculated using the battery’s cost divided by the number of charging/discharging cycles, i.e., an amortized cost \( C_B \) (in unit of dollars) incurred with each charging or discharging. Therefore, at each slot, an operating cost of \( C_B \) is incurred whenever the battery is charging \((P_B(t) > 0)\) or discharging \((P_B(t) < 0)\). Nevertheless, the results in the paper can be easily generalized to more complicated battery models.

B. Load Model

We denote by \( L(t) \) the residential load generated at slot \( t \). \( L(t) \) is assumed to be independent and identical distributed (i.i.d.) over time slots with some unknown probability distribution. Assume that \( L(t) \) is deterministically bounded by a finite constant \( L^{\text{max}} \), so that
\[ L(t) \leq L^{\text{max}}, \forall t. \] (6)

\( L^{\text{max}} \) can be determined by the total power consumption of all electrical appliances in the home.

With the battery, the total power used to serve the load is given by
\[ L(t) = P(t) - P_B(t), \] (7)
where \( P(t) \) denotes the power drawn from the grid at slot \( t \). Assume that the maximum amount of power that can be drawn from the grid in any slot is upper bounded by \( P^{\text{max}}, \) i.e.,
\[ 0 \leq P(t) \leq P^{\text{max}}, \forall t. \] (8)

Note that for the original scenario where the battery is not used, \( P^{\text{max}} \) must be larger than \( L^{\text{max}} \) in order to satisfy all residential loads. Therefore, we have \( P^{\text{max}} \geq L^{\text{max}} \).

C. Electricity Pricing Model

Since the electricity price is time-varying, e.g., time-of-use electricity pricing, the rechargeable battery offers an opportunity to reduce the cost of electricity, in addition to privacy protection. Let \( c(t) \) denote the cost per unit of power drawn from the grid at slot \( t \). We assume that \( c(t) \) evolves according to an i.i.d. process with some unknown distribution and is deterministically bounded by a finite constant \( c^{\text{max}} \), so that \( c(t) \leq c^{\text{max}} \).

Given the system model, our goal is to design a control algorithm that jointly optimizes the cost of power and smart meter data privacy, while meeting all the constraints. For ease of exposition, the load process and the electricity price process are assumed to be i.i.d. over slots. The algorithm developed for this case can be applied to non i.i.d. scenarios, as shown in Section V, where numerical studies are carried out in non i.i.d. environments. Our results can be generalized for the non i.i.d. case by using the delayed Lyapunov drift and \( T \) slot drift techniques developed in [16] and [17].

III. COST-EFFECTIVE AND PRIVACY-PRESERVING ENERGY MANAGEMENT

Based on the system model above, we now study cost-effective and privacy-preserving energy management.

A. Control Objective

With the use of a battery, the original load profile \( L(t) \) becomes \( P(t) = L(t) + P_B(t) \). Intuitively, in order to mask energy usage profiles, the modified load profile \( P(t) \) needs to be as “flat” as possible, as the privacy information is contained in appliance switch-on/off events, i.e., the differences between successive power measurements. If \( P(t) \) is equal to a constant value representing the average residential load, then all individual consumption events would be perfectly masked. Let \( T = \lim_{t \to \infty} \frac{1}{T} \sum_{\tau=1}^{T} L(\tau) \) denote the average residential load. In real time, by charging/discharging the battery, \( P(t) \) needs to be controlled with as little deviation from \( T \) as possible. Equivalently, the control objective is to minimize the “variance” of \( P(t) \), i.e.,
\[ \lim_{t \to \infty} \frac{1}{T} \sum_{\tau=1}^{T} \mathbb{E}\{(P(\tau) - T)^2\}. \] (9)

It can be seen that minimizing (9) generally requires the knowledge of all future consumption events due to \( T \). Heuristic algorithms (e.g., [9] and [10]) have been devised to mitigate this issue by charging/discharging the battery to make \( P(t) \) achieve a target load determined heuristically, which inevitably sacrifices performance. As discussed later in Proposition 1, by exploring the problem structure, this issue can be solved by transforming (9) into an equivalent problem.

Beyond privacy protection, the rechargeable battery can also be used to reduce the cost electricity, by utilizing the time-varying electricity price. In this paper, we jointly optimize the cost of electricity and smart meter data privacy. The
corresponding objective function can be expressed as
\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\left\{ c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta(P(\tau) - \bar{T})^2 \right\},
\]
where the indicator function \( \mathbf{1}_B(t) \) corresponds to the battery operation, i.e., \( \mathbf{1}_B(t) = 1 \) if \( P_B(t) \neq 0 \) (the battery is charging/discharging), otherwise \( \mathbf{1}_B(t) = 0 \). \( \beta \geq 0 \) is a parameter chosen by the customer to trade off between reducing the cost of electricity and protecting smart meter data privacy. For example, when \( \beta = 0 \), the customer optimizes only the cost of electricity; when \( \beta \) is large, the customer focuses more on protecting smart meter data privacy. Assuming that the limit exists in (10), our goal is to design a control algorithm that can minimize this time averaged cost subject to the constraints described in the system model.

B. Problem Formulation

The cost-effective and privacy-preserving energy management problem can be formulated as the following stochastic dynamic programming problem:

**P1:**
\[
\min \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\left\{ c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta(P(\tau) - \bar{T})^2 \right\}
\]
s.t. constraints (4), (5), (7), (8).

(11)

One major challenge of solving **P1** is the lack of knowledge of future time-varying load requirements and electricity prices. Moreover, due to the finite battery capacity constraints (4) and (5), the current control action would impact the future control actions, making it more challenging to solve **P1**. Note that the traditional approach based on dynamic programming (e.g., the approach in (11)) is difficult to apply, because it requires the statistics of load requirements and electricity prices, and the computational complexity may suffer from the curse of dimensionality. As the statistics of load requirements and electricity prices may not be known, in this paper, we develop an online control algorithm to solve this problem by using only the current observations, while taking into account the finite battery capacity constraints.

C. Problem Relaxation

To solve **P1**, we first consider a relaxed version of **P1**, by using (2) to relax the constraints (4) and (5). Define the average expected charging or discharging rate of the battery as
\[
\bar{T}_{B} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{P_B(\tau)\}.
\]
Since the battery energy evolves based on (1), summing over all \( t \) and taking expectation of both sides, we have
\[
\mathbb{E}\{B(t + 1)\} - B(1) = \sum_{\tau=1}^{t} \mathbb{E}\{P_B(\tau)\},
\]
where \( B(1) \) is the initial battery energy level. Since the battery capacity is finite, dividing both sides by \( t \), and taking \( t \to \infty \) in (13) yields
\[
\bar{T}_{B} = 0.
\]
(14)

Thus, we have the following relaxed problem:

**P2:**
\[
\min \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta(P(\tau) - \bar{T})^2\}
\]
s.t. constraints (2), (7), (8).

(15)

One main challenge of solving **P2** is that it requires the knowledge of all future consumption events to compute \( \bar{T} \). Now that \( \bar{T}_{B} = 0 \), the average charging power is equal to the average discharging power, indicating that the battery does not consume energy on average. By exploiting this structure, we show in the following proposition that the decision variable \( P(t) \) is independent of \( \bar{T} \). Therefore, **P2** can be solved without using the knowledge of \( \bar{T} \).

**Proposition 1:** Consider the following stochastic optimization problem:

**P3:**
\[
\min \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{c(\tau)P(\tau) + \mathbf{1}_B(\tau)C_B + \beta(P(\tau))^2\}
\]
s.t. constraints (2), (7), (8).

(16)

Any optimal solution to **P3** is also optimal for **P2** and vice versa.

**Proof:** Since any feasible solution to **P3** is also feasible for **P2** and vice versa, it is sufficient to show that the objective functions of **P3** and **P2** differ by at most a constant independent of \( P(t) \). Equivalently, it is sufficient to show that
\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{\bar{T} - \bar{T}P(\tau)\} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{P(\tau)\}
\]
(17)
is a constant independent of the choice of \( P(t) \). Since \( \bar{T} \) is a constant depending only on \( L(t) \), it is sufficient to show that
\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{P(\tau)\}
\]
is a constant independent of the choice of \( P(t) \).

Since \( P(t) = L(t) + P_B(t) \), summing over all \( t \), taking the expectation of both sides, dividing both sides by \( t \) and taking \( t \to \infty \), we have
\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{P(\tau)\} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{L(\tau)\}
\]
\[
+ \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{P_B(\tau)\}
\]
\[
= \bar{T} + \bar{T}_{B} = \bar{T}.
\]
(19)

Since \( \bar{T}_{B} = 0 \), we have
\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\{P(\tau)\} = \bar{T},
\]
(20)
thereby concluding the proof.

From Proposition 1, any feasible solution to **P1** is also a feasible solution to **P3**, since **P2** is equivalent to **P3** and the feasible set of **P2** is larger than that of **P1**. Let \( \phi_{\text{rel}} \) and \( \phi_{\text{opt}} \) denote the optimal objective value of the relaxed problem **P3** and that of **P1**, respectively. Since **P3** is less constrained than **P1**, the optimal value of **P3** will not exceed that of **P1** by
more than a constant $\beta T^2$ that can be computed based on Proposition 1, i.e., $\phi_{rel} \leq \phi_{opt} - \beta T^2$.

Note that $P_3$ is a decoupled control problem, in the sense that the optimal control actions $P(t)$ and $P_B(t)$ at each slot can be determined purely by the current state $L(t)$ and $c(t)$. Specifically, it can be shown that there exists a stationary, randomized policy that can achieve the optimal solution to $P_3$ as presented in the following lemma.

**Lemma 1**: If $L(t)$ and $c(t)$ are i.i.d. over time slots, then there exists a stationary, randomized policy that makes control decisions $P_{stat}(t)$ and $P_{stat}^B(t)$ at every slot purely as a function (possibly randomized) of the current state $L(t)$ and $c(t)$, while satisfying the constraints of $P_3$ and providing the following guarantees:

$$
\mathbb{E}(P_{stat}^B(t)) = 0,
$$
$$
\mathbb{E}(c(t)P_{stat}(t) + 1_{\{P_{stat}(t) \neq 0\}}C_B + \beta P_{stat}(t)^2) = \phi_{rel},
$$
where the expectations above are with respect to the stationary distributions of $L(t)$ and $c(t)$, and the randomized control decisions.

The proof of Lemma 1 follows from the framework in [17] and is omitted for brevity. Note that the above stationary, randomized policy may not be feasible for the original problem $P_1$, as the constraints (4) and (5) may be violated. However, the existence of such a policy can be used to design our online control policy that meets all constraints of $P_1$ and derive a performance guarantee for our algorithm as shown later in Theorem 1.

### IV. Optimal Online Control Algorithm

In this section, we design an online control algorithm that can achieve the optimal solution to $P_1$ asymptotically. The proposed online control algorithm is designed based on $P_3$, and uses a control parameter $V > 0$ to quantify the impact of the battery capacity on the performance of the algorithm as discussed later. The key idea of our algorithm is to construct a Lyapunov function with a perturbed weight for determining the control actions. By carefully designing the weight, we can show that whenever the battery is charged or discharged, the battery energy level always lies in the feasible region of $P_1$.

From Proposition 1, if the optimal solution to $P_3$ can be found and lies in the feasible region of $P_1$, it is also optimal for $P_1$.

To this end, we define a perturbed variable $U(t)$ to track the battery energy level as

$$
U(t) = B(t) - V(c^\text{max} + 2\beta L^\text{max}) - P_{\text{min}}^B.
$$

Note that $B(t)$ is the actual battery energy level at slot $t$ and $U(t)$ is simply a shifted version of $B(t)$ with the same dynamics:

$$
U(t + 1) = U(t) + P_B(t).
$$

#### A. Lyapunov Optimization

We define the following Lyapunov function: $Z(U(t)) = \frac{1}{2} U(t)^2$. The corresponding conditional Lyapunov drift can be defined as follows:

$$
\Delta(U(t)) = \mathbb{E}\{Z(U(t + 1)) - Z(U(t))|U(t)\}.
$$

Following the drift-plus-penalty framework [17], our online control algorithm is designed to minimize an upper bound on the following function, in order to achieve the optimal solution to $P_1$:

$$
\Delta(U(t)) + V \mathbb{E}\{c(t)P(t) + 1_B(t)C_B + \beta P(t)^2|U(t)\}.
$$

(24)

The following lemma provides an upper bound for (24).

**Lemma 2**: (Drift Bound) For any control policy that satisfies the constraints of $P_3$, we have

$$
\Delta(U(t)) + V \mathbb{E}\{c(t)P(t) + 1_B(t)C_B + \beta P(t)^2|U(t)\} \leq K + U(t)\mathbb{E}\{P_B(t)|U(t)\} + V \mathbb{E}\{c(t)P(t) + 1_B(t)C_B + \beta P(t)^2|U(t)\},
$$

(25)

where the constant $K$ is defined as

$$
K = \frac{1}{2} \max\{(P_{\text{min}}^B)^2, (P_{\text{max}}^B)^2\}.
$$

Proof: Squaring both sides of (22), dividing by 2, and rearranging, we have

$$
\frac{U(t+1)^2 - U(t)^2}{2} = \frac{1}{2} P_B(t)^2 + U(t)P_B(t).
$$

Since $-P_{\text{min}}^B \leq P_B(t) \leq P_{\text{max}}^B$, we have

$$
\frac{U(t+1)^2 - U(t)^2}{2} \leq \frac{1}{2} \max\{(P_{\text{min}}^B)^2, (P_{\text{max}}^B)^2\} + U(t)P_B(t).
$$

Taking conditional expectations of the above and adding $V \mathbb{E}\{c(t)P(t) + 1_B(t)C_B + \beta P(t)^2|U(t)\}$ to both sides, the proof is concluded.

#### B. Online Control Algorithm

The design principle of our online control algorithm is to minimize the right-hand-side of the drift-plus-penalty bound (25) subject to the constraints of $P_3$ at each slot $t$, by observing the current state $U(t)$, $L(t)$, and $c(t)$. The online control algorithm chooses the control actions $P_B(t)$ and $P(t)$ as the solution to the following optimization problem:

$$
P_4: \min U(t)P_B(t) + V(c(t)P(t) + 1_B(t)C_B + \beta P(t)^2) \text{ s.t. } -P_{\text{min}}^B \leq P_B(t) \leq P_{\text{max}}^B,
$$

$$
0 \leq P(t) \leq P_{\text{max}},
$$

$$
L(t) = P(t) - P_B(t).
$$

(27)

It can be observed that for each slot $t$, $P_4$ is a mixed-integer nonlinear program. To solve it, we consider the optimal values of the objective in $P_4$ for two modes with and without charging/discharging respectively, and then choose the mode that yields the lowest value of the objective. The corresponding control actions of the mode can then be implemented in real time.

Let $P_B^*(t)$ and $P^*(t)$ denote the optimal solution to $P_4$. For each case, $P_B^*(t)$ and $P^*(t)$ can be characterized by using the Karush-Kuhn-Tucker (KKT) conditions [22]. The optimal solution to $P_4$ is given as follows:

1) The case without charging/discharging ($1_B(t) = 0$): Let $\theta_1(t)$ denote the optimal value of the objective in $P_4$ without charging/discharging. In this case, we have $P_B^*(t) = 0$ and $P^*(t) = L(t)$. $\theta_1(t)$ can be calculated as

$$
\theta_1(t) = V(c(t)P^*(t) + \beta P^*(t)^2).
$$

(28)
Algorithm 1 Online control algorithm

**Initialization:** Given the initial battery charge level \(0 \leq B_{init} \leq B_{max}\), set \(t=1\) and compute \(U(1)\) based on (21).

**For each time slot \(t\)**

1. Compute \(\theta_1(t)\) and \(\theta_2(t)\) based on (28) and (29).
2. Choose the control decisions \(P_B^{*}(t)\) and \(P^{*}(t)\) associated with the lower value of \(\theta_1(t)\) and \(\theta_2(t)\).
3. Update \(U(t)\) according to (22).

2) The case with charging/discharging (1\(B(t) = 1\)):
Let \(\theta_2(t)\) denote the optimal value of the objective in \(P_4\) with charging/discharging. Letting \(UB = \min\{P_B^{max}, P_{max} - L(t)\}\) and \(LB = \max\{-P_{min}, -L(t)\}\), \(P_B^{*}(t)\) and \(P^{*}(t)\) can be calculated as follows:

- If \(\frac{U(t)+V(c(t)+2\beta(t)L(t))}{2}\) > \(UB\), then \(P_B^{*}(t) = UB\) and \(P^{*}(t) = P_B^{*}(t) + L(t)\).
- If \(UB \leq \frac{U(t)+V(c(t)+2\beta(t)L(t))}{2}\) < \(LB\), then \(P_B^{*}(t) = \frac{U(t)+V(c(t)+2\beta(t)L(t))}{2}\) and \(P^{*}(t) = P_B^{*}(t) + L(t)\).
- If \(UB \leq \frac{U(t)+V(c(t)+2\beta(t)L(t))}{2}\) ≤ \(LB\), then \(P_B^{*}(t) = LB\) and \(P^{*}(t) = P_B^{*}(t) + L(t)\).

Given \(P_B^{*}(t)\) and \(P^{*}(t)\), \(\theta_2(t)\) can be calculated as follows:

\[
\theta_2(t) = U(t)P_B^{*}(t) + V(C_B + c(t))P^{*}(t) + \beta P^{*}(t)^2. \tag{29}
\]

After computing \(\theta_1(t)\) and \(\theta_2(t)\), we choose the lower value of \(\theta_1(t)\) and \(\theta_2(t)\) and the corresponding control actions. A detailed description of the online control algorithm is given in Algorithm 1.

C. Performance Analysis

In this section, we analyze the feasibility and performance of our online control algorithm. We define an upper bound \(V_{max}\) on parameter \(V\) as follows:

\[
V_{max} = \frac{B_{max} - P_B^{max} - P_{min}}{c_{max} + 2\beta L_{max}}. \tag{30}
\]

Next, the optimal solution to \(P_4\) has the following properties that are useful for the performance analysis.

**Lemma 3:** The optimal solution to \(P_4\) has the following properties:

- If \(U(t) \geq 0\), then \(P_B^{*}(t) \leq 0\).
- If \(U(t) \leq -V(c_{max} + 2\beta L_{max})\), then \(P_B^{*}(t) \geq 0\).

**Proof:** If \(U(t) \geq 0\), then \(P_B^{*}(t)\) can be either \(LB\) or \(-\frac{U(t)+V(c(t)+2\beta L(t))}{2}\). Since \(LB \leq 0\) and \(-\frac{U(t)+V(c(t)+2\beta L(t))}{2}\) ≤ 0, we have \(P_B^{*}(t) \leq 0\).

If \(U(t) \leq -V(c_{max} + 2\beta L_{max})\), then \(P_B^{*}(t)\) can be either \(UB\) or \(-\frac{U(t)+V(c(t)+2\beta L(t))}{2}\). Since \(UB \geq 0\) and \(-\frac{U(t)+V(c(t)+2\beta L(t))}{2}\) ≥ 0, we have \(P_B^{*}(t) \geq 0\).

Then, we have the following results.

**Theorem 1:** Suppose the initial battery charge level \(B_{init}\) satisfies \(0 \leq B_{init} \leq B_{max}\). Implementing the above online control algorithm with any fixed parameter \(0 < V \leq V_{max}\) for all \(t\), we have the following performance guarantees:

1) The battery energy level \(B(t)\) is always in the range \(0 \leq B(t) \leq B_{max}\) for all \(t\).
2) All control decisions are feasible for \(P_1\).
3) If \(L(t)\) and \(c(t)\) are i.i.d. over slots, then the time-average expected cost under the proposed online control algorithm is within \(K/V\) of the optimal value:

\[
\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E}\left\{c(\tau)P(\tau) + 1_{B(\tau)C_B + \beta P(\tau)^2}\right\} \leq \mathbb{E}\left[Z(U(t))\right] + \mathbb{E}\left[Z(U(1))\right].
\]

where \(K\) is a constant given in (26).

**Proof:** 1) Proof of Part 1: To show \(0 \leq B(t) \leq B_{max}\) for all \(t\), it suffices to show that \(-V(c_{max} + 2\beta L_{max}) = P_{min}\). Since \(0 \leq B(t) \leq B_{max}\), it is easy to verify \(-V(c_{max} + 2\beta L_{max}) = P_{min}\) for all \(t\) according to (22). As \(0 \leq B_{init} \leq B_{max}\), we have the following performance guarantees:

\[
(U(t) \leq \frac{B_{max} - V(c_{max} + 2\beta L_{max})}{2}) \land (P_{min} \leq U(t) \leq B_{max} - V(c_{max} + 2\beta L_{max}) - P_{min}).
\]

Then, we show \(-V(c_{max} + 2\beta L_{max}) - P_{min} \leq U(t) \leq B_{max} - V(c_{max} + 2\beta L_{max}) - P_{min}\) by induction. Suppose that \(-V(c_{max} + 2\beta L_{max}) - P_{min} \leq U(t) \leq B_{max} - V(c_{max} + 2\beta L_{max}) - P_{min}\) holds for slot \(t\). We need to show that it also holds for slot \(t+1\).

First, if \(0 \leq U(t) \leq B_{max} - V(c_{max} + 2\beta L_{max}) - P_{min}\), then \(P_B^{*}(t) \leq 0\) based on Lemma 3. Therefore, using (22), we have \(U(t+1) \leq U(t) \leq B_{max} - V(c_{max} + 2\beta L_{max}) - P_{min}\). If \(U(t) \leq 0\), then \(U(t+1) \leq P_{max}\) based on (22), since the maximum discharging rate is \(P_B^{max}\). Using (30), for any \(0 < V \leq V_{max}\) we have \(B_{max} - V(c_{max} + 2\beta L_{max}) \geq B_{max} - P_{min} = V(c_{max} + 2\beta L_{max}) \geq P_{max}\). Therefore, we have \(U(t+1) \leq B_{max} - V(c_{max} + 2\beta L_{max})\).

Second, if \(-V(c_{max} + 2\beta L_{max}) - P_{min} \leq U(t) \leq -V(c_{max} + 2\beta L_{max})\), then \(P_B^{*}(t) \geq 0\) based on Lemma 3. Then, using (22), we have \(U(t+1) \geq U(t) \geq -V(c_{max} + 2\beta L_{max}) = P_{min}\). If \(U(t) \geq -V(c_{max} + 2\beta L_{max})\), \(U(t+1) \geq -V(c_{max} + 2\beta L_{max}) \geq P_{min}\) based on (22), since the maximum discharging rate is \(P_B^{max}\). Therefore, we have \(-V(c_{max} + 2\beta L_{max}) - P_{min} \leq U(t+1)\), which concludes the proof.

2) Proof of Part 2: From the proof above, the constraint on \(B(t)\) is satisfied for all \(t\). Since the control decisions we make satisfy all the constraints in \(P_3\), with \(0 \leq B(t) \leq B_{max}\) for all \(t\), the constraints in \(P_1\) are also satisfied. Therefore, our control decisions are feasible for \(P_1\).

3) Proof of Part 3: The proposed online control algorithm is designed to minimize the right-hand-side of (25) over all possible feasible control policies, including the optimal, stationary policy given in Lemma 1. Therefore, we have the following:

\[
\Delta(U(t)) + V\sum_{\tau=1}^{t} \mathbb{E}\left\{c(\tau)P(t) + 1_{B(t)C_B + \beta P(t)^2}(t)\right\} \leq Kt + V\sum_{\tau=1}^{t} \mathbb{E}\left[Z(U(t))\right] + \mathbb{E}\left[Z(U(1))\right].
\]

Taking the expectation of both sides, using the law of iterative expectation, and summing over all \(t\), we have:

\[
\sum_{\tau=1}^{t} \mathbb{E}\left\{c(\tau)P(t) + 1_{B(t)C_B + \beta P(t)^2}(t)\right\} \leq Kt + Vt\mathbb{E}\left[Z(U(t))\right] + \mathbb{E}\left[Z(U(1))\right].
\]

Dividing both sides by \(V\), taking the limit as \(t \to \infty\) and using the facts that \(\mathbb{E}\left[Z(U(t))\right] = \text{finite} \) and \(\mathbb{E}\left[Z(U(1))\right] = \text{finite}\).
nonnegative, we have
\[ \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=1}^{t} \mathbb{E} \{ c(\tau) P(\tau) + 1_B(\tau) C_B + \beta P(\tau)^2 \} \leq \phi_{rel} \frac{K}{V}. \]

Theorem 1 shows that by choosing a larger \( V \), the time-average expected cost under the proposed online control algorithm can be pushed closer to the optimal solution to P1. Since \( V^{\max} \) is determined by the battery capacity, (31) quantifies the impact of the battery capacity on the performance of the proposed online control algorithm. The larger the battery capacity is, the better the proposed online control algorithm can optimize smart meter data privacy and the cost of electricity.

Remarks: Note that Theorem 1 holds for all sample paths, including sample paths generated by \( L(t) \) and \( c(t) \) that are non i.i.d. over slots. The proposed online control algorithm can be applied to non i.i.d. scenarios, as demonstrated in Section V, where numerical studies are carried out in non i.i.d. environments. Our results can be generalized for the non i.i.d. case by using the delayed Lyapunov drift and \( T \) slot drift techniques developed in [16] and [17].

V. CASE STUDIES

A. Data and Simulation Setting

We evaluate the performance of the proposed algorithm by using the real-time measurements at a Georgian apartment [9] and the load profile constructed based on a domestic electricity demand model [23]. The time resolution of the load profile is one-minute. The actual load profile given by the real-time measurements represents the case in which the power usage is high, while the constructed load profile based on the domestic electricity demand model [23] represents the case in which the power usage is low. The electricity prices are set according to SRP’s residential time-of-use price plan [24]. The on-peak price is 21.09 cent per kWh during 1:00 PM to 8:00 PM, while the off-peak price is 7.04 cent per kWh for the remaining time of a day. In the simulation, we fix the parameters \( P_{B}^{\min} = P_{B}^{max} = 6 \) kW, \( P_{C}^{max} = 10 \) kW and \( C_B = 0.1 \) cent.

B. Privacy Protection and Power Cost Saving

Based on the two data sets, Fig. 2 compares load profiles given by our algorithm under different values of \( \beta \), where \( B^{\max} = 12 \) kWh, and \( V = V^{\max} \). when \( B^{\max} = 100 \) kWh and \( \beta = 1 \), where the slow-time-scale fluctuations are smoothed out.

When \( \beta = 0 \), our algorithm minimizes only the cost of electricity. As shown in Fig. 2, the proposed algorithm stores the energy in the battery in the off-peak price period and uses the energy in the battery as much as possible in the on-peak price period to reduce the cost of electricity.

Fig. 5 illustrates the tradeoff between the cost of electricity and smart meter data privacy under different values of \( \beta \) using the actual load profile. In Fig. 5, the privacy protection is quantified by using the standard deviation of \( P(t) \) given by the proposed algorithm. As \( \beta \) increases, the proposed algorithm focuses more on data privacy, and the resulting \( P(t) \) becomes more “flat”. Accordingly, the cost of electricity increases with \( \beta \). Fig. 5 can be used for the user to determine the value of \( \beta \) that he or she would consider appropriate.

C. Privacy Protection vs. Battery Capacity

As discussed, the privacy information is contained in appliance switch-on/off events, i.e., the differences between successive power measurements. Let \( dP(t) = P(t) - P(t-1) \) represent the difference between successive power measurements. NILM algorithms (e.g., [18]–[20]) explore the differences between successive power measurements to identify the
appliance switch-on/off events. These differences are called features. In Table I, we compare the number of features in the load profiles given by the proposed online control algorithm under different values of $\beta$ and battery capacity $B^{\max}$ with the NILL algorithm [10] and the best-effort algorithm [9], in which the differences less than 50 W (lights) are not accounted for. From Table I, it is observed that features are significantly reduced by the proposed online control algorithm as $\beta$ increases. When $\beta = 10^{-5}$ and $B^{\max} = 24$ kWh, only two features are left, compared with 393 features in the original load profile. Although in terms of feature reduction, the NILL algorithm [10] and the best-effort algorithm [9] can perform comparable to the proposed online control algorithm, the system cost given by the proposed online control algorithm is much lower than these algorithms as demonstrated in the next section. Moreover, as pointed out in [14], the methods proposed in [9] and [10] suffer from precise load change recovery attacks such that the original load profile can be recovered using the leakage of load-change information from these algorithms. In contrast, the proposed online control algorithm determines the observed load profile by solving a well-designed optimization problem with unobservable parameters ($U(t)$, $V$, and $\beta$), without which the original load profile cannot be recovered.

### D. System Cost vs. Battery Capacity

We now study the impact of the battery capacity on the performance of the proposed algorithm. As shown in Theorem 1, the system cost under the proposed online control algorithm converges to the optimal value as the battery capacity increases. Based on (30), if the electricity price is given, $V^{\max}$ is determined by the battery capacity. For different values of the battery capacity, the system cost is evaluated by using the actual load profile. To examine the convergence of the proposed algorithm, the system cost given by the proposed algorithm is normalized by the optimal offline solution to $P_1$.
The optimal offline solution to $P_1$ is solved by assuming that the load profile and the electricity price are known perfectly. Fig. 6 compares the normalized system cost given by the proposed algorithm with the NILL algorithm [10] and the best-effort algorithm [9]. As illustrated in Fig. 6, the normalized system cost given by the proposed algorithm converges to the optimal system cost. Furthermore, it is observed from Fig. 6 that the performance of the proposed algorithm with a battery of small capacity is reasonably good. When the battery capacity is 10 kWh, the system cost given by the proposed algorithm with a battery of small capacity is reasonably good.

VI. CONCLUSION

This paper has studied cost-effective smart meter data privacy protection by using batteries. A dynamic programming framework has been designed for consumers to jointly protect smart meter data privacy and reduce the cost of the electricity. By exploring the underlying structure of the original problem, an equivalent problem has been derived, which can be solved by using only the current observations. Then an online control algorithm has been developed to solve the equivalent problem based on the Lyapunov optimization technique. It has been shown that without requiring any knowledge of the statistics of the load requirements and electricity prices, the proposed online control algorithm is within $O(1/V)$ of the optimal solution to the original problem, where the maximum value of $V$ is limited by the battery capacity. Using real data, numerical results have corroborated that our algorithm can protect smart meter data privacy in a cost-effective manner.

For future work, it is of great interest to integrate demand response management into the current framework. Another interesting direction is to integrate renewable generation into the energy management to protect smart meter data privacy as considered in [12].

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REFERENCES


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