Design and Performance Analysis of Fiber Wireless Networks

by

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ABSTRACT

A Fiber-Wireless (FiWi) network integrates a passive optical network (PON) with wireless mesh networks (WMNs) to provide high speed backhaul via the PON while offering the flexibility and mobility of a WMN. Generally, increasing the size of a WMN leads to higher wireless interference and longer packet delays. The partitioning of a large WMN into several smaller WMN clusters, whereby each cluster is served by an Optical Network Unit (ONU) of the PON, is examined. Existing WMN throughput-delay analysis techniques considering the mean load of the nodes at a given hop distance from a gateway (ONU) are unsuitable for the heterogeneous nodal traffic loads arising from clustering. A simple analytical queuing model that considers the individual node loads to accurately characterize the throughput-delay performance of a clustered FiWi network is introduced. The accuracy of the model is verified through extensive simulations. It is found that with sufficient PON bandwidth, clustering substantially improves the FiWi network throughput-delay performance by employing the model to examine the impact of the number of clusters on the network throughput-delay performance. Different traffic models and network designs are also studied to improve the FiWi network performance.

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Chapter 1

INTRODUCTION

Fiber-Wireless (FiWi) networks have gained much attention in recent years due to their high-throughput and low-delay properties provided by the optical backhaul network while the wireless mesh network (WMN) provides easy set-up and flexible coverage in the last mile of the network (Ghazisaidi and Maier, 2011). For the optical backhaul network, the passive optical network (PON) is an important optical access technology and several PON technologies, such as Gigabit PON (GPON) and Ethernet PON (EPON), have been standardized (Effenberger *et al.*, 2007). For both GPON and EPON, time-division multiple access (TDMA) is applied to the upstream traffic and dynamic bandwidth allocation (DBA) may be applied to flexibly utilize the bandwidth (Aurzada *et al.*, 2011; Bontozoglou *et al.*, 2013; McGarry and Reisslein, 2012; Seoane *et al.*, 2012; Sivakumar *et al.*, 2013; Skubic *et al.*, 2009). GPON and EPON are both capable of providing service rates greater than 1 Gb/s and new architectures of next-generation PONs (NG-PONs) have been designed to provide larger throughput to satisfy the growing demand for bandwidth (Jimenez *et al.*, 2012; Maier *et al.*, 2012; Rawshan and Park, 2013; Sue *et al.*, 2014).

A WMN provides low cost, easy maintenance, robustness, and flexibility in the last mile of a FiWi network (Akyildiz *et al.*, 2005; Bruno *et al.*, 2005; Karrer *et al.*, 2004; Lee *et al.*, 2006). Since the WMN transports the upstream traffic to the optical backhaul network, its characteristics have a great effect on the FiWi network performance. It has been shown that the performance of a WMN is location dependent, whereby nodes with longer hop distance to the gateway (ONU) tend to suffer from higher delay and lower throughput (Gambiroza *et al.*, 2004; Lee *et al.*, 2008; Liu and

Liao, 2008). Multi-channel techniques are often applied with sophisticated routing mechanisms to reduce the delay (Draves *et al.*, 2004; Tang *et al.*, 2005). Another WMN research topic is the throughput-delay trade-off (Bansal and Liu, 2003; Gamal *et al.*, 2004; Grossglauser and Tse, 2002; Gupta and Kumar, 2000; Liu *et al.*, 2003). A throughput bound of a WMN is given in (Gupta and Kumar, 2000), while it is shown in (Grossglauser and Tse, 2002) that the per-node throughput increases by exploiting node mobility as multiuser diversity and an optimal throughput-delay tradeoff is derived in (Gamal *et al.*, 2004). The majority of the results in (Bansal and Liu, 2003; Gamal *et al.*, 2004; Grossglauser and Tse, 2002; Gupta and Kumar, 2000; Liu *et al.*, 2003) is for the asymptotic case and may not be suitable for analyzing a finite-size WMN.

Though many studies have examined research issues related to FiWi networks, as reviewed in Section 1.1, the effects of superimposing a FiWi network onto an existing WMN are still a relatively open research area. In this dissertation, we introduce a simple model to characterize the resulting clustered FiWi network. By modeling the wireless mesh nodes as queues, the traffic loads can be readily evaluated. Importantly, our investigations demonstrate that modeling WMN throughput-delay based on the average traffic load of the nodes at a given hop distance to the gateway (ONU) is inadequate for characterizing WMN clusters in FiWi networks with heterogeneous traffic loads at the nodes with a given hop distance to an ONU. We develop a novel WMN analysis based on the traffic loads at the individual nodes. The novel analysis employs elementary queueing theory yet gives a reasonably accurate characterization of the network behavior, as verified through simulation results. Through extensive numerical evaluations based on the novel analysis and verifying simulations we examine the trade-offs when superimposing the clustered FiWi network on a WMN. We find that with proper clustering, the clustered FiWi network substantially improves the throughput-delay performance compared to the existing WMN.

This dissertation is organized as follows. Section 1.1 gives a brief review of the related work on FiWi networks. In Chapter 2, the FiWi network model is described. Chapter 3 gives the mathematical delay and throughput analysis of the clustered FiWi network. In Chapter 4, we examine the accuracy of the proposed analytical model through comparisons with simulations and present discussions of network design strategies and guidelines. In Chapter 5, we study the analytical FiWi system performance with Poisson input traffic. The controlled input traffic design and network design at node level are presented in Chapter 6. In Chapter 7, the proposed network designs are examined via simulation and analytical results. Chapter 8 gives the conclusion of our study and the possible future work on of the clustered FiWi network.

1.1 Related Work

To increase the throughput of a WMN, different modified WMN architectures have been proposed and studied (Toumpis, 2004). One of the architectures is hybrid WMN, which consists of multiple wire-connected gateways and wireless mesh nodes. In (Agarwal and Kumar, 2004), it is shown that linear scaling of throughput can be approached in a two-tier hybrid network. Studies (Li *et al.*, 2009; Zemlianov and de Veciana, 2005) further studied the conditions for achieving the linear scaling of throughput. The number of hops, multi-hop uplinks, and failure tolerance in a hybrid WMN are examined in (Shila *et al.*, 2011). The downlink capacity of a hybrid cellular ad hoc network with fading channels is studied in (Law *et al.*, 2010). An asymptotic analysis of a hybrid WMN consisting of wireless mesh nodes and gateways has been conducted in (Wang and Abouzeid, 2011). The throughput of multi-tier hybrid WMN consisting of multiple gateways and different tiers of radio nodes is studied in (Zhao and Raychaudhuri, 2009; Zhou *et al.*, 2008). Most of the obtained results are for the asymptotic case, which studies the ideal number of gateways for an increasing number of nodes so as to ensure throughput scalability, and may not be applicable for the analysis for a fixed-size hybrid WMN.

Several related studies have focused on queueing analysis of specific MAC mechanisms in the contexts of wireless ad hoc and mesh networks, e.g., (Bisnik and Abouzeid, 2009; Fu *et al.*, 2008; Hu and Kuo, 2008; Lin *et al.*, 2012; Xie and Haenggi, 2009; Yang *et al.*, 2014). In contrast, our analysis considers a generic MAC model and focuses on the effects of partitioning a WMN into several clusters supported by ONUs. Another set of related studies has focused on the impact of routing. For instance, routing metrics for a WMN network have been defined in (Malnar *et al.*, 2014), while the QoS effects of multi-commodity flow modeling have been examined in (Liu *et al.*, 2014) and multicast is studied in (Liu and Liao, 2010). A capacity-aware route selection algorithm for increasing the throughput of a WMN has been proposed in (Bruno *et al.*, 2011). A strategy for redirecting traffic to different gateways has been proposed in (Lin *et al.*, 2011). Simulation evaluations of WMN routing have been reported in (Alwan, 2014; Ikeda *et al.*, 2013), while measurement evaluations have been conducted in (Ali *et al.*, 2008; Ho *et al.*, 2012).

Another set of related studies have developed queuing modeling approaches for WMNs. For instance, Wu et al. (Wu *et al.*, 2006) have derived bounds on the queueing delays in a WMN with specific linear and grid topology, while a tree topology is examined in (Liu *et al.*, 2012). Scheduling algorithms in WMNs are evaluated with an M/D/1 queue model for each WMN link in (Naeini, 2014). Chen et al. (Chen *et al.*, 2008) studied the delay bound violation probabilities of a WMN where each wireless mesh node is modeled as a single queue; in contrast we model the wireless mesh node as a combination of two $M/M/1/K$ queues so as to distinguish relay and locally generated traffic. Feng et al. (Feng *et al.*, 2009) studied the throughput and delay of a WMN with symmetric tree topology by applying a parallel-server queuing model. However, the parallel-server queuing model cannot describe the network behavior when the traffic loads are not balanced among the nodes with the same hop distance to the gateway. Asymptotic evaluations of the scaling behaviors of WMN have been examined in (Chien *et al.*, 2012), while a framework for WMN analysis based on network calculus is outlined in (Qi *et al.*, 2008; Wang *et al.*, 2012a). A clustering of a WMN with support from wired gateways similar to our study has been analyzed by Pandey et al. (Pandey *et al.*, 2013). Pandey et al. consider a specific load balancing approach and model only the gateway nodes, through M/M/1 queues. In contrast, we develop a more comprehensive queueing model encompassing all the wireless mesh network nodes and gateways.

A FiWi network is an example of a two-tier hybrid WMN. FiWi network technology choices and their implications for FiWi network structures have been extensively investigated (Ali *et al.*, 2010; Ghazisaidi and Maier, 2011; Sarkar *et al.*, 2007; Zheng *et al.*, 2009b). Specific routing and scheduling strategies for FiWi networks have been examined in (Dashti and Reisslein, 2014; He *et al.*, 2013; Honda *et al.*, 2011; Li *et al.*, 2011; Wang *et al.*, 2010; Zheng *et al.*, 2009a). Complementary throughput-delay analyses for specific medium access control and quality of service mechanisms in FiWi networks have been presented in (Aurzada *et al.*, 2014; Dhaini *et al.*, 2011, 2010; Fadlullah *et al.*, 2013; Lee *et al.*, 2014) We also briefly note for completeness that the analysis of energy saving mechanisms and their respective impact on FiWi network performance has gained increasing interest (Barradas *et al.*, 2013; Kantarci and Mouftah, 2012; Sankaran and Sivalingam, 2013; Togashi *et al.*, 2013). The present study complements the existing FiWi network literature in that it contributes a fundamental analysis of the throughput-delay implications of partitioning a given WMN into several WMN clusters, each supported by an ONU. The presented analysis thus provides an evaluation methodology for examining the implications of the WMN cluster structure in a FiWi network, and presents evaluation results for the trade-offs and interactions between WMN clusters and PON.

Chapter 2

WMN AND FIWI NETWORK ARCHITECTURE MODELS

In order to study the effect of superimposing the FiWi network onto an existing WMN, we first give the model of the existing WMN, which we refer to as the maternal network. We consider a maternal network consisting of N wireless mesh nodes and one gateway. All wireless mesh nodes operate on the same radio frequency and have transmission range r. The transmission rate of the wireless channel is W bits per second. Packets are forwarded (upstream) in a multihop fashion from a given source node to the gateway (downstream packets from the gateway to a destination node are not considered in our model).

To transform a WMN into a FiWi network, the maternal network is divided into Z non-overlapping clusters and one ONU serving as the gateway is placed within each cluster, see illustration for $Z = 3$ clusters in Fig. 2.1. We consider a clustering arrangement with a given number of clusters Z to be static, i.e., we do not consider dynamic on-the-fly changes of the clustering. We define a cluster as a contiguous region of the maternal network. Similar to the maternal network, packets are forwarded in a multihop fashion to the corresponding gateway within each cluster. In order to avoid an unfair advantage of the clustered FiWi network over the maternal network, we assume that all wireless nodes in both the maternal network and the clustered FiWi network still share the same radio frequency and wireless transmission bit rate W, and have the same transmission range r. When an upstream packet reaches the gateway (ONU), it enters a queue and waits for transmission out of the wireless mesh network. We note that the FiWi network is identical to the maternal network when

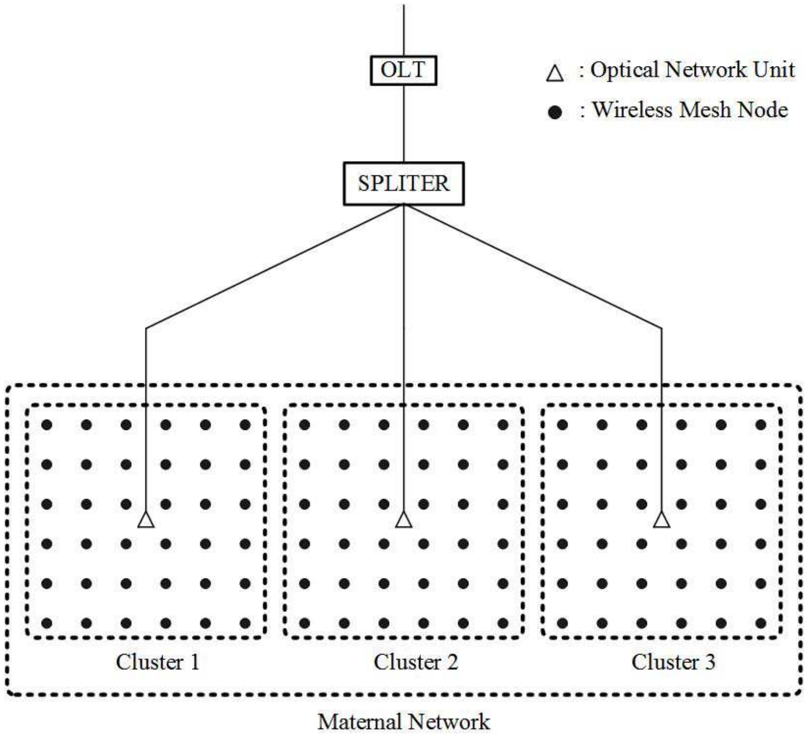


Figure 2.1: Illustration of a Clustered FiWi Network: The Original Maternal (Unclustered) Wireless Mesh Network (WMN) is Partitioned into $Z = 3$ Clusters. Each Cluster is Served by an Optical Network Unit (ONU) of the Passive Optical Network (PON).

$Z = 1$.

2.1 WIreless Mesh Node Model

We consider the heavy-loaded traffic model, which is commonly considered for tractability in WMN studies (Liu and Liao, 2008; Tu and Sreenan, 2008; Vieira *et al.*, 2012): each wireless mesh node is always backlogged with locally generated packets waiting to be transmitted. We model each wireless mesh node as the combination of two queues, as shown in Fig.2.2. Queue Q_r serves the relayed packets, while queue Q_s serves the locally generated packets (and is always backlogged). A given wireless mesh node $m_i,\ i = 1, \ldots, N$, forwards packets as follows:

1. If Q_r is empty, transmit a packet from the backlogged queue Q_s.

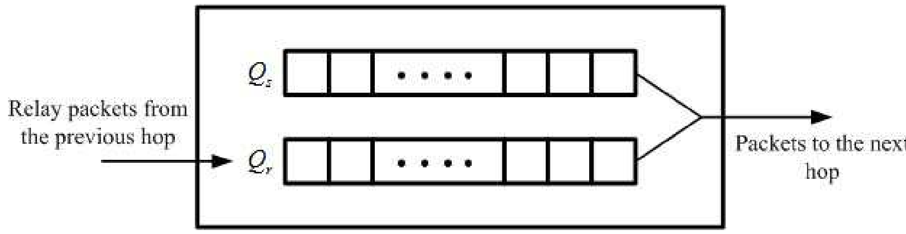


Figure 2.2: Queuing Model of a Wireless Mesh Node: Locally Generated (Source) Packets are Served by Queue Q_s, Whereas Queue Q_r Relays Packets from Other Nodes.

2. If Q_r is not empty, transmit a packet from Q_r with the forwarding probability q_i, or a packet from Q_s with probability $1 - q_i$.

2.2 Routing Protocol for Wireless Mesh Nodes and Relay Issues

Within each cluster, the shortest path routing protocol is applied in the WMN part and if one node has multiple next hop candidates, it randomly select one of them on a per-packet basis. Without loss of generality (Akyildiz *et al.*, 2005; Bruno *et al.*, 2005), we assume that the WMN part of each cluster is highly connected and robust and each wireless mesh node can find at least one path to its corresponding gateway in the cluster. We define an x-hop node as a wireless mesh node with hop distance x to its corresponding gateway. Liu and Liao (Liu and Liao, 2008) studied a pure WMN under the assumption that each x-hop node has to provide relay service for (nearly) the same number of $x+1$-hop nodes and can ask the same number of $x-1$-hop nodes to relay its outgoing packets. We do not consider this homogeneity assumption since it requires the WMNs to be set up in a specifically designed homogeneous topology and our results in Chapter 4 show that the homogeneity assumption fails to describe true WMN behaviors, especially when highly heterogenous WMNs are formed by dividing the maternal network into clusters.

2.3 Medium Access Control Protocol

2.3.1 WMN Part

We do not consider a specific MAC protocol for the WMN; instead, we consider the generic MAC model proposed in (Liu and Liao, 2008). The generic MAC model (Liu and Liao, 2008) describes the network behavior through the probability $p(x)$ of successful channel access. Specifically, $p(x)$ represents the probability of an x-hop node obtaining the transmission opportunity within one time slot for a time-division-multiplexing-access (TDMA) system. The values of the channel access probability $p(x)$ of a WMN are determined by many factors, including scheduling policies, interference from neighboring nodes, physical channel conditions, and MAC protocols (Bianchi, 2000). With proper measurements, one can find a matching set of $p(x)$ to describe a specific WMN. Due to the facts that (a) $p(x)$ is closely related to the throughput of both relayed and locally generated traffic at the x-hop nodes and (b) nodes with lower hop distance to the gateways have to provide relay service for heavier amounts of traffic, it is generally desired that the nodes with lower hop distances to the gateway have higher values of $p(x)$. In this dissertation, previously studied channel access probabilities $p(x)$ for a pure WMN (Liu and Liao, 2008) are examined to observe their effects in the FiWi environment.

2.3.2 Optical Part

For the ONUs serving as gateways at the clusters, we assume that all ONUs are identical and provide packet forwarding service at the same fixed transmission speed. We acknowledge that extensive research has examined dynamic bandwidth allocation (DBA) mechanisms for the upstream transmissions in PONs (Aurzada *et al.*, 2011; Bontozoglou *et al.*, 2013; McGarry and Reisslein, 2012; Seoane *et al.*, 2012; Sivakumar *et al.*, 2013; Skubic *et al.*, 2009). However, in an effort to expose the fundamental trade-offs between the wireless and optical parts in a tractable analysis, we consider an elementary PON upstream service, namely equal (fixed) bandwidth sharing by the ONUs. This simple model can be applied to a conventional WDM PON (Ma *et al.*, 2012; Rawshan and Park, 2013; Zhou *et al.*, 2012), whereby each ONU occupies a fixed portion of the total bandwidth of the upstream wavelength channels, or a TDMA PON, where each ONU is granted transmission permission during prescribed time slots. With the fixed transmission speed assumption, the ONUs can be modeled as $M/D/1/K$ queues. We note that this model applies also to other hybrid (wired-added) WMN networks with gateways operating at fixed transmission speed.

Chapter 3

MATHEMATICAL ANALYSIS OF THE FIWI NETWORK

In this chapter, we present the mathematical analysis of the FiWi network. First, we study the queuing at a specific wireless mesh node m_i, which leads to the derivation of the delay and throughput of the WMN. With the results obtained for the WMN, we further evaluate the delay and throughput of the PON. The overall performance of a FiWi network is obtained by combining results from both WMN and PON parts. The main analysis notations are listed in Table 3.1.

3.1 Packet Service Rates at Wireless Mesh Node m_i

We first study queuing behaviors at a given wireless mesh node m_i. A similar analysis based on the assumption of a homogeneous topology of WMN nodes that only considers the hop distance x from the gateway is conducted in (Liu and Liao, 2008). We generalize the analysis in (Liu and Liao, 2008) by considering individual nodes m_i so as to accurately model heterogeneous WMNs. We consider a TDMA-based system, where a packet is successfully transmitted within a time slot of duration t_c. The channel access probability p_i describes the probability that node m_i obtains the transmission opportunity within a given time slot. The length T_I of the random interval between two transmission opportunities at a given node m_i is then characterized by

$$P(T_I > kt_c) = (1 - p_i)^k, \tag{3.1}$$

Table 3.1: Summary of Main Notations

Notation	Definition
	Network structure
m_i	Wireless mesh node $i, i = 1, \ldots, N$
h_i	Hop distance from node m_i to the gateway
H	Largest hop distance of the network
S_x	$S_x = \{j : h_j = x \text{ for } j = 1, \ldots, N\}$ Set of indices of nodes with hop distance x
R_i	$R_i = \{j : m_i \text{ is a possible next hop of } m_j \text{ for } j = 1, \ldots, N\}$ Set of indices of possible previous hops of m_i
f_i	Number of possible next hops of node m_i
$N(x)$	Number of nodes with hop distance x
	Channel access and forwarding prob.
$p(x)$	Channel access probability of a wireless mesh node with hop distance x
$q(x)$	Forwarding probability of a wireless mesh node with hop distance x
	Packet traffic rates at node m_i
μ_i	Overall pkt. service rate, source + relay traffic
$\sigma_{r,i}$	Relay packet traffic output rate
$\sigma_{s,i}$	Source packet traffic output rate
λ_i	Relay packet traffic arrival rate
ρ_i	Relay packet traffic intensity at node m_i

where k denotes any positive integer. Replacing kt_c with t, we can rewrite Eqn. (3.1) as

$$P(T_I > t) = (1 - p_i)^{t/t_c}. \tag{3.2}$$

Binomial probabilities can be approximated by Poisson probabilities under appropriate conditions (Gross and Harris, 1998). In our network model, the arrival of transmission opportunities can be modeled as a Poisson process over a long time horizon. With this approximation, we can further rewrite Eqn. (3.2) as

$$P(T_I > t) \approx e^{-\mu_i t}, \tag{3.3}$$

where $\mu_i = 1/t_c \ln(1/(1 - p_i))$, thus, approximately,

$$\mu_i \approx \frac{p_i}{t_c}. \tag{3.4}$$

Note that μ_i denotes the arrival rate of transmission opportunities for node m_i and is equivalent to the service rate of packets of m_i. We note that the service rate μ_i is shared by the local packet queue Q_s and the relay packet queue Q_r of m_i.

As described in the network model in Section 2.1, the relay packet queue Q_r has probability of q_i to obtain a given transmission opportunity that has already been granted to node m_i. The service rate of relay packets in Q_r, is thus

$$\mu_{r,i} = \mu_i q_i.$$

When Q_r is empty, the transmission opportunity is automatically granted to Q_s. The effective relay packet traffic output rate $\sigma_{r,i}$ of Q_r, i.e., the actual output rate of relayed packets from Q_r at node m_i to the next hop, is equal to the service rate of Q_r multiplied by probability of Q_r being nonempty, i.e.,

$$\sigma_{r,i} = \mu_{r,i}(1 - P_{0,i}), \tag{3.5}$$

where $P_{0,i}$ (to be derived in Section 3.3) denotes the probability of Q_r of node m_i being empty.

A transmission opportunity is always granted to the source queue Q_s when the relay queue Q_r is empty. Thus, the service rate $\mu_{s,i}$ of locally generated packets in Q_s at node m_i is

$$\mu_{s,i} = \mu_i - \mu_{r,i}(1 - P_{0,i}). \tag{3.6}$$

Since Q_s is always backlogged for the considered heavy traffic model, a locally generated packet is transmitted when the transmission opportunity is granted to the source queue Q_s. The effective source packet traffic output rate $\sigma_{s,i}$ of Q_s is thus identical to the service rate $\mu_{s,i}$:

$$\sigma_{s,i} = \mu_{s,i}. \tag{3.7}$$

With Eqn. (3.7) and (3.5), we readily verify that the overall effective output rate $\sigma_i = \sigma_{s,i} + \sigma_{r,i}$ of node m_i is identical to the arrival rate μ_i of transmission opportunities to m_i. We note that in the presented model, (a) packet transmission opportunities arrive to a given node m_i according to a Poisson process (and all transmission opportunities are utilized for either source or relay packets), and (b) the outgoing Poisson traffic of node m_i is the potential incoming traffic of its next-hop nodes. Thus, the Poisson packet arrival and service processes at a wireless mesh node make the $M/M/1/K$ queue model applicable.

3.2 Packet Arrival Rate at Wireless Mesh Node m_i

In the proposed network model, a maternal WMN is divided into several clusters. Though we still assume that each wireless node can find at least one path to its corresponding gateway, the situation that typically each x-hop node has to provide relay

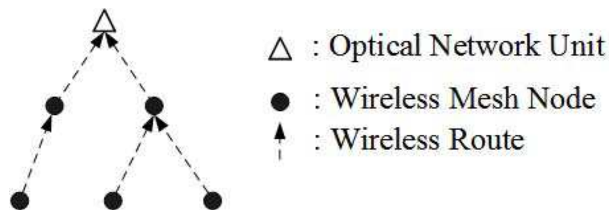


Figure 3.1: Illustration of Routing Scenario Leading to Heterogeneous Traffic Loads at Wireless Mesh Nodes

service for the same number of $x+1$-hop nodes and can ask the same number of $x-1$-hop nodes to relay its outgoing packets does not apply in the general case, i.e., the clustered FiWi network. With different numbers of clusters and dividing strategies, each cluster could be fragmented and the traffic loads are likely very different among nodes with the same hop count. Fig. 3.1 illustrates a simple example of a routing scenario resulting in heterogeneous traffic loads among the wireless mesh nodes. Assuming that all wireless mesh nodes share the same channel access probability p_i, the 1-hop node on the right in Fig. 3.1 has twice the traffic load of the 1-hop node on the left. In order to derive a mathematical analysis suitable for the heterogeneous traffic loads likely to arise from clustering a WMN, we study the input and output traffic loads for each individual wireless mesh node m_i and derive its traffic intensity, which is an essential parameter for the delay and throughput analysis.

We define the set of the node indices of the possible preceding nodes of a given node m_i as

$$\begin{aligned} R_i \quad = \quad & \{j : m_i \text{ is a possible next hop of } m_j, \\ & \text{for } j = 1, \ldots, N\}. \end{aligned} \tag{3.8}$$

According to the routing protocol in Section 2.2, a given node can choose the next hop randomly among its possible next hop candidates on a per-packet basis, i.e., all its next hop candidates share the same portion of the outgoing traffic. With this routing, we can express the arrival packet rate (of relay traffic) λ_i at node m_i as

$$\lambda_i = \sum_{j \in R_j} \frac{\mu_j}{f_j}, \tag{3.9}$$

where f_j is the number of next hop candidates of preceding node m_j and μ_j/f_j is the input rate of relayed packets from preceding node m_j to the considered node m_i. According to the properties of the exponential distribution, the distribution of the interarrival time between the packets transmitted from m_j to m_i is also exponential with mean f_j/μ_j, i.e., the incoming packet process at m_i is also a Poisson process. Since the superposition of independent Poisson processes is also a Poisson process, we conclude that the incoming process of all relayed packets is a Poisson process, whereby the distribution of the time between two incoming relayed packets is exponential with mean $1/\lambda_i$. With the property that both incoming and outgoing processes are Poisson, the relay queue Q_r of a wireless mesh node m_i can be modeled as an $M/M/1/K$ queue, where K denotes the buffer size in packets. We define the relay traffic intensity of m_i as

$$\rho_i = \frac{\lambda_i}{\mu_i}. \tag{3.10}$$

3.3 $M/M/1/K$ Queue Model for Relay Queue Q_r

As noted in the preceded sections, the relay queue Q_r in each wireless mesh node can be modeled as a $M/M/1/K$ queue. We briefly review the queueing theory for the $M/M/1/K$ queue in Appendix A.1. We note that for a fixed holding capacity of K packets, the mean waiting time $W_{\mathrm{M}}(\mu, \lambda, K)$ in Eqn. (A.5) in Appendix A.1 is a function of both the service rate μ and the arrival rate λ, while the probabilities of the queue being empty ($P_{\mathrm{M},0}$, see Eqn. (A.2) in Appendix A.1) and full ($P_{\mathrm{M},K}$, see Eqn. (A.1) in Appendix A.1) are functions of only the traffic intensity ρ. Thus, in order to correctly evaluate the delay in a given wireless mesh node, i.e., correctly evaluate average waiting time in the $M/M/1/K$ queue, we need to know the correct arrival and service rates of packets at each wireless mesh node m_i, which have been derived in the Section 3.2.

3.4 Throughput of the WMN Part

In the preceding Sections 3.2–3.3, we have studied the incoming and outgoing packet traffic processes of a wireless mesh node m_i. We have shown that the relay queue Q_r in a wireless mesh node m_i can be modeled as an $M/M/1/K$ queue. The exact analysis of the WMN part would require delay and throughput calculation for all possible node-to-gateway paths for all nodes in the WMN. This exhaustive evaluation could involve prohibitively high complexity for a large WMN since nodes with long hop distances tend to have many possible paths to the gateway. In this section, we propose an approximate, low-complexity evaluation of the throughput performance of the WMN part.

First, we study the end-to-end throughput of the WMN part. For the x-hop wireless mesh nodes, we define the source packet traffic throughput $T_{\mathrm{W}}(x)$ as the average number of packets generated by the x-hop wireless mesh nodes reaching the gateways per unit time. Mathematically, $T_{\mathrm{W}}(x)$ can be expressed as the total source packet traffic output rate of the x-hop nodes multiplied by the probability of the packets not being blocked at any of the intermediate relay nodes. Since the exhaustive evaluation of the blocking probabilities for all individual paths could be highly complex, we propose the following approximate method for evaluating the average blocking probability of the paths for the x-hop nodes. We first evaluate the average blocking probability on the wireless WMN path for the nodes with hop distance x as

$$P_{\mathrm{W},b}(x) = \sum_{i \in S_x} \frac{P_{\mathrm{M},K}(\rho_i, K)}{N(x)}, \tag{3.11}$$

where $S_x = \{i : h_j = x \text{ for } i = 1, \ldots, N\}$ is the set of nodes indices of the x-hop nodes, $N(x)$ denotes the number of x-hop nodes, and $P_{\mathrm{M},K}(\rho_i, K)$ is obtained from Eqn. (A.1) in Appendix A.1. For a packet generated at an x-hop node, we approximate the probability of reaching the gateway without blocking as

$$\prod_{h=1}^{x-1} [1 - P_{\mathrm{W},b}(h)], \tag{3.12}$$

since all packets generated at the x-hop nodes, with $x = 1, 2, \ldots, H$, have to pass through $x - 1$ relay nodes without blocking to reach the gateway. With the non-blocking probability obtained in Eqn. (3.12), the aggregate throughput of the x-hop nodes can be expressed as the product of the source packet traffic output rate of the x-hop nodes and the non-blocking probability. Specifically, we define the aggregate source packet traffic output rate of the x-hop nodes

$$\sigma_{s,\mathrm{agg}}(x) = \sum_{i \in S_x} \sigma_{s,i}. \tag{3.13}$$

Nodes with $x = 1$ hop to the gateway cannot be blocked at a relay node, while nodes with $x = 2, 3, \ldots, H$ hops need to be relayed by $x - 1$ nodes without blocking to reach the gateway, resulting in the source traffic throughput of x-hop nodes in the WMN

$$T_{\mathrm{W}}(x) = \begin{cases} \sigma_{s,\mathrm{agg}}(1), & x = 1 \\ \sigma_{s,\mathrm{agg}}(x) \prod_{h=1}^{x-1} [1 - P_{\mathrm{W},b}(h)], & x = 2, \ldots, H. \end{cases} \tag{3.14}$$

From Eqn. (3.14), we note that since source traffic output rate $\sigma_{s,i}$ and blocking probability $P_{\mathrm{W},b}(x)$ are functions of channel access probability $p(x)$ and forwarding probability $q(x)$, the throughput is also a function of both $p(x)$ and $q(x)$, as numerically studied in Chapter 4.

The aggregate WMN throughput is obtained by summing $T_{\mathrm{W}}(x)$ over the hop distance x:

$$T_{\mathrm{W,agg}} = \sum_{x=1}^{H} T_{\mathrm{W}}(x). \tag{3.15}$$

Note that all packets must be forwarded to the gateways by the 1-hop nodes, thus, the aggregate WMN throughput is also equal to the aggregate output rate of the 1-hop nodes, i.e., defining $\mu(1)$ as the service rate at the 1-hop nodes,

$$T_{\mathrm{W,agg}} = N(1)\mu(1). \tag{3.16}$$

The average per node WMN throughput is

$$T_{\mathrm{W,avg}} = \frac{T_{\mathrm{W,agg}}}{N}. \tag{3.17}$$

3.5 WMN Delay

Building on the traffic rates at the individual wireless mesh nodes m_i examined in the preceding sections, we derive in this section first the mean WMN delays under consideration of the individual heterogeneous traffic loads at the nodes. Subsequently, we contrast with the analysis approach of Liu and Liao (Liu and Liao, 2008) that considers only the mean traffic load of the nodes at a given hop distance.

3.5.1 Our Approach: Based on Individual Node Loads

We define the end-to-end delay of a packet in the WMN part as the time between when the first bit of the packet leaves the source node and when the last bit of the packet reaches the gateway. For a packet generated at an x-hop node, the end-to-end delay consists of the length of the x time slots for the packet transmissions and the queuing delays at the $x-1$ intermediate nodes providing relay service. For an x-hop node providing relay service, we approximate the average waiting time for the relayed packets in its relay queue Q_r as

$$W_{\mathrm{W,avg}}(x) = \sum_{i \in S_x} \frac{W_{\mathrm{M}}(\mu_i, \lambda_i, K)}{N(x)}, \tag{3.18}$$

whereby the mean waiting time $W_{\mathrm{M}}(\mu_i, \lambda_i, K)$ in an $M/M/1/K$ queue is obtained from Eqn. (A.5) in Appendix A.1.

With the knowledge of the average waiting time $W_{\mathrm{W,avg}}(x)$ in a given wireless mesh node with hop distance x, we obtain the expected end-to-end WMN delay as follows. A node with a hop distance $x = 1$ transmits a source packet only when a transmission opportunity at the node is not utilized by a relay packet. With the delay measurement starting when the first bit leaves the source node, the source packet traffic generated at 1-hop nodes experiences an end-to-end WMN delay corresponding to only the transmission delay t_c. Source packet traffic generated at x-hop nodes with $x = 2, 3, \ldots, H$, needs to be transmitted x times and incurs the relay queue waiting times $D_{\mathrm{W}}(x)$ at relay nodes that are $1, 2, \ldots, x-1$ hops from the gateway. In summary,

$$D_{\mathrm{W}}(x) = \begin{cases} t_c & \text{if } x = 1 \\ x t_c + \sum_{h=1}^{x-1} W_{\mathrm{W,avg}}(h) & \text{if } x = 2, \ldots, H. \end{cases} \tag{3.19}$$

The average end-to-end delay $D_{\mathrm{W,avg}}$ of the WMN part can be calculated by averaging the delays of packets reaching the gateways. Specifically, we weigh the delay $D_{\mathrm{W}}(x)$ experienced by x-hop nodes by the corresponding source traffic output rate $T_{\mathrm{W}}(x)$ of x-hop nodes:

$$D_{\mathrm{W,avg}} = \frac{\sum_{x=1}^{H} T_{\mathrm{W}}(x) D_{\mathrm{W}}(x)}{T_{\mathrm{W,agg}}}. \tag{3.20}$$

The analysis in this subsection provides the delay and throughput performance for the WMN part and we note that the analytical model does not limit the number Z of gateways, which makes this analysis applicable to other general WMNs.

3.5.2 *Contrast to Analysis Based on Mean Load at a Hop Distance*

Liu and Liao (Liu and Liao, 2008) presented a delay and throughout analysis for a WMN assuming that all nodes with a given hop distance have on average (nearly) the same input and output packet traffic rates (which lead to the same average traffic intensities). With this assumption, the average of the queuing behaviors of the nodes with the same hop distance is identical to the queuing behavior of a single node with the considered average of the input and output packet traffic rates at the individual nodes. For networks with heterogeneous input and output traffic rates at the different nodes (at the same hop distance), this assumption introduces large inaccuracies since the blocking probability and queue length of an $M/M/1/K$ queue are not linear functions of the traffic intensity. That is, averaging the input/output traffic rates and computing the blocking prob./queue length based on the average of the input/output traffic rates is not equivalent to averaging the blocking probs./queue lengths of the individual queues if the individual queues have substantially different input/output traffic rates.

3.6 Throughput-Delay Analysis for PON Part

When the packets are received by the gateways, they are immediately forwarded to the corresponding ONUs. Each ONU operates as a queue and transmits its queued packets to the OLT when transmission opportunities are given. Since all ONUs share the same physical optical bandwidth, several packet scheduling techniques have been proposed to efficiently utilize the bandwidth usage (Bontozoglou *et al.*, 2013; McGarry and Reisslein, 2012; Sue *et al.*, 2014; Zhou *et al.*, 2012). In this dissertation, we consider a basic model without any specific scheduling policies. We assume that the PON part operates in TDMA fashion and that each ONU can transmit its packets at specific time slots, which results in a deterministic service rate at each ONU. Similar to the wireless network, we denote t_{D} as the time slot duration needed to transmit a packet in the PON part. We proceed to show that the ONUs can be modeled as $M/D/1/K$ queues and derive the overall delay and throughput of the FiWi network.

To show that the ONU can be modeled as a $M/D/1/K$ queue, we first examine the packet arrival rates at the ONUs. At each ONU, the packets are forwarded directly from the corresponding gateway, i.e., an ONU and its corresponding gateway share the same input packet traffic. We define $g_z, z = 1, \ldots, Z$, as the number of 1-hop nodes in cluster z. Similar to the arguments in Section 3.2, we find that the incoming packet process at each gateway is Poisson since it is the superposition of several Poisson processes. Thus, the Poisson packet arrival rate at the gateway of cluster z is

$$\lambda_{\mathrm{D},z} = g_z \frac{p(1)}{t_c} = g_z \mu(1),$$

since there are g_z 1-hop nodes in cluster z and each 1-hop node feeds a traffic stream with rate $p(1)/t_c$ to the gateway. Considering that all Z ONUs operate at the same fixed rate (with fixed equal sharing of the total PON upstream bandwidth), each individual ONU can be modeled as an $M/D/1/K$ queue with service rate

$$\mu_{\mathrm{D},z} = \frac{1}{t_{\mathrm{D}} Z}. \tag{3.21}$$

The resulting traffic intensity of the ONU in cluster z is

$$\rho_{\mathrm{D},z} = \frac{g_z p(1) t_{\mathrm{D}} Z}{t_{\mathrm{c}}}.$$

For a FiWi network serving both wireless users as well as wired users that are directly connected to an ONU, e.g., through fiber to the home (FTTH), the traffic load (intensity) of an ONU is the sum of traffic loads from wireless and wired users.

Based on the queueing theory for the $M/D/1/K$ queue, as reviewed in Appendix A.2, we proceed to analyze the delay and throughput for the PON part.

We define the aggregate throughput $T_{\text{O,agg}}$ of the PON part as the average number of packets reaching the OLT per unit time. The aggregate throughput $T_{\text{O,agg}}$ is the sum of the effective output rates of the ONUs

$$T_{\text{O,agg}} = \sum_{z=1}^{Z} g_z \mu(1)[1 - P_{\text{D},K}(\rho_{\text{D},z}, K)], \tag{3.22}$$

where $g_z \mu(1)$ is the input rate at the ONU of cluster z and $1 - P_{\text{D},K}(\rho_{\text{D},z}, K)$ is the probability that the packets are not blocked. We note that the throughput of the PON part is also the throughput of the FiWi network.

The average delay at the ONUs is obtained by weighing the delays $W_{\text{D}}(\mu_{\text{D},z}, \lambda_{\text{D},z}, K)$ at the individual ONUs z, $z = 1, \ldots, Z$, by the corresponding packet output rates $g_z \mu(1)(1 - P_{\text{D},K}(\rho_{\text{D},z}, K))$:

$$W_{\text{O}} = \frac{\sum_{z=1}^{Z} W_{\text{D}}(\mu_{\text{D},z}, \lambda_{\text{D},z}, K) g_z \mu(1)[1 - P_{\text{D},K}(\rho_{\text{D},z}, K)]}{T_{\text{O,agg}}}.$$

3.7 Performance Analysis of Clustered FiWi Network

With the performance analysis for both the WMN and PON part derived in the preceding sections, we can obtain the overall performance of the FiWi network. The aggregate FiWi throughput T_{F} is equal to the aggregate throughput of the PON part as given by Eqn. (3.22). Similar to the WMN analysis, we define the overall end-to-end delay of a packet in the FiWi network as the time between when the first bit of the packet leaves the source node and when the last bit of the packet reaches the OLT. The delay can be calculated by adding the average delays generated at the wireless mesh nodes and the ONUs. For the packets generated at the x-hop node, the average delay $D_{\text{F}}(x)$ is

$$D_{\text{F}}(x) = D_{\text{W}}(x) + W_{\text{O}} + t_{\text{D}}, \tag{3.23}$$

where t_{D} is the transmission delay at the ONU. The average end-to-end delay of a packet can be calculated as

$$D_{\mathrm{F,avg}} = D_{\mathrm{W,avg}} + W_{\mathrm{O}} + t_{\mathrm{D}}, \tag{3.24}$$

with the average WMN delay $D_{\mathrm{W,avg}}$ given in Eqn. (3.20) and the average PON delay W_{O} given in Eqn. (3.23).

Chapter 4

NUMERICAL EVALUATION

For the numerical evaluations, we set the packet size to 1500 Byte and the time slot lengths for both the WMN and PON part are set to the time needed to transmit one packet. The buffer of the relay queues Q_r of the wireless mesh nodes and the ONUs are set to $K = 64$ packets. All simulation results have been obtained with 98 % confidence intervals that are less than 2 % of the corresponding sample means and are too small to be visible in the plots.

4.1 Network Topology

We consider a topology with 126 wireless mesh nodes distributed on 6 rings, as also considered in (Liu and Liao, 2008) and illustrated in Fig. 4.1. Ring h has a radius of $(55h)$ m and $6h$ wireless mesh nodes are located with even spacing on the ring. Each wireless mesh node has a transmission range of $r = 100$ m. The wireless mesh nodes are static and from the maternal network. This design ensures that (a) Each wireless mesh node can find at least one node within its transmission range on both its inner and outer rings, but it cannot find any node within its transmission range that is two or more rings away. (b) Each wireless mesh node can communicate with its two neighbors on the same ring. This design ensures robustness of the network even when it is divided into clusters. To divide the maternal network into a FiWi network with Z clusters, the maternal network is cut into Z even circular sectors and the gateways are located in the centroid of each circular sector. For the WMN case, i.e., $Z = 1$, the gateway is placed in the center of the rings.

We first briefly examine elementary characteristics of the considered network.

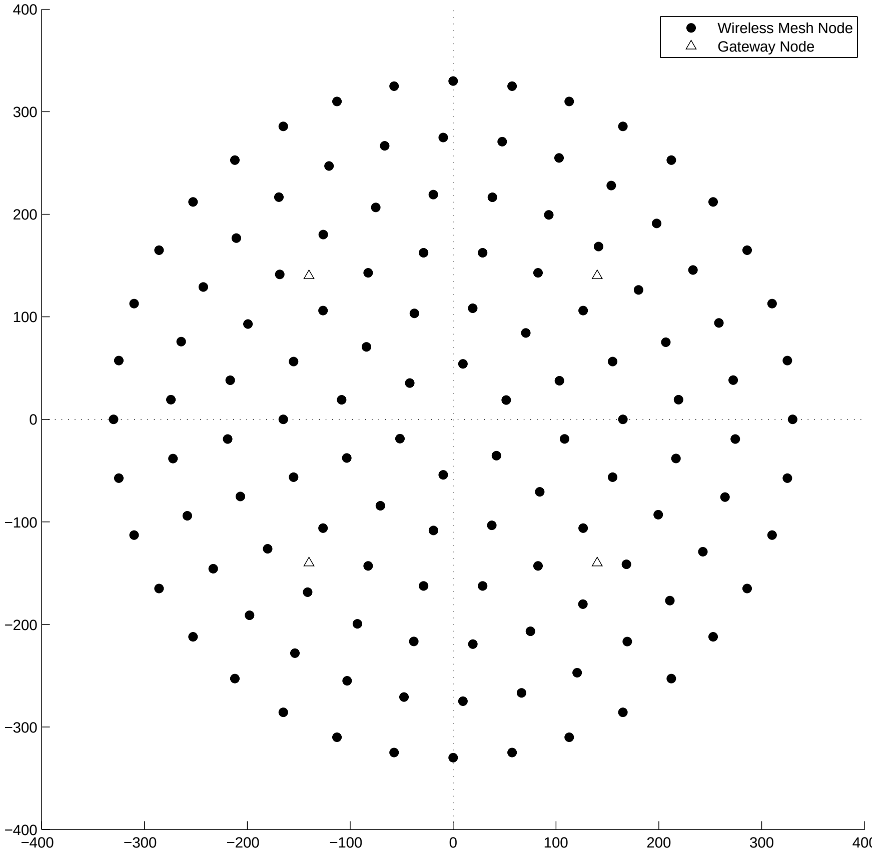


Figure 4.1: Network Topology with 126 Wireless Mesh Nodes Placed on Rings h, $h = 1, 2, \ldots, 6$, with Radius $(55h)$ m. The Illustration Shows the Simulated FiWi Network with $Z = 4$ Clusters.

Table 4.1 gives the average hop distance from a node to its corresponding ONU. Note from Eqn. (3.16) that the aggregate output of the WMN part is in proportion to the number $N(1)$ of nodes with a hop distance of one to their gateway and the table shows that the $N(1)$ values increase with the number of clusters Z, which indicates that dividing the maternal network into more clusters increases the aggregate throughput of the WMN part (and in turn increases the traffic load of the PON part). Table 4.1 also indicates that the average hop count of the wireless mesh nodes decreases as the number of clusters Z increases and Eqn. (3.19) shows that packets from higher hop

Table 4.1: Characteristics of FiWi Network for Different Number of Clusters Z: Average Hop Distance to ONU, Maximum Hop Distance H in Network, and Number of Nodes $N(1)$ With One Hop to an ONU.

Z	avg. dist	H	$N(1)$
1	4.333	6	6
2	2.714	5	20
3	2.143	4	33
4	1.810	3	42
5	1.667	3	52
6	1.667	3	54
7	1.540	3	64
8	1.508	3	68
9	1.500	3	69
10	1.476	3	72

count nodes suffer from higher delays due to more relay hops.

4.2 Channel Access Prob. p_i and Forwarding Prob. q_i

We consider example scenarios where all x-hop nodes have the same channel access probability p_i and forwarding probability q_i, i.e.,

$$\begin{aligned} p_i &= p(x) \ \forall\ i \text{ such that } h_i = x \\ q_i &= q(x) \ \forall\ i \text{ such that } h_i = x, \end{aligned}$$

where h_i is the hop distance of node m_i to its corresponding gateway. Each setting satisfies

$$\sum_{x=1}^{H} N(x)p(x) = 1, \tag{4.1}$$

which guarantees that at least one wireless mesh node is granted the transmission opportunity per time slot. Specifically, we consider three different settings for the channel access probability $p(x)$ and the forwarding probability $q(x)$, which effectively control the bandwidth allocation:

p07: Each wireless mesh node has the same channel access probability, i.e., $p(1) = p(2) = \cdots = p(H) = 1/126$, and the same forwarding probability $q(x) = 0.7$.

pth: $p(x)$ is set according to Eqn. (A.9) in Appendix A.3 and $q(x)$ is set to the lower bound in (A.11), see Table 4.2.

pde: $p(x)$ is set according to Eqn. (A.9), see Table 4.2(a), and $q(x)$ is set to 0.975, which is higher than the lower bound in (A.11), cf. Table 4.2(b), further reducing the delay of the WMN part.

4.3 WMN Throughput and Delay

4.3.1 Comparison of Individual Load and Mean Load Analyses

We initially set the wireless transmission bit rate to 100 Mb/s and the PON transmission bit rate to 1 Gb/s. In Fig. 4.2(a) and (b) we compare mean throughput and delay obtained with simulations, our analysis based on individual node traffic loads, and the analysis in (Liu and Liao, 2008) based on the mean node traffic load at a given hop distance. Specifically, in Fig. 4.2(a) we plot the source packet traffic throughput of the 2-hop nodes $T_{\mathrm{W}}(2)$ in the WMN. We observe that our analytical method (labeled with suffix "-the") provides good prediction of the simulation results (labeled with suffix "-sim"), while the analytical results of (Liu and Liao, 2008) (labeled with suffix "-the[Liu]") fail to describe the accurate throughput behavior when the number Z of clusters increases.

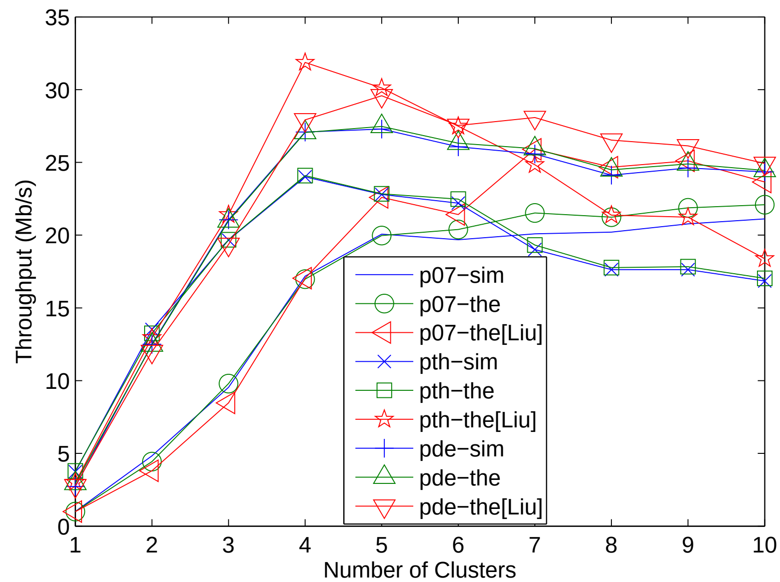
Table 4.2: Channel Access Prob. $p(x)$ and Forwarding Prob. $q(x)$ as a Function of Hop Distance x to Gateway for Pth Setting for Varying Number of Clusters Z.

(a) Channel access prob. $p(x)$ from (A.9) in Appendix A.3 and (4.1)

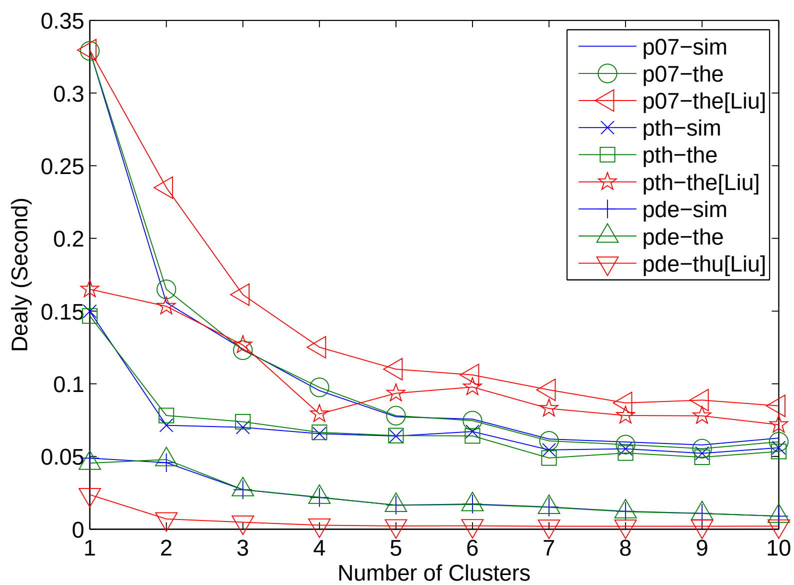
Z	$p(1)$	$p(2)$	$p(3)$	$p(4)$	$p(5)$	$p(6)$
1	0.0410	0.0186	0.0109	0.0067	0.0039	0.0017
2	0.0203	0.0091	0.0048	0.0030	0.0024	n/a
3	0.0152	0.0065	0.0042	0.0033	n/a	n/a
4	0.0136	0.0053	0.0042	n/a	n/a	n/a
5	0.0118	0.0053	0.0046	n/a	n/a	n/a
6	0.0114	0.0055	0.0045	n/a	n/a	n/a
7	0.0103	0.0055	0.0050	n/a	n/a	n/a
8	0.0099	0.0056	0.0051	n/a	n/a	n/a
9	0.0098	0.0057	0.0051	n/a	n/a	n/a
10	0.0096	0.0058	0.0051	n/a	n/a	n/a

(b) Forwarding prob. $q(x)$ from (A.11) in Appendix A.3

Z	$q(1)$	$q(2)$	$q(3)$	$q(4)$	$q(5)$
1	0.9203	0.8795	0.8221	0.7289	0.545455
2	0.7707	0.6170	0.4128	0.1875	n/a
3	0.6649	0.4190	0.2142	n/a	n/a
4	0.6220	0.2142	n/a	n/a	n/a
5	0.5565	0.1351	n/a	n/a	n/a
6	0.5351	0.1666	n/a	n/a	n/a
7	0.4699	0.0967	n/a	n/a	n/a
8	0.4375	0.1034	n/a	n/a	n/a
9	0.4295	0.1052	n/a	n/a	n/a
10	0.4055	0.1111	n/a	n/a	n/a



(a) 2-hop node source throughput $T_W(2)$



(b) Average per node delay

Figure 4.2: Mean Throughput and Delay Characteristics of WMN with 100 Mb/s Wireless Transmission Bit Rate as a Function of Number of Clusters Z. Our Analysis (the) Based on Individual Node Traffic Loads Closely Matches the Simulations (sim), While the Analysis (Liu and Liao, 2008) (the[Liu]) Based on the Mean of the Traffic Loads of the Nodes at a Given Hop Distance Deviates Significantly from Simulations, Especially for Large Number of Clusters Z.

For a low number Z of clusters, the level of heterogeneity of the traffic loads of the nodes at a given hop distance is relatively low. For instance, for the maternal network $Z = 1$ of the network topology illustrated in Fig. 4.1, all nodes on the first ring ($h = 1$) receive relay traffic from two nodes in the second ring. However, nodes on the second ring ($h = 2$) receive relay traffic from either two or three nodes in the third ring; and this pattern of receiving relay traffic from either two or three nodes continues for nodes on rings $h = 3, 4$, and 5. In contrast, for $Z = 4$ clusters, each of the ONUs illustrated in Fig. 4.1 serves a quarter sector of the network. The wireless mesh nodes located between a given ONU and the outer edge of the original network have now significantly more relay traffic than the wireless mesh nodes with the same hop distance located between the ONU and the center of the original network. Thus, the level of heterogeneity of the traffic loads of the nodes at a given hop distance increases with increasing number of clusters Z.

As noted in Subsection 3.5.2, the analysis approach in (Liu and Liao, 2008) averages the traffic loads of the nodes at a given hop distance x to the gateway. The average traffic load is then employed to obtain the blocking probability $P_{M,K}$ through Eqn. (A.1) in Appendix A.1, which governs the throughput, see Section 3.4. Generally, the blocking probability $P_{M,K}$ viewed as a function of the load ρ has two near-linear segments, namely for very low loads ($\rho \to 0$) and for very high loads ($\rho \to \infty$) (Gross and Harris, 1998). Thus, if all individual node loads lie in one of the near-linear segments, then the blocking probability for the average of the loads closely approximates the average of the blocking probabilities evaluated for the individual loads. Thus, the approach in (Liu and Liao, 2008) gives increasing discrepancies from the true mean throughput as the traffic loads of the nodes at a given hop distance become increasingly heterogeneous.

Similarly, we observe for the mean WMN delay plotted in Fig. 4.2(b) that our analytical method precisely describes the WMN delay, while the analytical results of (Liu and Liao, 2008) generally diverge substantially from the simulation results. Only for $Z = 1$, for the p07 setting does the (Liu and Liao, 2008) approach accurately give the mean delay because all the queues are very highly loaded in this scenario (i.e., are operating in a near linear segment of W_{M}). Even for low cluster numbers Z, the delay analysis (Liu and Liao, 2008) differs substantially from the simulations. This is mainly because the mean WMN delay evaluation considers the entire range of hop distances. For the (Liu and Liao, 2008) analysis, the traffic load variations at each of the hop distances would need to fall into a near-linear segment of the W_{M} curve, which is highly unlikely. We thus conclude that the consideration of the individual node loads at each hop distance level, as considered in our analysis, is required for accurate throughput-delay evaluation of a WMN with heterogeneous node traffic loads.

4.3.2 Impact of Channel Access and Forwarding Prob.

We observe from Fig. 4.2(b) that the pth setting provides lower WMN delays than the p07 setting, while the pde setting further reduces the delay. The pde setting has higher forwarding probabilities $q(x)$ than the pth setting. The higher forwarding probabilities $q(x)$ provide higher service rates to the relayed traffic, which reduces the delay for relayed traffic, resulting in lower WMN delay. The p07 setting has the same forwarding probability of 0.7 for each hop distance, resulting in bottlenecks and high delays as packets approach the gateway.

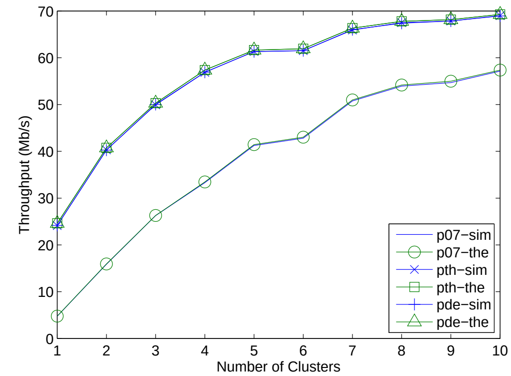
We note that the pth and pde settings have the same channel access probabilities $p(x)$ and would (for the considered continuously backlogged sources, see Section 2.1) result in the same aggregate throughput $T_{\mathrm{W,agg}}$ of the WMN part, see Eqn. (3.16). In order to provide detailed insight into the throughput characteristics of the different channel access and forwarding probability settings p07, pth, and pde, we present results for the 2-hop node source packet traffic throughput $T_W(2)$ in this subsection. We observe from Fig. 4.2(a) that the source packet traffic throughput of all 2-hop nodes $T_W(2)$ first generally increases with the number of clusters Z and then reaches a plateau or slightly decreases for large Z. These overall dynamics of the throughput curves are mainly due to the number of 2-hop nodes in the network, which initially grows and then slightly decreases as the number of clusters Z increases.

Next, we observe from Fig. 4.2(a) that the pde setting achieves the highest 2-hop source packet traffic throughput $T_W(2)$. Moreover, the pth setting achieves higher $T_W(2)$ than the p07 setting for a small number of clusters Z; however, for large Z, the $T_W(2)$ of pth drops below the $T_W(2)$ of the p07 setting. These differences are primarily due to the forwarding probabilities $q(x)$. The pde setting has the highest forwarding probabilities $q(x)$, which prioritize the transmission of the relayed packets (so that they are rarely blocked at the 1-hop nodes). With the pth setting, the forwarding probability (at a given hop distance x) $q(x)$ generally decreases as the number of clusters Z increases, see Table 4.2(b). As a result, for a large number of clusters Z, the pth setting gives lower priority to the relayed packets, leading to a decrease in the 2-hop node source throughput $T_W(2)$. In contrast, the p07 setting has constant forwarding probability $q(x)$, irrespective of the number of clusters Z, and thus achieves higher $T_W(2)$ throughput than the pth setting for large Z.

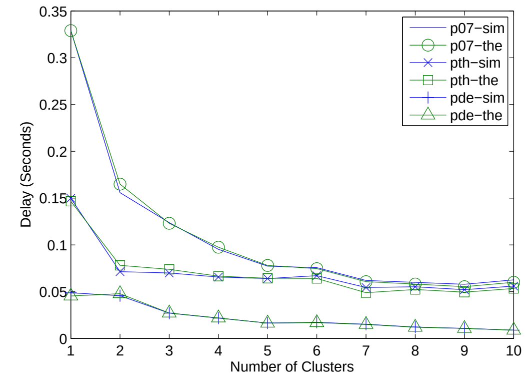
In the following sections, we consider the aggregate throughput $T_{W,agg}$ of the WMN part as well as FiWi network throughput $T_F = T_{O,agg}$ (3.22).

4.4 Throughput and Delay of Clustered FiWi Network

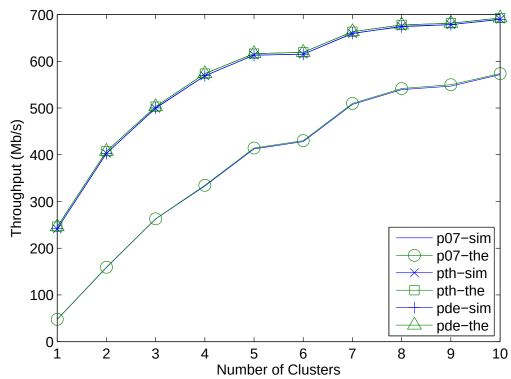
In Fig. 4.3 we plot the mean FiWi network throughput $T_F = T_{O,agg}$ (3.22) and mean FiWi network delay $D_{F,avg}$ (3.24).



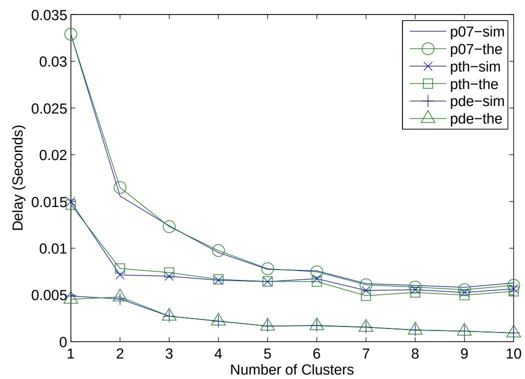
(a) Throughput, 1 Gb/s opt.–100 Mb/s wirel.



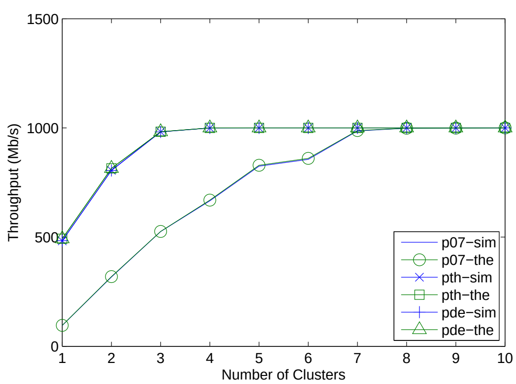
(b) Delay, 1 Gb/s opt.–100 Mb/s wirel.



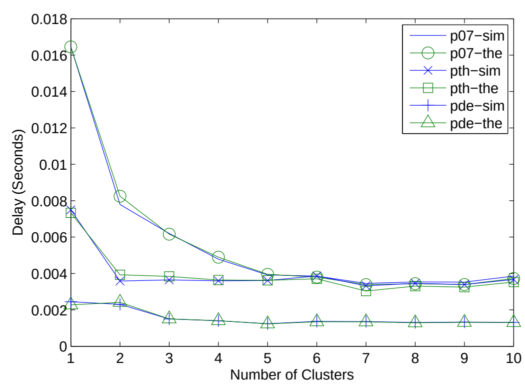
(c) Throughput, 1 Gb/s opt.–1 Gb/s wirel.



(d) Delay, 1 Gb/s opt.–1 Gb/s wirel.



(e) Throughput, 1 Gb/s opt.–2 Gb/s wirel.



(f) Delay, 1 Gb/s opt.–2 Gb/s wirel.

Figure 4.3: Mean FiWi Network Throughput-Delay Performance as a Function of Number of Clusters Z for 1 Gb/s Optical Transmission Rate Combined with 100 Mb/s, 1 Gb/s, or 2 Gb/s Wireless Transmission Rate.

4.4.1 Channel Access and Forwarding Prob.

We observe from Fig. 4.3(a), (c), and (e) that for a given fixed number of clusters Z, the pth and pde settings give the same throughput, while the p07 setting gives lower throughput. The pth and pde settings have the same channel access probabilities $p(x)$. Thus, as noted in Subsection 4.3.2, both settings result in the same aggregate WMN throughput $T_{\mathrm{W,agg}}$ as the continuously backlogged 1-hop nodes utilize any available transmission opportunities for their source traffic. The uniform channel access probabilities $p(x)$ for the different hop distances x with the p07 setting give rise to bottlenecks at the nodes close to the gateway (ONU), as observed previously in (Gambiroza *et al.*, 2004; Lee *et al.*, 2008; Liu and Liao, 2008), limiting the throughput.

We observe from Fig. 4.3(b), (d), and (f) that for fixed Z, the pde setting gives the lowest delay, followed by pth, and then p07. The pde setting prioritizes the forwarding of relay traffic, reducing the delay compared to the pth setting. The high delays with the p07 setting are due to bottlenecks close to the ONUs.

4.4.2 Number of Clusters Z

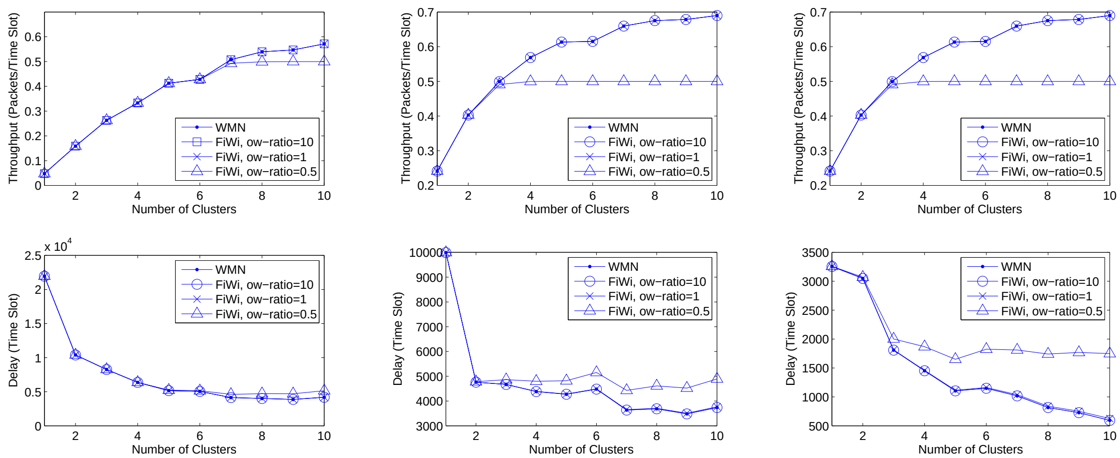
For all channel access/forwarding probability (i.e., bandwidth allocation) settings, we observe from Fig. 4.3 that the overall throughput-delay performance levels generally improve with increasing number of clusters Z. The throughput-delay improvements reaped from increasing the number of clusters Z are most pronounced for small Z, i.e., for FiWi networks with up to four or five clusters. Increasing the number of clusters beyond $Z = 5$, brings small improvements, especially when the ratio of optical transmission rate to wireless transmission rate is low, as examined in more detail in the next subsection.

4.4.3 *Optical to Wireless Transmission Bit Rate Ratio (ow-ratio)*

Advancing wireless transmission technologies may increase the transmission bit rates in the WMN relative to the transmission bit rate on the PON. For instance, the different WMN clusters of the FiWi network could operate on different transmission channels, thus vastly increasing the effective wireless transmission bit rates. We model such advances through varying the ratio of optical to wireless transmission bit rate (ow-ratio) for the considered network operating on a single radio frequency (see Subsection 2.3.1).

In Fig. 4.3(c) and (d) we increase the wireless transmission bit rate tenfold, i.e., to 1 Gb/s, compared to Fig. 4.3(a) and (b), i.e., the ow-ratio is reduced from ten in Fig. 4.3(a) and (b) to one in Fig. 4.3(c) and (d). We observe that while the curves in these two pairs of plots have the same shape, the FiWi network with 1Gb/s wireless transmission rate in Fig. 4.3(c) and (d) provides close to ten times the throughput while reducing the delay to a tenth compared to the FiWi network with 100Mb/s wireless transmission rate in Fig. 4.3(a) and (b). This improvement in the absolute throughput-delay values while maintaining the same shapes of the throughput and delay curves as a function of the number of clusters Z is mainly due the WMN part limiting the overall performance in both FiWi networks. The FiWi network with 1Gb/s wireless transmission rate can essentially fully utilize the wireless transmission bit rate increase to increase the overall network performance. That is, there is effectively no penalty due to the increasing load on the PON part operating at 1 Gb/s.

However, reducing the ow-ratio further to 0.5 in Fig. 4.3(e) and (f), we observe that the pth and pde settings reach the 1 Gb/s transmission bit rate limit of the PON part with $Z = 3$ clusters in Fig. 4.3(e), while p07 reaches the limit with $Z = 7$



(a) p07 setting (b) pth setting (c) pde setting

Figure 4.4: Comparison of Throughput-Delay Performance of WMN part and Overall FiWi Network as a Function of the Number of Clusters Z for Different Channel Access/Forwarding Probability Settings P07, Pth, and Pde and Optical to Wireless Transmission Bit Rate Ratios (ow-ratios).

clusters. Similarly, we observe from Fig. 4.3(f) that the mean FiWi delays are not further reduced for growing number of clusters Z. For the ow-ratio $= 0.5$, the PON becomes the bottleneck as the number of clusters Z increases, as further examined in Fig. 4.4.

We examine the interplay between the limitations of the WMN part and the PON part in more detail by comparing the mean throughput and delay (from simulations) of only the WMN part and the overall FiWi network (i.e., the combination of WMN and PON parts) in Fig. 4.4. The WMN part accounts for the throughput and delays up to the point when the packets reach the gateways (ONUs). The delay unit is the length of the wireless time slot and the throughput unit is the number of packets per wireless time slot.

We observe from Fig. 4.4(a) that for the p07 setting and the ow-ratios 10 and 1, the delay and throughput of the WMN part and the FiWi network are essentially identical. This indicates that the FiWi network performance is limited by the WMN part, while the PON part blocks almost no packets and introduces negligible delay. For the ow-ratio 0.5, the FiWi network performance remains essentially identical to the WMN part for six or fewer clusters. When the number of clusters reaches $Z = 7$, the throughput of the WMN part, i.e., the input traffic speed to the PON part, begins to exceed the transmission bit rate of the PON part, and the PON part begins to limit the FiWi network throughput. When the throughput of the WMN part exceeds the transmission bit rate of the PON part, the packets begin to be stored in the queues of the ONUs and the delay caused by the PON part significantly contributes to the overall FiWi network delay. Since the FiWi network delay is the sum of the delay of the WMN part and the delay of the PON part, the delay of the PON part can be observed as the difference between the FiWi network and WMN delay curves. This difference becomes visible in the tail of the delay curves for ow-ratio=0.5 for the p07 setting in Fig. 4.4(a).

We observe from Fig. 4.4(b) and (c) that for the pth and pde settings, the PON part limits the throughput of the FiWi network for the ow-ratio 0.5 as soon as the number of clusters exceeds two. Examining closer the difference between the FiWi network and WMN delay curves, we observe that for the ow-ratio 0.5, increasing the number of clusters beyond two leads to a widening gap of the delay curves, i.e., increasing PON delay. With increasing the number of clusters Z, each ONUs is allocated less bandwidth, i.e., lower service rate, while the traffic intensity is increased due to the increasing WMN throughput. The resulting growing queues in the ONUs increase the PON delay. The increasing PON delay is essentially compensated by the decreasing WMN delay, resulting in nearly steady FiWi network delay for increasing number of clusters Z in Fig. 4.4(b) and (c). However, the growing gaps between the WMN and FiWi network throughput curves in Fig. 4.4(b) and (c) indicate increasing packet drop probabilities for increasing Z for the ow-ratio 0.5.

We conclude the evaluation chapter by illustrating a design example of a QoS-aware FiWi network which requires a FiWi network throughput around 50 % of the wireless channel bit rate. The aggregate throughput of the FiWi network, which is equivalent to the throughput of the PON part, is given by Eqn. (3.22). Based on Eqn. (3.22) we can determine the throughput as a function of the number of clusters Z, as illustrated in Fig. 4.4. Fig. 4.4 indicates that increasing the number of clusters Z generally increases the throughput. Specifically, we observe from Fig. 4.4 that for the p07 channel access and forwarding probability setting, we need $Z = 7$ clusters to satisfy the desired throughput criterion; whereas for the pth and pde settings, $Z = 3$ is sufficient.

Chapter 5

FIWI NETWORKS WITH POISSON INPUT TRAFFIC

In (Chen and Reisslein, 2015), we studied the delay and throughput behaviors of a FiWi with heavy-loaded traffic. Due to the nature of the heavy-loaded traffic, the packet delay at the local queues is not considered in (Chen and Reisslein, 2015). In this chapter, we consider the Poisson input traffic model, which allow us to consider the delay at the local queues, and it also shows that the heavy-loaded traffic can be approximated by the Poisson traffic model with proper traffic loads. With the Poisson traffic model, we are also able to study the input traffic design strategy according to different network environments to utilize the system throughput without saturating the delay performance.

5.1 Network Modeling with Poisson Input Traffic

We consider the locally generated traffic at each wireless mesh node to be Poisson distributed and model each wireless mesh node as the combination of two queues, as shown in Fig.2.2. Queue Q_r serves the relayed packets, while queue Q_s serves the locally generated (source) packets. A given wireless mesh node m_i, $i = 1, \ldots, N$, forwards packets as follows:

1. If Q_r and Q_s are not empty, transmit a packet from Q_r with probability q_i, or a packet from Q_s with probability $1 - q_i$.

2. If Q_r is empty and Q_s is not empty, transmit a packet from Q_s.

3. If Q_s is empty and Q_r is not empty, transmit a packet from Q_r.

4. If Q_r and Q_s are both empty, do nothing.

In this packet forwarding model, we do not consider any adaptive bandwidth usage for the WMN part. If the transmission opportunity is given to a wireless mesh node with no packets at both Q_r and Q_s, the transmission opportunity is wasted and reduce the bandwidth usage. In (Liu and Liao, 2008) and (Chen and Reisslein, 2015), It has shown that heavy-loaded traffic, which assumes that Q_s is always full, can utilize the bandwidth usage, but it also introduces the highest delay at the Q_S. The network design aiming to solve the high delay issue is presented later in the dissertation.

5.1.1 Packet Service Rates at Wireless Mesh Node m_i

As described in the wireless mesh node model in Section 5.1, the relay packet queue Q_r can obtain a given transmission opportunity that has already been granted to node m_i under two circumstances: 1) The transmission opportunity is directly given to Q_r. 2) The transmission opportunity is first given to the empty Q_s and then given back to Q_r. The service rate of relay packets in Q_r is thus

$$\mu_{r,i} = \mu_i q_i + \mu_i(1 - q_i)P_{0,s,i}, \tag{5.1}$$

where $P_{0,s,i}$ is the probability of Q_s of m_i being empty, q_i is the probability of the transmission opportunity being directly given to Q_r and $(1 - q_i)P_{0,s,i}$ is the probability of the transmission opportunities being first given to the empty Q_s and then given back to Q_r. With such expression, we can conclude that the arrival of transmission opportunities at Q_r is also a Poisson process since the arrival of transmission opportunities at Q_r is part of the original Poisson process with rate μ_i. With similar approach, we obtain the service rate of the local packet queue Q_s as

$$\mu_{s,i} = \mu_i(1 - q_i) + \mu_i q_i P_{0,r,i}, \tag{5.2}$$

where $P_{0,r,i}$ is the probability of Q_r of m_i being empty. Similar to Q_r, the arrival of transmission opportunities at Q_s is also a Poisson process. With the knowledge of the service rates of Q_r and Q_s, we can further obtain the actual output rate of both queues

$$\begin{aligned} \sigma_{r,i} &= \mu_{r,i}(1 - P_{0,r,i}) \\ \sigma_{s,i} &= \mu_{s,i}(1 - P_{0,s,i}), \end{aligned} \tag{5.3}$$

where $\sigma_{r,i}$ is the actual output rate of Q_r of m_i and $\sigma_{s,i}$ is the actual output rate of Q_s of m_i. The actual output rate of m_i, denoted as σ_i, is the sum of the output rates of its Q_r and Q_s. With Eqn. (5.1), (5.2) and (5.3), σ_i can be expressed as

$$\begin{aligned} \sigma_i &= \sigma_{r,i} + \sigma_{s,i} \\ &= \mu_i(1 - P_{0,r,i} \cdot P_{0,s,i}). \end{aligned} \tag{5.4}$$

Eqn. (5.4) shows that the transmission opportunities given to m_i would not be used only when both Q_r and Q_s are empty, which verified the forwarding policy described in Section 5.1 and it also shows that the output traffic of m_i is a Poisson process.

5.1.2 Packet Arrival Rates at Wireless Mesh Node m_i

According to the routing protocol in Section 2.2, a given node can choose the next hop randomly among its possible next hop candidates on a per-packet basis, i.e., all its next hop candidates share the same portion of the outgoing traffic. With this routing, we can express the arrival packet rate (of relay traffic) $\lambda_{r,i}$ at node m_i as

$$\lambda_{r,i} = \sum_{j \in R_j} \frac{\sigma_j}{f_j}, \tag{5.5}$$

where f_j is the number of next hop candidates of preceding node m_j and σ_j/f_j is the input rate of relayed packets from preceding node m_j to the Q_r of considered node m_i. According to the properties of the exponential distribution, the distribution of the interarrival time between the packets transmitted from m_j to m_i is also exponential with mean f_j/σ_j, i.e., the incoming packet process at m_i is also a Poisson process. Since the superposition of independent Poisson processes is also a Poisson process, we conclude that the incoming process of all relayed packets is a Poisson process, whereby the distribution of the time between two incoming relayed packets is exponential with mean $1/\lambda_{r,i}$. With the property that both incoming and outgoing processes are Poisson, the relay queue Q_r of a wireless mesh node m_i can be modeled as an $M/M/1/K$ queue, where K denotes the buffer size in packets. We define the relay traffic intensity of Q_r of m_i as

$$\rho_{r,i} = \frac{\lambda_{r,i}}{\mu_{r,i}}. \tag{5.6}$$

For the local queue Q_s of node m_i, we assume that the input traffic is Poisson distributed where the distribution of the time between two incoming relayed packets is exponential with mean $1/\lambda_{s,i}$. Since the arrival of transmission opportunity at Q_s is shown to be a Poisson Process in Section 3.1, the local queue Q_s of node m_i can also be modeled as a $M/M/1/K$ queue with arrival rate $\lambda_{s,i}$ and service rate $\mu_{s,i}$. We define the relay traffic intensity of Q_s of m_i as

$$\rho_{s,i} = \frac{\lambda_{s,i}}{\mu_{s,i}}. \tag{5.7}$$

5.1.3 $M/M/1/K$ Queue Model for Relay Queue Q_r and Local Queue Q_s

As noted in the preceded subsections, the relay queue Q_r and local queue Q_s in each wireless mesh node can be modeled as $M/M/1/K$ queues. We briefly review the queueing theory for the $M/M/1/K$ queue in Appendix A. We note that for a fixed holding capacity of K packets, the mean waiting time $W_{\mathrm{M}}(\mu, \lambda, K)$ in Eqn. (A.5) in Appendix A is a function of both the service rate μ and the arrival rate λ, while the probabilities of the queue being empty ($P_{\mathrm{M},0}$, see Eqn. (A.2) in Appendix A) and full ($P_{\mathrm{M},K}$, see Eqn. (A.1) in Appendix A) are functions of only the traffic intensity

ρ. Thus, in order to correctly evaluate the delay in a given wireless mesh node, i.e., correctly evaluate average waiting time in the $M/M/1/K$ queue, we need to know the correct arrival and service rates of packets at both Q_r and Q_s at wireless mesh node m_i, which have been derived in the Section 3.2.

5.1.4 Dynamic Bandwidth Adjustment at Wireless Mesh Nodes

In this subsection, we show that the bandwidth given to a wireless mesh node m_i is dynamically adjusted if the input traffic is properly controlled. This property will be used in the channel access probability design and the input traffic control design proposed later in this paper. Let us consider the case that $\lambda_{r,i} < \mu_i q_i$, which indicates that the incoming relay traffic rate is lower than the highest relay packet service rate, and study the service rate of local queue Q_s. With Eqn. (5.1), (5.2) and (A.2), we obtain

$$\begin{aligned}
\mu_{s,i} &= \mu_i(1-q_i) + \mu_i q_i P_{0,r,i} \\
&= \mu_i(1-q_i) + \mu_i q_i \frac{1 - \frac{\lambda_{r,i}}{\mu_i q_i + \mu_i(1-q_i)P_{0,s,i}}}{1 - (\frac{\lambda_{r,i}}{\mu_i q_i + \mu_i(1-q_i)P_{0,s,i}})^{k+1}} \\
&> \mu_i(1-q_i) + \mu_i q_i \frac{1 - \frac{\lambda_{r,i}}{\mu_i q_i}}{1 - (\frac{\lambda_{r,i}}{\mu_i q_i})^{k+1}} \\
&> \mu_i(1-q_i) + \mu_i q_i \left(1 - \frac{\lambda_{r,i}}{\mu_i q_i}\right) = \mu_i - \lambda_{r,i}.
\end{aligned} \tag{5.8}$$

Eqn. (5.8) shows that if the incoming relay traffic rate $\lambda_{r,i}$ is lower than the highest relay packet service rate $\mu_i q_i$, the service rate $\mu_{s,i}$ of the local queue Q_s adjusts to the lower incoming relay traffic rate and provides higher service rate to the source packets automatically. Similar argument can also be made for the case that $\lambda_{s,i} < \mu_i(1-q_i)$

following the same procedure and we get the following property

$$\mu_{s,i} > \mu_i - \lambda_{r,i} \quad \text{if} \quad \lambda_{r,i} < \mu_i q_i \tag{5.9}$$

$$\mu_{r,i} > \mu_i - \lambda_{s,i} \quad \text{if} \quad \lambda_{s,i} < \mu_i(1 - q_i), \tag{5.10}$$

which proves that the transmission opportunities given to a wireless mesh node can dynamically shared among its two queues.

5.2 Mathematical Analysis of the FiWi Network with Poisson Input Traffic

5.2.1 Throughput of the WMN Part Poisson Input Traffic

In the preceding Subsections 5.1.2–5.1.3, we have studied the incoming and outgoing packet traffic processes of a wireless mesh node m_i. We have shown that the relay queue Q_r and local queue Q_s in a wireless mesh node m_i can be modeled as $M/M/1/K$ queues. The exact analysis of the WMN part would require delay and throughput calculation for all possible node-to-gateway paths for all nodes in the WMN. This exhaustive evaluation could involve prohibitively high complexity for a large WMN since nodes with long hop distances tend to have many possible paths to the gateway. In this subsection, we propose an approximate, low-complexity evaluation of the throughput performance of the WMN part.

First, we study the end-to-end throughput of the WMN part. For the x-hop wireless mesh nodes, we define the local packet traffic throughput $T_{\mathrm{W}}(x)$ as the average number of packets generated by the x-hop wireless mesh nodes reaching the gateways per unit time. Mathematically, $T_{\mathrm{W}}(x)$ can be expressed as the total source packet traffic output rate of the x-hop nodes multiplied by the probability of the packets not being blocked at its local node and any of the intermediate relay nodes. Since the exhaustive evaluation of the blocking probabilities for all individual paths could be highly complex, we propose the following approximate method for evaluating the average blocking probability of the paths for the x-hop nodes. We first evaluate the weighted average relay packet blocking probability on the wireless WMN path for the nodes with hop distance x as

$$P_{\mathrm{W},r,b}(x) = \frac{\sum_{i \in S_x} \lambda_{r,i} P_{\mathrm{M},K}(\rho_{r,i}, K)}{\sum_{i \in S_x} \lambda_{r,i}}, \tag{5.11}$$

where $S_x = \{i : h_j = x \text{ for } i = 1, \ldots, N\}$ is the set of nodes indices of the x-hop nodes and $P_{\mathrm{M},K}(\rho_i, K)$ is obtained from Eqn. (A.1) in Appendix A. Similarly, we obtain the weighted average source packet blocking probability on the wireless WMN path for the nodes with hop distance x as

$$P_{\mathrm{W},s,b}(x) = \frac{\sum_{i \in S_x} \lambda_{s,i} P_{\mathrm{M},K}(\rho_{s,i}, K)}{\sum_{i \in S_x} \lambda_{s,i}}. \tag{5.12}$$

For a packet generated at an x-hop node, we approximate the probability of reaching the gateway without blocking as

$$[1 - P_{\mathrm{W},s,b}(x)] \prod_{h=1}^{x-1} [1 - P_{\mathrm{W},r,b}(h)], \tag{5.13}$$

since all packets generated at the x-hop nodes, with $x = 1, 2, \ldots, H$, have to pass through its source node and $x - 1$ relay nodes without blocking to reach the gateway. With the non-blocking probability obtained in Eqn. (5.13), the aggregate throughput of the x-hop nodes can be expressed as the product of the source packet traffic input rate of the x-hop nodes and the non-blocking probability. Specifically, we define the aggregate source packet traffic output rate of the x-hop nodes

$$\sigma_{s,\mathrm{agg}}(x) = \sum_{i \in S_x} \sigma_{s,i} = \sum_{i \in S_x} \lambda_{s,i} [1 - P_{\mathrm{W},s,b}(x)], \tag{5.14}$$

which indicates the rate of source packets leaving the x-hop source nodes and would become the relay traffic in the intermediate relay nodes. Nodes with $x = 1$ hop to the gateway cannot be blocked at a relay node, while nodes with $x = 2, 3, \ldots, H$ hops need to be relayed by $x-1$ nodes without blocking to reach the gateway, resulting in the source traffic throughput of x-hop nodes in the WMN

$$T_{\mathrm{W}}(x) = \begin{cases} \sigma_{s,\mathrm{agg}}(1), & x = 1 \\ \sigma_{s,\mathrm{agg}}(x) \prod_{h=1}^{x-1} [1 - P_{\mathrm{W},b}(h)], & x = 2, \ldots, H. \end{cases} \tag{5.15}$$

From Eqn. (5.15), we note that since source traffic output rate $\sigma_{s,i}$ and blocking probability $P_{\mathrm{W},b}(x)$ are functions of channel access probability p_i and forwarding probability q_i, the throughput is also a function of both p_i and q_i, as numerically studied in Chapter 7.

The aggregate WMN throughput is obtained by summing $T_{\mathrm{W}}(x)$ over the hop distance x:

$$T_{\mathrm{W,agg}} = \sum_{x=1}^{H} T_{\mathrm{W}}(x). \tag{5.16}$$

5.2.2 *Delay of of the WMN Part with Poisson Input Traffic*

Building on the traffic rates at the individual wireless mesh nodes m_i examined in the preceding subsections, we derive in this subsection first the mean WMN delays under consideration of the individual heterogeneous traffic loads at the nodes.

We define the end-to-end delay of a packet in the WMN part as the time between when the first bit of the packet enters the local queue of the source node and when the last bit of the packet reaches the gateway. For a packet generated at an x-hop node, the end-to-end delay consists of the length of the x time slots for the packet transmissions and the queuing delays at the $x-1$ intermediate nodes providing relay service. For an x-hop node providing relay service, we approximate the average waiting time for the relayed packets in its relay queue Q_r as

$$W_{\mathrm{W,r,avg}}(x) = \sum_{i \in S_x} \frac{\sigma_{r,i} W_{\mathrm{M}}(\mu_{r,i}, \lambda_{r,i}, K)}{\sigma_{r,i}}, \tag{5.17}$$

whereby the mean waiting time $W_{\mathrm{M}}(\mu_i, \lambda_i, K)$ in an $M/M/1/K$ queue is obtained from Eqn. (A.5) in Appendix A. Similarly, we obtain the average waiting time for the source packets in its source queue $Q_r s$ as

$$W_{\mathrm{W,s,avg}}(x) = \sum_{i \in S_x} \frac{\sigma_{s,i} W_{\mathrm{M}}(\mu_{s,i}, \lambda_{s,i}, K)}{\sigma_{s,i}}. \tag{5.18}$$

With the knowledge of the average relay packet waiting time $W_{\mathrm{W,r,avg}}(x)$ and average source packet waiting time $W_{\mathrm{W,s,avg}}(x)$ in a given wireless mesh node with hop distance x, we obtain the expected end-to-end WMN delay as follows. A node with a hop distance $x = 1$ transmits a source packet to the gateway node when the transmission opportunity is granted to its Q_s. With the delay measurement starting when the first bit leaves the source node, the source packet traffic generated at 1-hop nodes experiences an end-to-end WMN delay corresponding to its queuing delay at the local queue plus transmission delay t_c. Source packet traffic generated at x-hop nodes with $x = 2, 3, \ldots, H$, needs to be queued in its Q_s and then transmitted x times and incurs the total queue waiting times $D_{\mathrm{W}}(x)$ at it source node and the relay nodes that are $1, 2, \ldots, x - 1$ hops from the gateway. In summary,

$$D_{\mathrm{W}}(x) = \begin{cases} t_c + W_{\mathrm{W,s,avg}}(x) & \text{if } x = 1 \\ x t_c + W_{\mathrm{W,s,avg}}(x) + \sum_{h=1}^{x-1} W_{\mathrm{W,avg}}(h) & \text{if } x = 2, \ldots, H. \end{cases} \tag{5.19}$$

The average end-to-end delay $D_{\mathrm{W,avg}}$ of the WMN part can be calculated by averaging the delays of packets reaching the gateways. Specifically, we weigh the delay $D_{\mathrm{W}}(x)$ experienced by x-hop nodes by the corresponding source traffic output rate $T_{\mathrm{W}}(x)$ of x-hop nodes:

$$D_{\mathrm{W,avg}} = \frac{\sum_{x=1}^{H} T_{\mathrm{W}}(x) D_{\mathrm{W}}(x)}{T_{\mathrm{W,agg}}}. \tag{5.20}$$

The analysis in this subsection provides the delay and throughput performance for the WMN part and we note that the analytical model does not limit the number Z of gateways, which makes this analysis applicable to other general WMNs.

5.2.3 Throughput-Delay Analysis for PON Part with Dynamic Bandwidth Allocation

When the packets are received by the gateways, they are immediately forwarded to the corresponding ONUs. Each ONU operates as a queue and transmits its queued packets to the OLT when transmission opportunities are given. Since all ONUs share the same physical optical bandwidth, several packet scheduling techniques have been proposed to efficiently utilize the bandwidth usage (Bontozoglou *et al.*, 2013; McGarry and Reisslein, 2012; Sue *et al.*, 2014; Zhou *et al.*, 2012). In this subsection, we study the performance of the gated DBA scheme.

First we study the packet arrival rates at the ONUs. At each ONU, the packets are forwarded directly from the corresponding gateway, i.e., an ONU and its corresponding gateway share the same input packet traffic. We define $g_z, z = 1, \ldots, Z$, as the number of 1-hop nodes in cluster z. We find that the incoming packet process at each gateway is Poisson since it is the superposition of several Poisson processes. Thus, the Poisson packet arrival rate at the gateway of cluster z is

$$\lambda_{\mathrm{D},z} = \sum_{j \in C_z} \sigma_j,$$

where C_z is the index of 1-hop nodes in cluster z.

For the dynamic bandwidth allocation (DBA) schemes, each ONU sends a Report message at the end of its packet transmission reporting the queue length as the request for bandwidth for its next packet transmission. After the OLT receives all Report messages form the ONUs, the transmission schedules for the next transmission of each ONU, named the Grant messages, are sent to the ONUs. With the Report and Grant messages, the optical bandwidth can be dynamically allocated among all ONUs.

Let us consider the gated DBA scheme where the OLT grants the ONUs with the sufficient bandwidth to transmit all packets in their Report messages. With gated DBA, an ONU is capable of utilizing the entire optical bandwidth which is not used by other ONUs. If the total WMN output rate is lower than the optical bandwidth and each ONU is equipped with sufficient large queue size, we can expect the output rates of all ONUs to be same as their input rates with gated DBA scheme. We assume the bandwidth used by Report and Grant messages can be ignored and with the above observations, we obtain the following equation

$$\mu_{\mathrm{D},z} = \frac{1}{t_{\mathrm{D}}} - \sum_{i=1, i \neq z}^{Z} \lambda_{\mathrm{D},i}, \text{if } \frac{1}{t_{\mathrm{D}}} > \sum_{i=1,}^{Z} \lambda_{\mathrm{D},i}, \tag{5.21}$$

where $\sum_{i=1,i\neq z}^{Z} \lambda_{\mathrm{D},i}$ is the optical bandwidth occupied by other ONU transmissions. Eqn. (5.21) shows that one ONU can have higher service rate if other ONUs have lower input traffic rates, which characterize the purpose of DBAs.

For the saturation cases where the total WMN output rate is higher than the optical bandwidth, all the ONUs will eventually reach the state where the traffic intensities are greater than 1. Since the ONUs with higher input traffic rates tend to request for higher bandwidth, the optical bandwidth is shared among all ONUs in proportion to their input traffic rates where

$$\mu_{\mathrm{D},z} = \frac{1}{t_{\mathrm{D}}} \frac{\lambda_{\mathrm{D},z}}{\sum_{i=1,}^{Z} \lambda_{\mathrm{D},i}}, \text{if} \frac{1}{t_{\mathrm{D}}} < \sum_{i=1,}^{Z} \lambda_{\mathrm{D},i}. \tag{5.22}$$

Similar to the argument made for the non DBA case, the input traffic of the ONUs with DBA is also Poisson. It is noted that the arrival of transmission opportunities at each ONU is random but not Poisson (since they would arrive consecutively within the granted transmission period) and we should model the ONUs as $M/G/1/K$ queues. But using the $M/G/1/K$ model requires the coefficient of variation of the service process, which is not available for the analysis. Hence we chose to approximate the ONUs as $M/M/1/K$ queues with service rate shown in Eqn. (5.21) and (5.22) since the traffic intensities and service rates are the more determinant factors in the queuing analysis. It would be shown in Chapter 7 that the $M/M/1/K$ queue approach still provides good estimation of the system performance.

5.2.4 Performance Analysis of Clustered FiWi Network

With the performance analysis for both the WMN and PON part derived in the preceding subsections, we can obtain the overall performance of the FiWi network. The aggregate FiWi throughput T_{F} is equal to the aggregate throughput of the PON part as given by

$$T_{\mathrm{O,agg}} = \sum_{z=1}^{Z} \lambda_{\mathrm{D},z}[1 - P_{\mathrm{M},K}(\rho_{\mathrm{D},z}, K)]. \tag{5.23}$$

Similar to the WMN analysis, we define the overall end-to-end delay of a packet in the FiWi network as the time between when the first bit of the packet enters the source node and when the last bit of the packet reaches the OLT. The delay can be calculated by adding the average delays generated at the wireless mesh nodes and the ONUs. For the packets generated at the x-hop node, the average delay $D_{\mathrm{F}}(x)$ is

$$D_{\mathrm{F}}(x) = D_{\mathrm{W}}(x) + W_{\mathrm{O}} + t_{\mathrm{D}}, \tag{5.24}$$

where t_{D} is the transmission delay at the ONU. The average end-to-end delay of a packet can be calculated as

$$D_{\mathrm{F,avg}} = D_{\mathrm{W,avg}} + W_{\mathrm{O}} + t_{\mathrm{D}}, \tag{5.25}$$

with the average WMN delay $D_{\mathrm{W,avg}}$ given in Eqn. (3.20) , and the average PON delay W_{O} is given as

$$W_{\mathrm{O}} = \frac{\sum_{z=1}^{Z} W_{\mathrm{M}}(\mu_{\mathrm{D},z}, \lambda_{\mathrm{D},z}, K)\lambda_{\mathrm{D},z}[1 - P_{\mathrm{M},K}(\rho_{\mathrm{D},z}, K)]}{T_{\mathrm{O,agg}}}. \tag{5.26}$$

Chapter 6

NETWORK DESIGN STRATEGY

In (Chen and Reisslein, 2015), we studied the FiWi performance with heavy-loaded input traffic. With the heavy-loaded traffic, all transmission opportunities given to the wireless mesh nodes are utilized, which gives the highest throughput of the network but also introduces the highest delay performance. With the knowledge of Poisson input traffic studied in the Chapter 4, we propose (a) the input traffic design strategy for any given channel access probability design and (b) the channel access probability design that can be used in both Poisson and heavy-loaded input traffic.

6.1 Input Traffic Rate Design

With the knowledge of the queuing theory, we know that the average queue size and actual output rate increase when the input rate increases and service rate remains fixed. As the input rate reaches the value where $\rho > 1$, the actual output rate would be limited by the service rate and the average queue size increases rapidly until it approaches the value of k. Subsection 5.1.4 also shows that the transmission opportunities given to a wireless node will be dynamically distributed between its local and relay queues if the source or relay packet traffics do not exceed certain thresholds. Such design shows that for a given wireless mesh node, the output rate can be maximized while maintaining a reasonable delay if the sum of source and relay packet traffic rates is equal to the total packet service rate given to the node. First we propose the following design strategy in the hop distance perspective where the wireless mesh nodes with the same hop distance have the same channel access probability, channel forward probability and local input traffic rate,

$$
\begin{aligned}
p_i &= p(x) \ \forall \ h_i = x \\
q_i &= q(x) \ \forall \ h_i = x \\
\lambda_{s,i} &= \lambda_s(x) \ \forall \ h_i = x.
\end{aligned}
$$

It starts with edge nodes, i.e., the H-hop nodes, which only have to serve the source traffics. Our goal is to limit the total input traffic rate at the H-hop nodes to be lower than the total service rate given to H-hop nodes:

$$
N(H)\lambda_s(H) \le N(H)\frac{p(H)}{t_c}, \tag{6.1}
$$

where $N(H)p(H)/t_c$ is the total service rate of the H-hop nodes. For the $H-1$ hop nodes, total of $N(H-1)p(H-1)/t_c$ services rates are available for the source packets and the relay packets from the H-hop nodes. The goal is also to limit the total traffic rate of the source packets at the $H-1$-hop nodes plus the relay traffic form the H-hop nodes to be lower than the total service rate given to $H-1$-hop nodes:

$$
N(H)\lambda_s(H) + N(H-1)\lambda_s(H-1) \le N(H-1)\frac{P(H-1)}{t_c},
$$

where $N(H)\lambda_s(H)$ is the maximum possible total relay traffic input rate form the H-hop nodes which is also restricted with the maximum value $N(H)P(H)/t_c$ according to Eqn. (6.1). Similar strategies can be applied to the following x-hop nodes where $1 \le x \le H$ and we obtain the following liner program

$$
\begin{aligned}
\text{Maximize} \quad & \sum_{x=1}^{H} N(x)\lambda_s(x) \\
\text{Subject to} \quad & \sum_{i=x}^{H} N(x)\lambda_s(x) \le \frac{N(x)P(x)}{t_c}, x = 1, \cdots, H,
\end{aligned}
$$

where the maximum objective is to maximize the input rate while not violating the bandwidth restrictions in hope that the system throughput can be maximized while not overflowing the buffers constantly. If all the wireless mesh nodes in the network have the same source traffic input rate, the maximized source traffic input rate solution of the linear program is

$$\lambda_{s,\text{opt}} = \min\left[\frac{p(x)/t_c}{1+\sum_{i=x+1}^{H}\frac{N(i)}{N(x)}}, x = 1, \ldots, H\right]. \tag{6.2}$$

If the network is perfectly designed that all relay nodes have the same relay traffic input rates, the proposed linear programming is capable of approaching the maximum system throughput. In Chapter 7, we show that such strategy can still reduce the delay significantly while maintain a high throughout of the network when the network is not perfectly designed where uneven traffic loads could be found among the nodes with the same hop distances.

6.2 Channel Access Opportunity Design at Hop Distance Level

In the proceeding section, we proposed the Poisson input traffic rate design strategy for FiWi networks. We can observe that higher transmission opportunities should be given to the nodes with lower hop distances since they have to provide relay service for more packets form the nodes with higher hop distances. We propose an easy channel access probability set design inspired by the result obtained in Eqn. (6.2). Let us consider a network where the service rate at the H-hop nodes is equal to $\lambda_s(H)$. The total service rate at the H-hop nodes is also equal to the total source input rate, which gives

$$N(H)\frac{p(H)}{t_c} = N(H)\lambda_s(H).$$

For the $(H-1)$-hop nodes, they have to provide sufficient service rate to the relay packets form the H-hop nodes and the their source packets, which gives

$$N(H-1)\frac{p(H-1)}{t_c} = N(H-1)\lambda_s(H-1) + N(H)\frac{p(H)}{t_c} \tag{6.3}$$

where $N(H)p(H)/t_c$ is the highest possible relay packet input rate for $(H-1)$-hop nodes. With the same logic we obtain the channel access probability design strategy for all $p(x)$. If we consider the case where all wireless mesh nodes have the same local input traffic rate λ_s, we can further simplify Eqn. (6.3) as

$$N(H-1)\frac{p(H-1)}{t_c} = [N(H-1)+N(H)]\frac{p(H)}{t_c},$$

and further obtain the following design rule

$$p(x) = \frac{\sum_{i=x}^{H} N(i)}{N(x)} p(H), x = 1, \ldots, H. \tag{6.4}$$

Form the above derivations, we observe that the total channel access probability of the x-hop nodes grows as x decreases, which match the results obtained in (Chen and Reisslein, 2015)(Liu and Liao, 2008). We should note that special case of Eqn. (6.4) gives the same $p(x)$ values as Eqn. (A.9) but with more straight forward forms and less calculations. For the forwarding probability set, our design is to give fair share of bandwidth for traffics form nodes with different hop distances. Since all nodes are assumed to have same source input rate, we can simply calculate traffic amount in terns of node numbers and hence

$$q(x) = \frac{\sum_{i=x+1}^{H} N(i)}{\sum_{i=x}^{H} N(i)}, x = 1, \ldots, H, \tag{6.5}$$

where $\sum_{i=x+1}^{H} N(i)$ is the number of nodes with hop distance higher than x and such $q(x)$ gives the portion of relay traffic to the total input traffic.

6.3 Channel Access Opportunity Design at Node Level

In the proceeding section, we introduced the network design strategy at the hop distance level. In real network topologies, the hop-level design may introduce performance degradation since wireless mesh nodes with same hop distance could have different relay traffic input rates. If the service rate is lower than the input traffic rate, lower throughout and higher delay are expected. Hence we propose the wireless network bandwidth design based on individual nodes. The main purpose of the individual node level design shares the same criteria as the hop distance level design, which equips each node with same service rate as its input traffic rate. For wireless mesh node m_i, the channel access probability design is

$$\frac{p_i}{t_c} = \sum_{x \in R_i} \frac{p_x}{f_i t_c} + \lambda_i, \tag{6.6}$$

where $\sum_{x \in R_i} p_x/(f_i t_c)$ is the highest relay traffic input rate. We note that for the edge nodes with hop distance H, $p_i = \lambda_i \cdot t_c$ since they do not have any relay traffic. For the forwarding probability, our design is also to give fair share of bandwidth for relay traffic,

$$q_i = \frac{\sum_{x \in R_i} \frac{p_x}{f_i t_c}}{\sum_{x \in R_i} \frac{p_x}{f_i t_c} + \lambda_i}. \tag{6.7}$$

In Chapter 7, we show that such node level design can provide better performance than the hop distance level designs.

Chapter 7

NUMERICAL EVALUATION OF FIWI NETWORKS WITH POISSON INPUT TRAFFIC

In this chapter, we study the performance of FiWi networks with Poisson input traffic. The effect of the network designs proposed in Chapter 6 is also examined. For the simulation, we use the same network setup in Chapter 4 for consistency purpose. The results show that our proposed controlled input traffic and node level network design are both capable of improving the FiWi performance.

7.1 Impact of Controlled Input Traffic

In the section, we study the effect of controlled input traffic rates with hop distance level network designs. For the hop distance level network designs, we use the pth and pde settings in Chapter 4 where the channel access probabilities can be obtained via both Eqn. (6.4) and (A.9). According to the channel access probabilities and Eqn. (6.2), the controlled input traffic rates are obtained and listed in Table 7.1. We note that since pth and pde settings have the same controlled input traffic rates since they also have the same channel access probability values.

Fig. 7.1 shows the delay and throughput performance of pth and pde settings with controlled and heavy-loaded input traffic. It is noted that the heavy-loaded traffic is capable of reaching the maximum system throughput since the local traffic would ensure that no channel access probability would be unused and fulfill the output rate of the wireless mesh nodes (Chen and Reisslein, 2015). In Fig. 7.1, We observe that for both pth and pde settings, the proposed controlled input traffic method is able to reduce around 50% of the delay while maintaining about 80% of throughput

Table 7.1: Controlled Input Traffic Rates of Pth and Pde Settings for Varying Numbers for Clusters.

Z	pth and pde
1	0.0018
2	0.0029
3	0.0037
4	0.0043
5	0.0048
6	0.0048
7	0.0052
8	0.0053
9	0.0053
10	0.0054

comparing to the heavy-loaded traffic. For the controlled input traffic method for pth and pde settings, total input traffic rate is equal to the highest possible system throughput and it can not guarantee that all wireless mesh nodes can provide output rates equals to their service rates and hence decreases of the system throughput.

In (Liu and Liao, 2008)(Chen and Reisslein, 2015), it is stated that pde setting can provide lower delay while providing same system throughput comparing to pth setting (see Fig. 4.2). It is noted that (Liu and Liao, 2008)(Chen and Reisslein, 2015) both consider only the delay at the relay queues. In Fig. 7.1, we show that pth and pde would have similar delay performance if the delays at the local queues are also considered. The above scenario can be explained as follows. Since the transmission opportunities of a wireless mesh node is shared by its local and relay queues, giving the transmission opportunity to one queue would introduce delay to the other queue.

Since pde provides extreme high transmission priority to the relay queue, it also introduces high delay to the local queue. If only the delays at the relay queues are considered, pde does reduce the relay delay but if we also consider the delays at the local queues, the overall delay may not be improved. For the overall network performance, pde does not provide significant improvement comparing to pth, but it does provide performance improvement to the node with higher hop distances and the improvements would be shown later in Section 7.4.

7.2 Controlled Input Traffic and Channel Access Probability Design at Node Level

In this section, we study the system performance of channel access probability design at node level, which allows all wireless mesh nodes to have different channel access probability p_i and forwarding probability q_i. Similar to the hop distance design, we assume

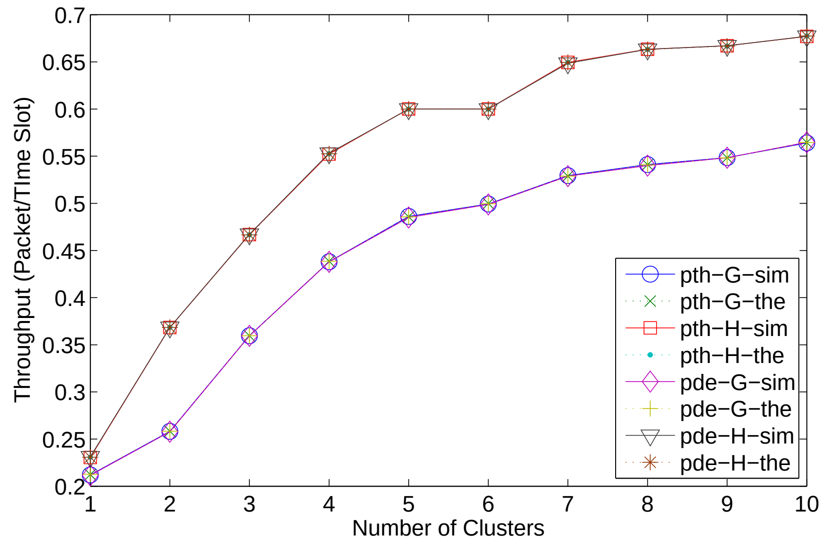
$$\sum_{i=1}^{N} p_i = 1. \tag{7.1}$$

and the values of p_i and q_i are designed according to Eqn. (6.6) and (6.7) with $\lambda_{s,i} = \lambda_s$. We find that with

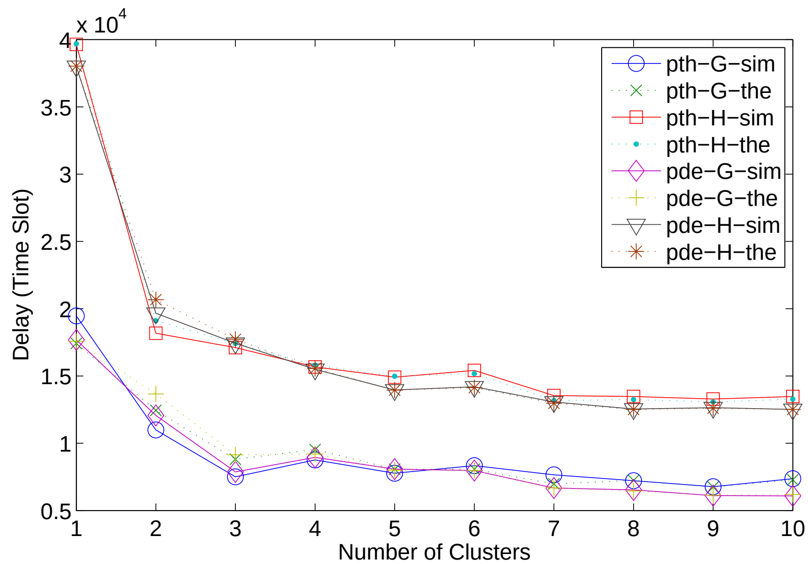
$$\lambda_s = \frac{1}{\sum_{i=1}^{N} h_i},$$

Eqn. (7.1) is satisfied. we name the above node level network design as pop. For the the controlled input traffic, $\lambda_i = \lambda_s$ and for the heavy-loaded input traffic $\lambda_i = 5p_i$.

Fig. 7.2 shows the FiWi delay and throughput performance of the pop setting. It is shown that the pop setting with controlled input traffic is capable of approaching the maximum system throughput while reduce about 70% of the delay comparing to the heavy-loaded input traffic. Fig 7.3 shows the performance comparison between the pop and pth settings and we find that pop and pth settings have the same maximum

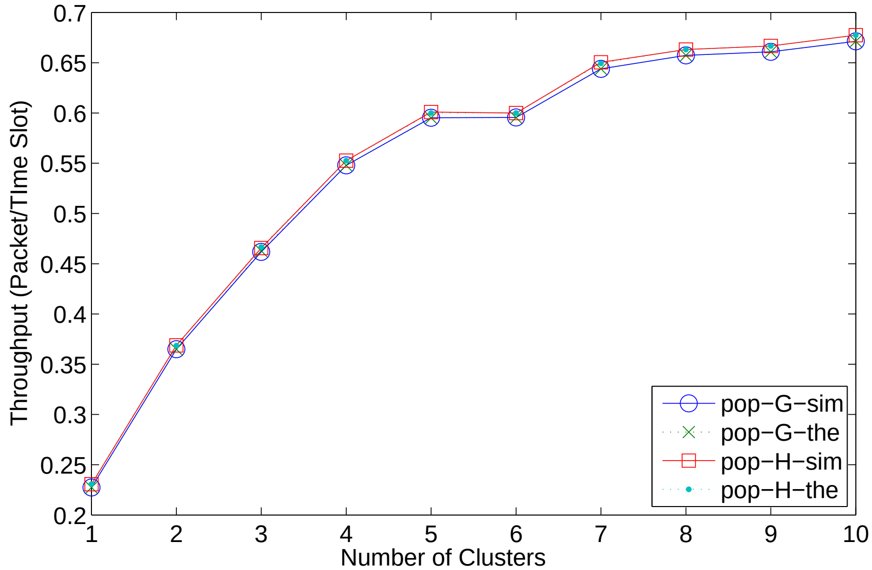


(a) Throughput of FiWi with pth and pde settings and Controlled and heavy-loaded traffic

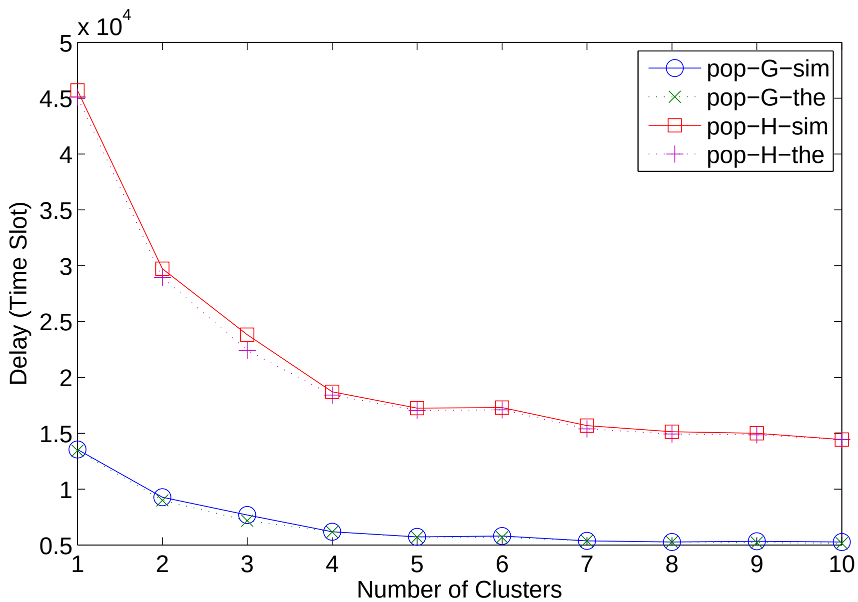


(b) Delay of FiWi with pth and pde settings and Controlled and heavy-loaded traffic

Figure 7.1: Mean Throughput and Delay Characteristics of FiWi as a Function of Number of Clusters Z with Hop Distance Level Design.



(a) Throughput of FiWi with pop setting and Controlled and heavy-loaded traffic



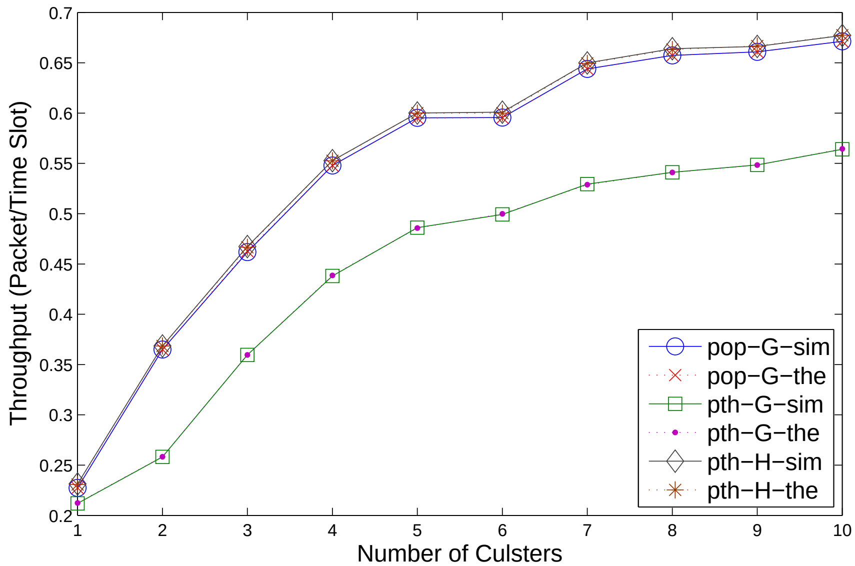
(b) Delay of FiWi with pop setting and Controlled and heavy-loaded traffic

Figure 7.2: Mean Throughput and Delay Characteristics of FiWi as a Function of Number of Clusters Z with Node Level Design.

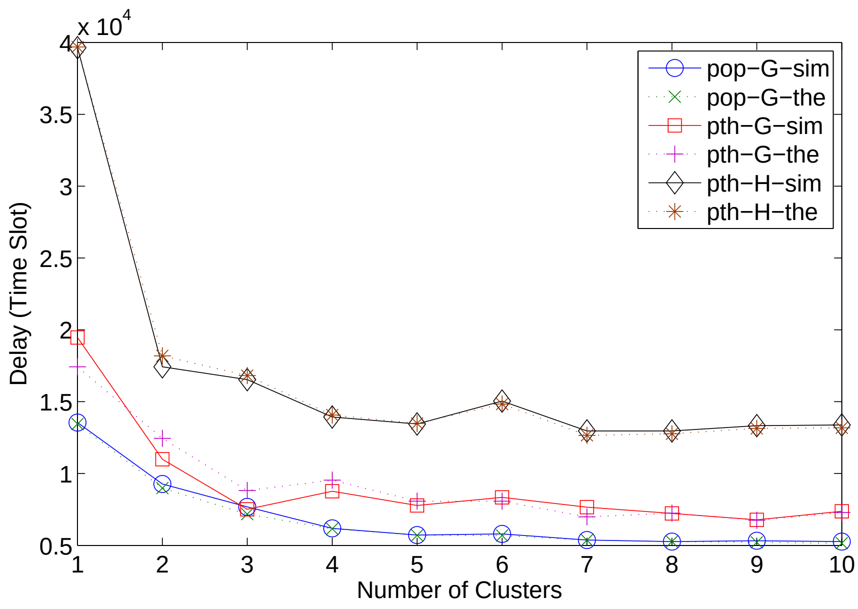
system throughput since they have similar design strategy and pop setting is capable of achieving the maximum throughput while greatly reducing the delay comparing to pth setting. It shows that assigning the channel access probability according to the traffic characteristics of each individual node could benefit in both delay and throughput performance.

7.3 DBA performance Analysis for PON Part

In Subsection 4.3.2, we show that the effect of PON part only becomes noticeable when the WMN output rate is close or larger than the optical service rate. In this section, we use the pth setting with controlled input traffic to demonstrate the effect of the gated DBA scheme. We assume that the PON service rate is 1/2 of the warless bandwidth (ow-ratio=0.5). Fig. 7.4 shows the delay and throughput performance of the PON part. For the scenarios where the WMN output rates are relative lower than the optical service rate ($Z = 1 \sim 3$), the PON introduces low delay and no packet blocking for both gated DBA and non-DBA schemes. For $Z = 4 \sim 6$ where the WMN output rates are close to the optical service rate, it is shown that gated DBA is capable of reducing the optical delay by assigning the bandwidth according to the input traffic of each ONU. For $Z \leq 7$ it is show that the output rates of gated DBA are capable of reaching the service rate but introduces higher optical delay. It is due to the fact that for the non DBA scheme, some ONUs have traffic intensity larger than 1 while other ONUs have intensity lower than 1. Such scenario results in lower output rate and lower delay comparing to the gated DBA scheme, where all ONUs have traffic intensities larger than 1. In conclusion, we show that DBA is able to reduce the optical delay but it is still desired to design the system that the PON service rate being higher than the WMN output rate. The results also show that our $M/M/1/K$ queue approach is capable of estimating the gated DBA behaviors.

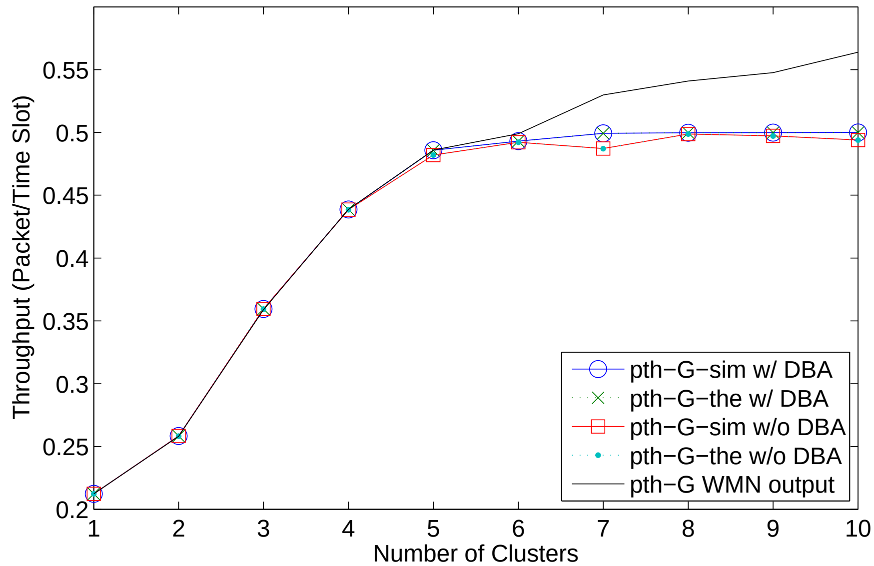


(a) Throughput comparison of pop and pth setting

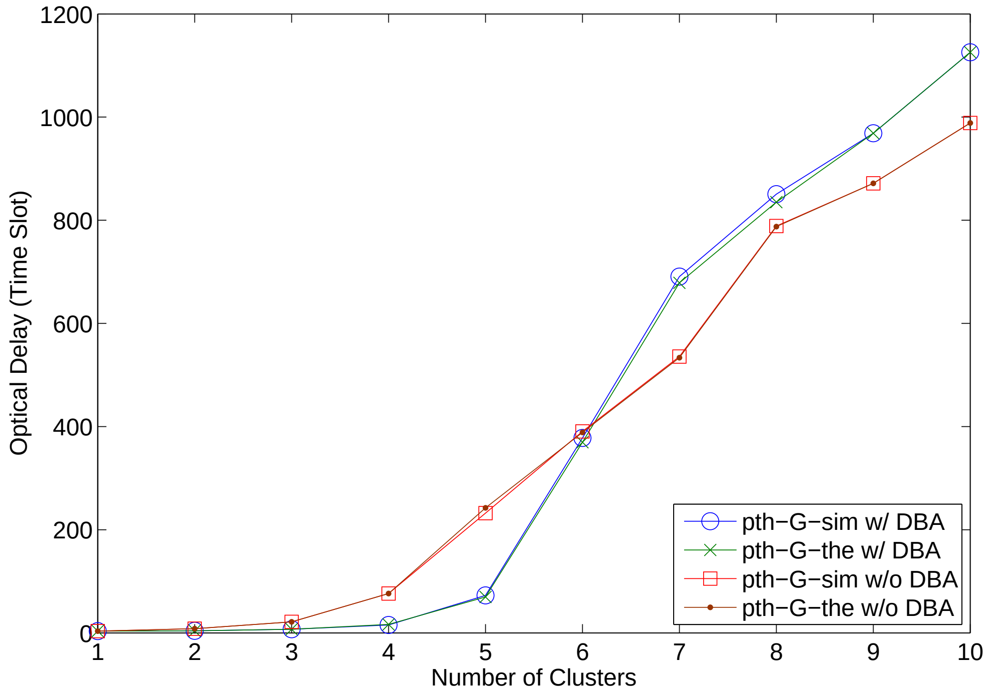


(b) Delay comparison of pop and pth setting

Figure 7.3: FiWi Performance Comparison of Node level and Hop Distance Level Design



(a) Optical throughput with pth setting and controlled input traffic

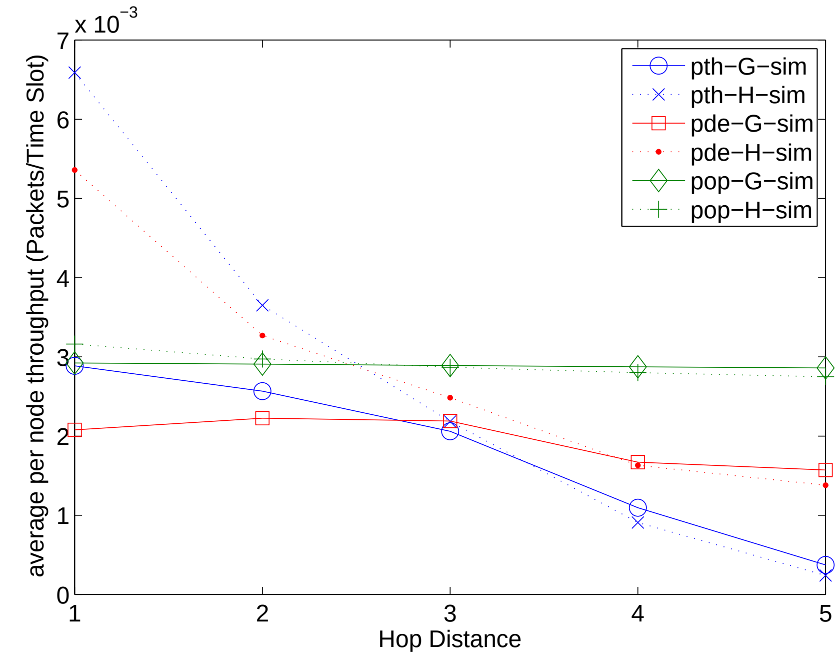


(b) Optical delay with pth setting and controlled input traffic

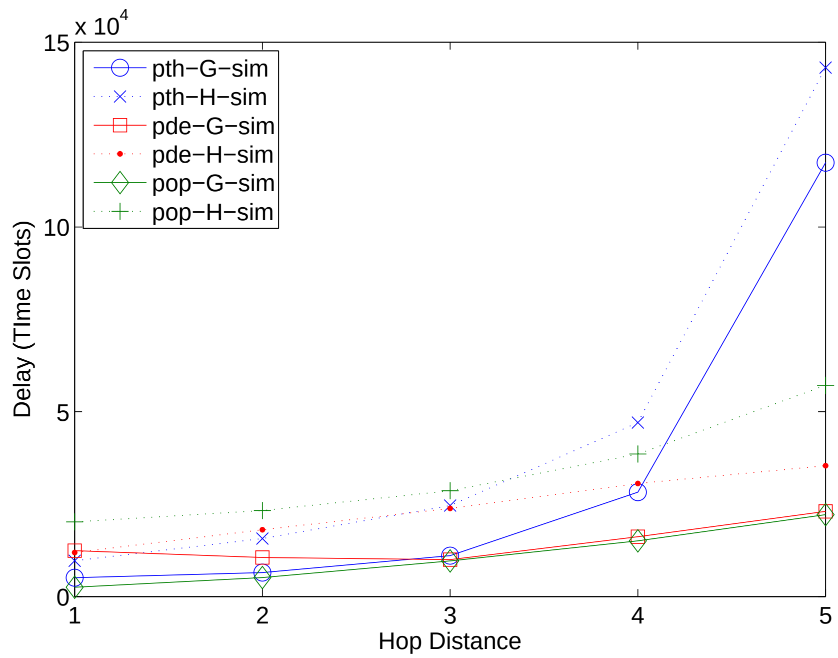
Figure 7.4: Mean Throughput and Delay Characteristics of PON as a Function of Number of Clusters.

7.4 QoS Study: Performance of Nodes Different Hop Distances

In this section, we study the delay and throughput performance based on the hop distances. Fig. 7.5 and 7.6 show the average delay and throughput of nodes with different hop distances of FiWi networks with 2 and 3 clusters. In the figures, we find that pth setting has the worst performance balance since the nodes with lower hop distances would have much higher per node throughput and average delay than nodes with higher hop distances and the unbalance worsens with the heavy-loaded input traffic. Pde setting has better performance balance comparing to pth, but nodes with lower hop distances still tend to have better performance in both throughput and delay. Pop setting provides fair per node throughput to nodes with all hop distances and the delay unbalance is the lightest. We note that the nodes with higher hop distances always tend to have higher delay and it is the nature of WMN that can hardly be solved. If a balanced FiWi system performance is desired, the node level channel access probability and controlled input traffic design is recommended.

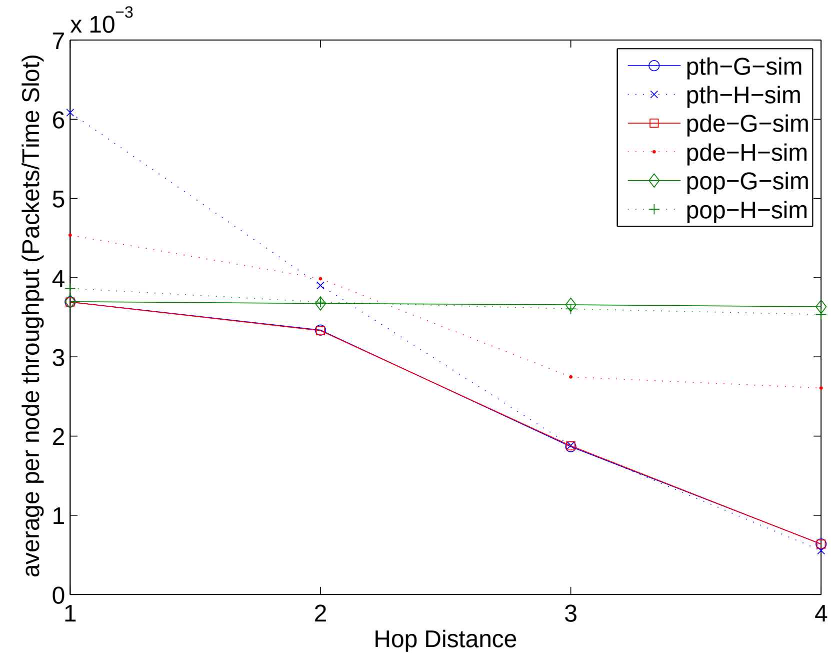


(a) average per node throughput of different hop distance and Z=2

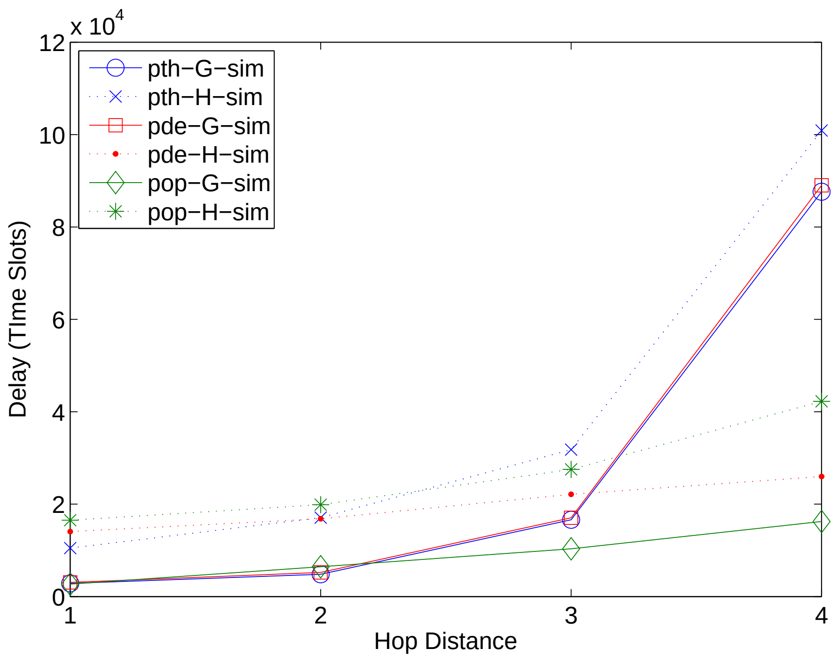


(b) average packet delay of different hop distance and Z=2

Figure 7.5: Mean Throughput and Delay Characteristics of Nodes with Different Hop Distances and Z=2.



(a) average per node throughput of different hop distance and Z=3



(b) average packet delay of different hop distance and Z=3

Figure 7.6: Mean Throughput and Delay Characteristics of Nodes with Different Hop Distances and Z=3.

Chapter 8

CONCLUSION AND FUTURE WORKS

We have developed a low-complexity, yet reasonably accurate analytical model for the throughput-delay evaluation of a clustered Fiber-Wireless (FiWi) network. The partitioning of a Wireless Mesh Network (WMN) into several small clusters, each supported by an Optical Network Unit (ONU) of a Passive Optical Network (PON) leads typically to highly heterogeneous traffic loads at the wireless mesh nodes with a prescribed hop distance to the ONU. Previous WMN analysis techniques based on the mean traffic load at the nodes with a given hop distance to the gateway fail to model such heterogeneous node traffic loads. We introduced a throughput-delay analysis based on the individual nodal traffic loads so as to enable the evaluation of FiWi networks consisting of WMNs with heterogeneous node traffic loads.

Our evaluations of the effects of superimposing a FiWi network onto an existing WMN indicated that partitioning a WMN into an increasing number of clusters generally improves the throughput-delay performance, particularly compared to a WMN without clusters or a small number of clusters. However, dividing a WMN into many clusters does not always improve performance. Rather, the FiWi performance is limited by the WMN part when the throughput of the WMN part is lower than the PON transmission rate. When the throughput of the WMN part exceeds the PON transmission rate, the WMN delay decrease achieved by increasing the number of clusters Z, can be counter-compensated by increasing delay in the PON part. Also, the limitation of the FiWi network throughput by the PON bandwidth can cause increasing packet drop probabilities as the WMN throughput is increased by increasing the number of clusters Z. The input traffic control and the network design proposed in this dissertation further utilize the FiWi network performance. We also show that the fairness in average per node throughput can be achieved by the node level network design.

There are many exciting directions for future research on clustered FiWi networks. One important direction is to examine the network planning issues arising from clustered FiWi networks, i.e., the specific planning of the node clusters and placement of the ONUs according to traffic demands and the constraints of existing infrastructure (Liu *et al.*, 2013, 2011; Sarkar *et al.*, 2009). Moreover, it is important to study the integration of the traffic flows from the WMN clusters into the overall traffic management of optical PON access networks and their interconnection to optical metropolitan area networks, e.g., (Ahmed and Shami, 2012; Castoldi *et al.*, 2009; Scheutzow *et al.*, 2003). Another direction is to explore the interactions between the bandwidth allocations to the ONUs of the PON, the WMN clusters, and the source nodes, such as individual wireless local area networks or sensor networks feeding traffic into the WMN clusters, see e.g. (Wang *et al.*, 2012b; Zaker *et al.*, 2014).

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APPENDIX A

QUEUING THEORY AND APPLICATION REVIEWS

A.1 Review of $M/M/1/K$ Queue

Define the traffic intensity as $\rho = \lambda/\mu$, where λ and μ are the packet arrival rate and the packet service rate of the queue holding at most K packets. The queue holds K packets, i.e., blocks newly arriving packets, with probability (Gross and Harris, 1998):

$$P_{\mathrm{M},K}(\rho, K) = \begin{cases} \frac{(1-\rho)\rho^K}{1-\rho^{K+1}} & \text{if } \rho \neq 1 \\ \frac{1}{K+1} & \text{if } \rho = 1. \end{cases} \tag{A.1}$$

The probability of the queue being empty is

$$P_{\mathrm{M},0}(\rho, K) = \begin{cases} \frac{1-\rho}{1-\rho^{K+1}} & \text{if } \rho \neq 1 \\ \frac{1}{K+1} & \text{if } \rho = 1 \end{cases} \tag{A.2}$$

and $P_{0,i}$ in Eqn. (3.5) can be obtained as

$$P_{0,i} = P_{\mathrm{M},0}(\rho_i, K). \tag{A.3}$$

The average queue length is (Gross and Harris, 1998):

$$L_{\mathrm{M}}(\rho, K) = \begin{cases} \frac{\rho}{1-\rho} - \frac{\rho(K\rho^K+1)}{1-\rho^{K+1}} & \text{if } \rho \neq 1 \\ \frac{K(K-1)}{2(K+1)} & \text{if } \rho = 1. \end{cases} \tag{A.4}$$

The average waiting time is

$$W_{\mathrm{M}}(\mu, \lambda, K) = \frac{1}{\mu} + \frac{L_{\mathrm{M}}(\rho, K)}{\lambda[1 - P_{\mathrm{M},K}(\rho, K)]}. \tag{A.5}$$

A.2 Review of $M/D/1/K$ Queue

Define input packet rate λ, output packet rate μ, and traffic intensity $\rho = \lambda/\mu$. Denote $P_{\mathrm{D},k}(\rho, K),\ k = 0, \ldots, K$, for the stationary state probabilities of holding k packets in the queue. For $0 \leq k \leq K-1$, the steady state probability can be obtained with the recursion (Gross and Harris, 1998):

$$P_{\mathrm{D},k}(\rho, K) = \lambda a_{k-1} P_{\mathrm{D},0}(\rho, K) + \lambda \sum_{j=1}^{k} a_{k-j} P_{\mathrm{D},j}(\rho, K), \tag{A.6}$$

where $a_n = \frac{1}{\lambda}(1 - \sum_{j=1}^{n} e^{-\rho}\rho^j/j!)$. The Kth state probability, i.e., the blocking probability, is

$$P_{\mathrm{D},K}(\rho, K) = \rho P_{\mathrm{D},0}(\rho, K) - (1-\rho) \sum_{j=1}^{K-1} P_{\mathrm{D},j}(\rho, K). \tag{A.7}$$

The recursion starts with $P_{\mathrm{D},0} = 1$ and the state probabilities are normalized with the equation $\sum_{i=0}^{K} P_{\mathrm{D},i}(\rho, K) = 1$. An explicit formula for $P_{\mathrm{D},k}(\rho, K)$ is derived in (Brun and Garcia, 2000), but the calculation process involves a large number operations for large K and may not be suitable for computational work (Tijms, 2006). With the state probabilities, the average waiting time $W_{\mathrm{D}}(\mu, \lambda, K)$ of an $M/D/1/K$ queue can be evaluated by applying Little's law:

$$W_{\mathrm{D}}(\mu, \lambda, K) = \frac{1}{\mu} + \frac{L_{\mathrm{D}}(\rho, K)}{\lambda[1 - P_{\mathrm{D},K}(\rho, K)]}, \tag{A.8}$$

where $L_{\mathrm{D}}(\rho, K) = \sum_{k=0}^{K} k P_{\mathrm{D},k}(\rho, K)$ is the average length of the $M/D/1/K$ queue.

A.3 Bandwidth Fair Sharing for WMN

One of the major problems of a WMN is the fairness share problem (Gambiroza *et al.*, 2004; Lee *et al.*, 2008) where the nodes with higher hop distance suffer from lower throughput compared to the nodes with lower hop distance. For a TDMA system, it is desired that the wireless nodes closer to the gateways should be allocated more radio resources, i.e., higher channel access probability $p(x)$ for lower hop count x, since they have to provide more relay services. If the scheduling scheme failed to provide sufficient radio resources to the wireless nodes closer to the gateways to maintain a reasonable relay traffic intensity, then low throughout and high delay would occur due to frequent buffer overflow and further affect the overall performance of the WMN. Liu and Liao (Liu and Liao, 2008) proposed the following wireless channel allocation scheme which we apply to the FiWi network:

$$\frac{p(x)}{p(x+1)} = N_r(x)\left[1 + \frac{1}{R(x)}\right], \; x = 1, 2, \ldots, H-1. \tag{A.9}$$

where

$$R(x) = \sum_{i=x}^{H} \prod_{j=x}^{i} N_r(i), \quad x = 1, 2, \ldots, H-1,$$

and

$$N_r(x) = \begin{cases} \frac{N(x+1)}{N(x)} & \text{if } x = 1, \ldots, H-1 \\ 0 & \text{if } x = H. \end{cases} \tag{A.10}$$

Equation (A.9) gives the $p(x)$ design criteria which provide fair throughput to all wireless mesh nodes regardless of the hop distances under the assumption that the relayed traffic is distributed evenly among the wireless mesh nodes. Inequality

$$q(x) > 1 - \frac{1}{1 + R(x)}. \tag{A.11}$$

specifies a lower bound for the forwarding probability $q(x)$ ensuring that an x-hop node is capable of providing fair bandwidth allocation (Liu and Liao, 2008).