Park-and-Ride Facilities Design for Special Events

Using Space-Time Network Models

by

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ABSTRACT

Given that more and more planned special events are hosted in urban areas, during which travel demand is considerably higher than usual, it is one of the most effective strategies opening public rapid transit lines and building park-and-ride facilities to allow visitors to park their cars and take buses to the event sites. In the meantime, special event workforce often needs to make balances among the limitations of construction budget, land use and targeted travel time budgets for visitors. As such, optimizing the park-and-ride locations and capacities is critical in this process of transportation management during planned special event. It is also known as park-and-ride facility design problem.

This thesis formulates and solves the park-and-ride facility design problem for special events based on space-time network models. The general network design process with park-and-ride facilities location design is first elaborated and then mathematical programming formulation is established for special events. Meanwhile with the purpose of relax some certain hard constraints in this problem, a transformed network model which the hard park-and-ride constraints are pre-built into the new network is constructed and solved with the similar solution algorithm. In doing so, the number of hard constraints and level of complexity of the studied problem can be considerable reduced in some cases. Through two case studies, it is proven that the proposed formulation and solution algorithms can provide effective decision supports in selecting the locations and capabilities of park-and-ride facilities for special events.
DEDICATION

I dedicate this work to, and would like to thank my loving parents, husband Pengfei and my beloved daughter Acadia for supporting me throughout the entire course of my master study. All of them have been my best support. I also dedicate this thesis to my many friends who have supported me throughout the process and I will always appreciate all they have done.
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The same appreciation goes to my committee members: Dr. Mikhail Chester and Dr. Yingyan Lou. I thank them for serving as my committee members and providing insightful suggestions to my thesis.

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CHAPTER

1 INTRODUCTION

Background

Over the last thirty years the transportation planning domain has been attracting more and more research efforts which in turn considerably relieved traffic congestion in many metropolitan areas. Among the efforts devoted in this area, a majority is related to forecasting and managing daily traffic demand in the near future as well as in a long-range transportation horizon. In the meantime, traffic operations under special conditions have become increasingly crucial to creating a safe and efficient traffic environment. Those special conditions include but are not limited to: sporting events, conventions, emergent conditions. The travel demand under these special conditions is also quite different from that under normal conditions, and therefore needs full recognition for its complex structural deviations.

Motivation and Problem Illustration

During special events, facilities are usually faced by higher traffic demand volume than normal, accompanied with special spatial distribution and traffic pattern. Since a large number of governmental agencies are paying more and more attention to establishing proactive traffic management plans for all kinds of traffic situations, it is necessary to study special event traffic planning with more detailed investigation and rigorous forecasting.

A large-scale special event may attract a few weeks of super-imposed traffic demand on urban traffic network with extraordinarily high concentrations. For example,
an Olympic Game is typically able to attract 500,000 visitors per day and 200,000 more
of logistics personnel daily, which in total leads to nearly 150 million in a single day
(Bovy, 2003). Therefore, the transportation organization/authorities are typically required
to develop special traffic management plans and coordinate among multi-mode
infrastructure and network service during those special events in order to mitigate
potential traffic congestion while still maintaining appropriate accessibility to the event
sites.

To maintain a high level of accessibility to special event sites, the locations and
capacities of park-and-ride stations are critically important. Multi-modal traffic pattern
can facilitate the interchange between private/lower occupancy traffic modes (e.g., cars)
to public/higher occupancy modes (e.g., buses) and further help to complete trip chains
through a sustainable multi-modal service network (Spillar, 1997). In the recent Olympic
Games from in Atlanta in 1996 to in London in 2012, the park-and-ride mode has been
widely used and demonstrated its potential in integrating an accessible public transport
system to a well-planned and comprehensive transportation system for managing
complex traffic demand at mega events (Currie, 2012).

In this thesis, the focus is on how to optimize the locations and additional parking
capacity of park-and-ride facilities. This goal is to allow a large number of tourists to
successfully complete their trip within a reasonable travel time budget and travel chain.

Objectives

The objectives of this study are to construct space-time network models to
evaluate park-and-ride facilities during special events, to develop mathematical
programming models to investigate the maximal accessibility issues and to design mixed linear integer programs and develop solution algorithms. Secondly, it is one of the objectives to employ the proposed solution framework in two numerical experiments: a simple three-corridor network and a realistic case study. The Lagrangian relaxation and decomposition is also utilized in this study to solve the integer linear programming problem. It is expected that the insights, analysis, modeling and conceptual description for the accessibility-based network design can provide inspiration and guidelines for researches in this area.
2 LITERATURE REVIEW ON NETWORK DESIGN PROBLEM FORMULATION AND SOLUTIONS

This study focuses on developing space-time network models for park-and-ride facilities design to maximize their accessibility during special events. The literature review starts with general network design problems and followed by previous research efforts in space-time network modeling.

General Network Design Problems and Mathematic Programming Methods

Network design problem was first proposed by Dantzig (1965) as a fixed cost transshipment problem. It was solved by linear programming to determine transporting activities with non-negative constraints satisfying material balance and minimizing transportation cost. Inspired by this early research, this categorical mathematic problem has been well developed in full range at strategic, tactical and operational planning levels. Many researchers have made great progress in transportation planning disciplines based on network design problems with various specific transportation topics which theoretically aims to find optimal locations and utilization of resources to achieve certain objectives (Crainic, 2000). A comprehensive review was conducted by Magnanti and Wong (1984), in which integer programming-based approaches, as well as several discrete and continuous choice models of network design problems were evaluated. The authors further elaborated their usages and limitations respectively. Magnanti and Wong (1984) also examined general versions of network design problems followed by several specializations of network design models which were listed in Table 1.
Table 1

Specializations and Variations of the Transportation Network Design Models

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Demand Structure</th>
<th>Objective Function</th>
<th>Capacities</th>
<th>Side Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal spanning tree</td>
<td>Complete (Undirected network)</td>
<td>Linear in design variables, no flow costs</td>
<td>Uncapacitated</td>
<td>None</td>
</tr>
<tr>
<td>Shortest Path</td>
<td>Arbitrary</td>
<td>Linear in flows, no design costs</td>
<td>Uncapacitated</td>
<td>None</td>
</tr>
<tr>
<td>Steiner tree problem</td>
<td>Complete on a subset of nodes (Undirected network)</td>
<td>Linear in design variables, no flow costs</td>
<td>Uncapacitated</td>
<td>None</td>
</tr>
<tr>
<td>Multi-commodity flow problem</td>
<td>Arbitrary</td>
<td>(Non) Linear flow costs, no design costs</td>
<td>Arbitrary</td>
<td>None</td>
</tr>
<tr>
<td>Minimal directed spanning problem</td>
<td>Single source</td>
<td>Linear in design variables, no flow costs</td>
<td>Uncapacitated</td>
<td>None</td>
</tr>
<tr>
<td>Traveling salesmen problem</td>
<td>Complete</td>
<td>Linear in design and flow variables, large constant fixed costs</td>
<td>Uncapacitated</td>
<td>Assignment constraints on design variables</td>
</tr>
<tr>
<td>Vehicle routing problem</td>
<td>Single source</td>
<td>Linear in design variables, no flow costs</td>
<td>Fixed capacity on all arcs</td>
<td>None</td>
</tr>
<tr>
<td>Facility location problem</td>
<td>Arbitrary</td>
<td>Linear in flow variables, fixed costs on split nodes</td>
<td>Capacities on split nodes</td>
<td>None</td>
</tr>
<tr>
<td>Fixed charge network design problem</td>
<td>Arbitrary</td>
<td>Linear in design and flow variables</td>
<td>Uncapacitated</td>
<td>None</td>
</tr>
<tr>
<td>Budget design problem</td>
<td>Arbitrary</td>
<td>Linear in flows, no design costs</td>
<td>Uncapacitated</td>
<td>Budget constraint on design costs</td>
</tr>
<tr>
<td>Network design traffic equilibrium problem</td>
<td>Arbitrary</td>
<td>Arbitrary</td>
<td>Uncapacitated</td>
<td>Minimum cost route choice for each commodity</td>
</tr>
</tbody>
</table>
Solution algorithms to network design problems were summarized as well, which included linear costs, heuristic solutions and nonlinear routing costs categories as illustrated in Table 2.

**Table 2**

Solution Algorithms for Different Types of Transportation Network Design Models

<table>
<thead>
<tr>
<th>Network design problem with</th>
<th>Problem Type</th>
<th>Solution Algorithm</th>
</tr>
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<tbody>
<tr>
<td>Linear costs</td>
<td>Budget</td>
<td>Branch and bound</td>
</tr>
<tr>
<td></td>
<td>Budget-convex routing costs</td>
<td>Branch and bound</td>
</tr>
<tr>
<td></td>
<td>Fixed charge</td>
<td>Branch and bound</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Benders decomposition</td>
</tr>
<tr>
<td>Heuristics</td>
<td>Fixed charge</td>
<td>Add and delete</td>
</tr>
<tr>
<td></td>
<td>Budget design</td>
<td>Delete and interchange</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Modified tree search</td>
</tr>
<tr>
<td></td>
<td>K-median</td>
<td>aggregation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>modified honeycomb</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Honeycomb, dynamic programming based</td>
</tr>
<tr>
<td>Nonlinear routing costs</td>
<td>Uncapacitated budget design with convex routing costs</td>
<td>Generalized Benders decomposition (heuristic)</td>
</tr>
<tr>
<td></td>
<td>Budget design with convex routing costs</td>
<td>Steenbrink decomposition</td>
</tr>
<tr>
<td></td>
<td>Uncapacitated design with convex routing costs</td>
<td>Delete heuristic</td>
</tr>
<tr>
<td></td>
<td>Unicapacitated budget design with convex routing costs</td>
<td>Generalized Benders decomposition</td>
</tr>
<tr>
<td></td>
<td>Convex routing costs with limited no. of paths</td>
<td>Linear programming generalized upper bounding code</td>
</tr>
<tr>
<td></td>
<td>Convex routing cost</td>
<td>Steenbrink decomposition (heuristic)</td>
</tr>
</tbody>
</table>
There have been numerous research efforts based on theoretical approaches and furthermore transportation network design problems have developed into three categories in the past few years (Kang, et.al, 2013), including operational network design based on dynamic traffic assignment considering peak period efforts, tactical service network design with schedule-based demand, and facility location planning. Service network design generally refers to freight transportation, which has been reviewed by Crainic (2000) who distinguished between the frequency (either as decision variables or derived outputs) and dynamic service network models in order to clearly represent the service network design classifications in the tactical planning procedure.

Transit network design and scheduling problems (TNDSP) falls into the category of general transportation network design and a comprehensive review has been addressed by Guihaire et.al (2008). In terms of complexity and multi-processes, Figure 1 illustrates the overall framework of transit network problem formulation with three basic components of design (TNDP), frequency setting (TNFSP) and timetabling (TNTP) were centrally allocated and two combined problems of design and frequencies setting (TNDFSP = TNDP + TNFSP) and scheduling (TNSP = TNFSP + TNTP). Furthermore, Guihaire et.al (2008) formulated the above problems as quadratic semi-assignment problems, mixed integer non-linear problems for transit timetabling problem and multi-commodity flow models. Solution methods were also classified in four categories as follows:

1) Specific and ad-hoc greedy heuristics;
2) Neighborhood search (i.e. simulated annealing and Tabu search);
3) Evolutionary search (e.g., genetic algorithms); and
4) Hybrid search combined with additional solutions methods.

Figure 1
Transit Network Problems Structure

More recently, Farahani et al., (2013) reviewed the up-to-date urban transportation network design problems which included integrated coverage of definitions, classifications, objectives, constraints, network topology decision variables and solution methods. In the urban network design category, strategic level and tactical level decisions in terms of network topology and its configuration were the main focus of researchers and a summary of the related practical problems is presented in Figure 2. Based on the existing literature, Farahani et al., (2013) also suggested future research directions: one aspect is the consideration of realistic policy requirements and integrated travel behavior, and the other is more efficient solution methods in view of rapid computational technology. Among the suggestions, inter-modal connectivity and park-and-ride were emphasized because of their potentials in improving urban transportation network efficiency as well as the lack of efforts in this category. Future challenges
include modeling service nodes, within which travelers could transfer conveniently form one mode to another, and allocating service facility and capacity.

### Figure 2

**Example of Decisions in Urban Network Design Problems**

The tactical level network design problems emphasize assumptions on explicit time scheduling of travelers, and congested situation on road ways as well. In order to take the impact of traffic demand into consideration, Kang et.al (2013) proposed an integrated model incorporating the demand-side schedules of the travelers/users as endogenous components in the design problem, namely activity-based network design problem. Inspired by the formulation of location routing problem, the proposed activity-based network design problem was expressed as a bi-level formulation composed of a network design and a shortest path problem. On the upper level, a set of disaggregate household itinerary optimization were solved, while the household activity pattern model was employed to determine the demand for the lower level. To solve this NP-hard lower level problem, a heuristic algorithm of decomposition was introduced based on the location routing problem and further numerical examples had demonstrated the sufficient accuracy of the solution algorithm.
Space-Time Concept and Modeling Framework

Hagerstrand (1970) first introduced the space-time concept by stating that for better understanding individual behaviors in the regional science, one should consider not only space coordinates but also time coordinates. This emphasized that accessibility should be measured in both space and time horizons. The introduction of space-time framework, to a certain extent, addressed the lack of micro-level resolution of large aggregated models. In addition, Hagerstrand (1970) also described three aggregations of constraints within the proposed the time-space network, namely, capability constraints, coupling constraints, and authority constraints. Based on the original idea of space-time framework, a wide variety of models have been developed by many researchers to represent human behaviors as well as to support planning and facility decision-making in transportation infrastructure. Miller (1991) extended the framework to a space-time prism within geographic information systems, and to measure the limitation of an individual’s ability to participate in activities in a certain location and a given amount of time. Based on available travel time budget and feasible travel velocity, only a limited range can be reached by a person, and only activities located within the range are accessible. Therefore, these time budgets and reachable distances work as constraints in the space-time model built to describe travel patterns of individuals. The space-time prism is constructed in geometry with the prism delimiting individual reaching specified locations during a given interval of time (Lenntorp 1976).
Dynamic Network Loading (DNL) Models

Dynamic network loading (DNL) models play a critical role in the network design problem. In general, compared with classic vehicle flow models, the DNL models in essence relax two hard constraints: First-In-First-Out rule for vehicles and road capacity constraints which are too complex to solve in the network design problem. They have the potential to process large-scale and time-dependent problems. In doing so, the DNL models significantly reduced the complexity of obtaining a feasible solution to the primal (original) problems. Certain relaxation techniques are often used in network flow problems to obtain the lower bound of the optimum, and the DNL models are effective in obtaining the upper bound of the optimum. Through reducing the gap between the lower and the upper bounds, it is possible to reach the global optimum or quansi-optimum solutions. The core of the DNL model is the vehicle flow modeling for traffic assignment problems. Two classic vehicle flow models are worth mentioning in particular because many related research findings were based on these two models. Gazis et al (1974) developed the linear programming model, referred to as the store-and-forward model (D'Ans and Gazis, 1976; Gazis, 1974), which takes exogenously pre-determined traffic assignment. Papageorgiou designed the store-and-forward model containing dynamic traffic assignment (Papageorgiou, 1990) and later implemented it in a traffic routing and control simulation package (Messner and Papageorgiou, 1990). Daganzo developed the cell transmission model (CTM) to divide the road network into many atomic cells equal to the distance a vehicle will travel within one time interval (Daganzo, 1994, 1995). The core part of the CTM model is a discrete approximation of the continuity equation for traffic flow conservation law defined in the Lighthill-Whitham-Richards model (Lighthill
and Whitham, 1955; Richards, 1956). Closely related to the CTM model that aims to capture traffic evolution over time, there are also other dynamic network models based on a discrete approximation of the kinematic wave theory at link level (Han et al., 2012; Yperman et al., 2005).

Another modeling technique for dynamic traffic assignment is a space-time network representation that divides the planning horizon into small time intervals. The space-time network was first proposed by Cooke and Halsey in which Bellman’s principle of optimality (Bellman, 1958) was modified to define a network containing both spaces and time (Cooke and Halsey, 1966). In light of the concept of space-time networks, travelers can be defined as explicit agents with their specific departure time and the shortest path from origins to destinations. The advantage of the space-time network over the physical network is that the space-time network representation is flexible in holding vehicles by adding waiting links. This is an important function in a congested network which contains bottlenecks and signal-controlled intersections.

Transportation Management for Special Events

The definition of special events can be drawn as certain sites where much more than usual traffic is attracted and spread which is typically infrequent. On an abstract level, there are two categories of special events: planned special events and unexpected emergencies. For the planned special events, various types of activities can be illustrated through the following lists:

- Festivals and fairs;
- Regularly or specially scheduled sporting events;
- Concerts;
• Olympics or World Expos;
• Conventions and exhibitions;
• Parades; and
• Fireworks;

Planned special events are characterized with known locations, scheduled time periods, and operational and management plans. In contrast, emergencies such as severe weather conditions or major catastrophes, which mostly occur at random with no pre-warning, usually lead to extremely high evacuation demands. In this study, the focus is planned special events, and a normal situation based analytical method is going to be developed. The term special event herein refers to planned special event.

Holding special events potentially affects a number of components in urban transportation systems including highway, public transit, pedestrian, parking facilities, as well as air quality, all of which are important and essential parts of urban transportation systems. Special events require a comprehensive transportation operational plan because the events usually generate an increase of travel demand and, as a result, would reduce available roadway capacity for other traffic. The Federal Highway Administration (FHWA) issued the Managing Travel for Planned Special Events (FHWA, 2003), defining special events as those activities of known locations, scheduled times of occurrence, associated operating characteristics such as increased travel demand and may possible road closures. Operational challenges to the hosts of a special event include: managing a high travel demand, minimizing the impact on adjacent roadways, providing various travel options to the event venues and accommodating high pedestrian volumes.
Table 3.1 summarizes transportation management plans for special events from various perspectives.

Table 3

Transportation Management Plan for Special Event

(Source: Managing Travel for Planned Special Events, September 2003, FHWA)

<table>
<thead>
<tr>
<th>STATE-OF-THE-PRACTICE</th>
<th>STATE-OF-THE-ART</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Institutional</strong></td>
<td></td>
</tr>
<tr>
<td>Manage traffic and parking for planned special events.</td>
<td>Manage travel for planned special events by adopting an inter-modal approach and utilizing travel demand management strategies.</td>
</tr>
<tr>
<td>Focus on traffic management team needs.</td>
<td>Form multidisciplinary stakeholder groups and solicit public input.</td>
</tr>
<tr>
<td>Secure verbal coordination between stakeholders.</td>
<td>Develop a joint operations policy or mutual-aid agreement between stakeholders.</td>
</tr>
<tr>
<td>Focus on single planned special events.</td>
<td>Create a committee on planned special events to monitor and plan travel management activities for all special events that occur within a region.</td>
</tr>
<tr>
<td><strong>Organizational</strong></td>
<td></td>
</tr>
<tr>
<td>Conduct periodic ad-hoc event planning.</td>
<td>Follow an established event operations planning process. Develop standard street use event routes and traffic flow routes.</td>
</tr>
<tr>
<td>Focus on event-specific planning and operations only.</td>
<td>Integrate event evaluation results into future planning activities to facilitate continuous improvement of transportation system performance.</td>
</tr>
<tr>
<td>Obtain periodic participation and contribution from community interest and event support stakeholders.</td>
<td>Establish stakeholder groups specific to advance planning and day-of-event activities to strengthen stakeholder coordination and commitment.</td>
</tr>
<tr>
<td><strong>Technical</strong></td>
<td></td>
</tr>
</tbody>
</table>
| Utilize fixed freeway and arterial management infrastructure to monitor and manage traffic during a planned special event. | Utilize mobile devices:  
- Portable traffic management systems (closed-circuit television, detectors, changeable message signs)  
- Portable traffic signals  
- Portable traffic management centers |
| Conduct point traffic and parking management using field personnel | Deploy automated systems:  
- Parking management systems  
- Dynamic trailblazer signs  
- Lane control signs  
- Blank-out signs |

Generally, there are three major objectives with event-based transportation management planning: traffic management plan, transit plan, and travel demand.
initiatives. As stated above, in order to accommodate intense demand of trips attracted by special event, one of the most efficient strategies is to provide multi-modal transportation accessibility to the event venue. More specifically, high-occupancy transit buses should be encouraged instead of private cars. In the meantime, since all travelers in the system would choose an optimal mode for the selfish theory in reality, minimizing the total travel cost should be taken into as a simultaneous objective in the problem. In the literature of multi-modal transportation planning problems, several research findings have been presented including allocating exclusive bus/bicycle lanes (Seo et al. 2005, Elshafei 2006, Mesbah et al. 2008, Li et al. 2009), determining bus routing and/or bus frequencies (Lee et al. 2005, Cipriani et al. 2006, Fan et al. 2006a, 2006b, Fan et al. 2008, Beltran et al. 2009, Gallo et al. 2011), determining signal (priority) setting and parking spaces (Cantarella et al. 2006), determining one-way street layout and lane additions (Szeto et al. 2006, Miandoabchi et al. 2011a, 2011b). As Farahani et al. (2013) stated, however, there was limited literature related to the multi-modal transportation planning problems, and researchers were more concentrated on the single mode network and inter-modal activities, such as park-and-ride mode.

Demand Management Strategies

In event-based transportation planning and management circumstance, the core challenge is to accommodate the intense travel demand from event attendees. Park-and-ride mode is widely considered an efficient way to relieve potential congestion by researchers as well as practitioners.
Park-and-ride was first introduced in North America in the 1930’s for work-orientated commuting trip to provide people lived in suburban area with an alternative travel mode (Spiller 1997). As some metropolitan areas and cities rapidly developed in United States and other developed countries, more urban congestion had emerged and various approaches of mitigation including park-and-ride had been explored.

Park-and-ride is widely adopted as one of the travel demand management (TDM) strategies and can be deployed as part of integrated TDM, which has been sufficiently studied. A number of policies as well as design guidelines have been published by various U.S. governmental agencies, metropolitan planning organizations and other policy offices. American Association of State Highway and Transportation Officials (AASHTO) has published a series of policy and design guidebooks (AASHTO 1992, 2004) covering design and planning of park-and-ride facilities, such as architectural design, impact analysis, and maintenance instructions. Figure 3 illustrates a comprehensive workflow for park-and-ride project developing process.
Figure 3
Park-and-Ride Program Development Workflow

Figure 3 shows the general development workflow of park-and-ride program. As one strategy in traffic demand management, the planning objectives of park-and-ride facilities span from social-economic considerations, accessibility, to environmental impacts, fair opportunities, safety and security. There is a rich body of best practices in North America and around the world showing that park-and-ride is an efficient TDM measure and is effective in alleviating urban traffic congestion. It is worth noting that park-and-ride facility planning contains two types of scenarios: extending existing facility and proposing new facility connecting transit, carpooling or vanpooling.

| Initiation                   | • Traffic demand management strategy  
|                             | • Land use considerations  
|                             | • Demand expectation  
| Planning stage              | • Integrated with transportation system planning  
|                             | • Element in transit system planning  
|                             | • Incorporate with ITS system  
|                             | • Encouragement for transit and carpooling  
| Design stage                | • Determining locations and capacity, pricing  
|                             | • Traffic considerations  
|                             | • Physical design  
| Implementation and Maintenance | • Traffic operational plan  
|                             | • Environmental concerns  

Network Design Problems with Park-and-Ride Facilities

A number of valuable research in the field of network design problems specifically studying location and/or capacity determination of park-and-ride facilities have been published. There are a number of related studies in two categories, including survey-based empirical investigation and model-based optimization analysis.

In the survey-based category, a number of studies examined the practical effectiveness of park-and-ride facilities. For example, Meek et al. (2008) suggested that park-and-ride might increase the average travel distance due to low load factors of dedicated buses, and trip generation. Horner (2004) took the advantages of Geographic Information System (GIS) to evaluate and compare several locations of potential alternatives, and to represent commuter coverage using an index based on accessibility and other factors. Based on the proposed method, it would help decision-makers to find the optimal locations of park-and-ride terminals along urban metro/light rail corridors and further deployment in the forecasting ridership process.

A multi-objective spatial model was developed by Farhan et al. (2008) as they emphasized three major considerations in the context of location modeling including maximizing demand coverage, minimizing distance between locations and major roadways as well as minimizing the cost of rebuilding existing traffic facilities.

Among the research efforts on park-and-ride problems, the concept of potential catchment area has been emphasized since it is an important factor to evaluate the performance of a proposed site. Potential catchment area for a particular park-and-ride location can be understood as the area of land use where the park-and ride facility could draw users from. There are empirical methods and analytical models to describe
catchment area in general, and the former is more common in previous research. The most common description of catchment area uses various shapes indicated by one or a set of parameters including empirical experience, survey data, geographic factors, accessibility measurements, etc. In terms of the describable shapes Holguin-Veras et al. (2012) presented an illustration in their literature review as shown in Figure 4, where a park-and-ride site served the central business district (CBD) area or downtown area; and pear-, parabolic- and elliptical-shaped catchment areas were summarized prospectively from different research efforts. However, Holguin-Veras et al. (2012) argued that there was still confusion about methods used in the process of drawing conclusions and analytical results were quite different from each other for different shapes.

![Figure 4](image)

Three Types of Shapes to Describe Catchment Area (Holguin-Veras et al., 2012)

Holguin-Veras et al. (2012) proposed an analytical method to describe the catchment area for park-and-ride facilities. They compared generalized cost for both
park-and-ride and driving-only travel modes, and defined the catchment area of a certain park-and-ride facility as the area where the generalized cost of the park-and-ride facility (including social-economics, construction, trip travel time and charges, etc.) is less than that of driving-only.

Research efforts in model-based optimization have mainly focused on trip patterns with a sufficient number of alternative locations. Considering the optimal location and pricing of a park-and-ride facility simultaneously in a linear monocentric city, Wang et al. (2004) aimed to find a deterministic mode choice equilibrium that maximizes object profit and minimizes social cost. Liu et al. (2009) proposed an improved model based on deterministic continuum equilibrium that can be formulated through a super-network approach.

Related research mainly focuses on location optimization, capacity constraints, and service efficiency for regular demand patterns. However, very limited attention is paid to the intermodal infrastructure and service network design for special event management which has its own unique characteristics. For example, compared to the common system-optimal objective that minimizes total travel time for a given traffic demand, a special event organizer typically wants to attract more visitors from different origins. Indeed, travel times are considered reasonable as long as travel time budgets for event attendees are satisfied. In this case, the accessibility to the special event sites is more relevant or important as an overall management goal, compared to the simple mobility measure. While there is a wide range of studies (e.g., Litman et al. (2003), Handy (2005), Litman et al. (2011)) examining accessibility-oriented strategies, only a few researchers have started systematically incorporating accessibility/connectivity
measures in a network design modeling framework. For example, Viswanath and Peeta (2003) proposed a mathematical model to minimize travel time and maximize connectivity at each demand center after an earthquake. Santos et al. (2008) introduced a transportation network design problem based on equity and accessibility. The activity-based network design problem studied by Kang et al. (2013) aimed to minimize both the network design costs and activity-related disutility using a bi-level model.
3 NETWORK DESIGN MODELING AND SOLUTIONS WITH PARK-AND-RIDE FACILITIES FOR SPECIAL EVENTS

This chapter elaborates details on the formulation of the park-and-ride facility design problem based on space-time network models as well as the development of a heuristic solution framework. Firstly, it is necessary to point out several considerations or constraints affecting travelers’ decisions:

1. **Travel time budget (TTB) constraints**: When people plan to attend a particular event, the first important factor is the time investment, including the time spent on the way to and back from the event location, as well as time duration of the event itself. To better understand this consideration, previously researchers have introduced the concept of travel time budget, could be treated as one of the parameters in network design problems.

2. **Construction budget constraints**: It is apparent that building as many as possible park-and-ride facilities with large capacities will help maximize the park-and-ride accessibility. However, in practice, available budget and land resources are limited. As a result, only a limited number of park-and-ride facilities with limited capacity can be constructed.

3. **Capacity constraints**: Another type of constraint is the road capacity constraints. It is desired that traffic from special events should not superimpose too much delay to existing traffic. Therefore, it is important to ensure that travel demand will not exceed the temporal capacity and road spatial capacity of surrounding roads to the event sites.

4. **Special event constraints**: In a feasible solution, the itinerary of travelers to the
events must include the event sites and they must stay there for a certain amount of time.

5. **Park-and-Ride constraints**: Once a traveler chooses the park-and-ride mode to attend the events, he must return to the same park-and-ride station to pick up his private vehicle.

**Concept Illustration Using a Simplified Example**

It is assumed that there are three potential park-and-ride facilities that serve a special event destination, from which public transit provides connection services. Figure 5 illustrates the elements necessary for modeling the problem as an inter-modal transportation network.

![Figure 5: Park-and-Ride Facilities Optimization Problem in an Intermodal Network](image)

---

**Figure 5**

Park-and-Ride Facilities Optimization Problem in an Intermodal Network
Specifically, travelers wish to travel from multiple origins \((O_1, O_2, O_3, O_4)\) to the special event site \((D)\). They can reach \(D\) either by driving along path \((L_1, L_2, L_3, L_4)\), or connecting to transit station and taking transit along path \((T_1, T_2)\), or take the park-and-ride mode that combines driving and transit. Park-and-ride lots \(P_1, P_2, P_3\) connect the road network and the transit network, and the parking lot on link \(L_4\) allows those travelers who directly drive all the way to the event site to park their cars and then walk to the event site. In such a complex context, a traveler who plans to go to the event needs to make a series of decisions including his departure time, mode choice (e.g., car vs. transit) and route decision to minimize his travel cost involving travel time/delay and transit and parking fares. When many travelers travel for the same purpose, a user equilibrium condition will be gradually reached within the scope of network. Apparently, solving such a problem satisfactorily is challenging.

In this thesis, the system-optimum objective functions are selected to minimize the total travel times of all travelers as well as to maximize the number of travelers who can finish their trips within their time budgets through the best intermodal trip option (e.g., driving all the way, taking bus all the way or parking and riding). The second objective is referred to as “Accessibility Maximization Problem” in other literature. Toward those goals, traffic management agencies typically have multiple options with various configurations of park-and-ride facilities’ locations and capacities as well as the capacity and schedule of transit services. As an illustration, a visitor in Figure 5 from origin \(O_1\) can drive along the road network \((O_1 \rightarrow L_1 \rightarrow L_2 \rightarrow L_3 \rightarrow L_4 \rightarrow D)\). If link \(L_3\) is congested or parking lot on link \(L_4\) is saturated, the traveler may consider an intermodal option through route \((O_1 \rightarrow L_1 \rightarrow L_2 \rightarrow P_2 \rightarrow T_2 \rightarrow D)\) by parking the car at park-and-
ride lot P2. Unless the capacity at parking lot P2 is still sufficient, the traveler may also consider driving a short distance to P1 through route \((O_1 \rightarrow L_1 \rightarrow P_1 \rightarrow T_1 \rightarrow T_2 \rightarrow D)\), or taking a transit only route through route\((O_1 \rightarrow T_1 \rightarrow T_2 \rightarrow D)\).

To solve this problem appropriately, it is necessary to formulate this problem as an optimization problem. One needs to identify the decision variables, objective function and constraint(s) in the context of space-time networks. In addition, efficient solution algorithms are also critical to quickly reach an optimal or sub-optimal solution when scenarios are given.

Methodology

Vehicle Trajectory Representation

In this section, the concept of the space-time network is first elaborated and how to represent a vehicle trajectory in the space-time network is illustrated. The concept of space-time networks aims to integrate physical transportation networks with travelers’ time-dependent trajectories. The first step of constructing a space-time network is to discretize the time into time intervals of equal length \(\sigma\). As shown in Figure 6, a physical transportation network is first shown on the upper portion, while the lower part shows how to extend the physical network with a series of space-time vertices, travel arcs and waiting arcs, each with different spatial and temporal characteristics. Specifically, while a traveler is traveling in this simple three-node network, he can depart from origin node \(o\) at \(t_0 + \sigma\) and arrive at node \(a_1\) at \(t_0 + 2\sigma\). The traveler finally reaches at the third node \(a_2\) at \((t_0 + 6\sigma)\). The route choice decision can be described as a sequence of selected arcs, including both travel arcs and waiting arcs, in this space-time network. As an
illustration, the traveler’s chosen route in Figure 6 is arcs(o, a₁, t₀ + σ, t₀ + 2σ) → (a₁, a₁, t₀ + 2σ, t₀ + 3σ) → (a₁, a₂, t₀ + 3σ, t₀ + 6σ). Figure 7 shows another alternative space-time network representation inspired by Hägerstrand (1970).

Figure 6
Illustration of Physical Network and the Extended Space-Time Network
Accessibility maximization for travelers is one of the objectives for park-and-ride facility design during special events. In a space-time network, a location is defined as accessible to travelers if travelers can reach that location within their time budget $T\sigma$ with reasonable traveling speeds. As shown in Figure 8, it can be seen, starting from o, the earliest possible times arriving at node $a_1$ and node $a_2$ are $t_0 + 2\sigma$ and $t_0 + 5\sigma$ respectively. If the travel time budget (TTB) is set as $4\sigma$, then node $a_1$ is accessible while node $a_2$ is inaccessible for travelers departing from node o.
In a more general case, a traveler departs from his origin location (e.g. home) and arrives at intermediate destination(s) before finishing the entire trip (e.g. workplace, shopping mall, hospital), performs an activity at those intermediate locations, and finally returns to the origin location. To represent the trip completely, Figure 9 illustrates the moving sequence of a traveler with one activity performing time of $2\sigma$ and TTB of $7\sigma$. It is apparent that activity location $a_1$ is accessible given the TTB of $7\sigma$ whereas node $a_2$ is inaccessible. To ensure that a complete trip always satisfies the flow balance requirement for the network flow model, it is assumed that a traveler stays at the origin node after he finishes his trip before the end of time horizon. This assumption can be reflected by selecting the waiting arc $(o, o, t_0 + 6\sigma, t_0 + 7\sigma)$ from the arrival time of $6\sigma$ to the TTB of $7\sigma$ in Figure 9.
Accessible and Inaccessible Nodes for Travelers with Traveling Time Constraints

An important concept for analyzing travelers’ accessibility is space-time prism (STP) proposed by Miller (1991), which is in essence the envelope of all possible space-time paths between two space-time vertices. Figure 10 illustrates a simple STP in planar space with zero activity time at intermediate locations. The spatial and temporal region bounded by the prism or the potential path space (PPS) measures the ability to reach vertices in space and time, given the locations and durations of fixed activities. Projecting PPS to the two-dimensional geographical plane will creates the potential path area (PPA) within which all the geographical locations can be occupied by travelers (Wu and Miller, 2001; Miller, 2005).
A Simple Space-Time Prism (Miller, 2005)

Figure 11 shows a space-time prism with constant activity time (the cylinder represents constant activity time) and it also highlights accessible/inaccessible activity locations within the prism. As an example, Figure 11 describes the prism of everyday commuters who have a total time budget $T$ for both travel and work tasks. If commuters need to work for $T_w$ hours in their offices, then the total time of $T - T_w$ is remains for them to travel between a pair of the work and home locations within their TTB.

Figure 11

Mapping Accessible/ Inaccessible Locations with Respect to Potential Activity Area
The latest theoretical development of space-time prism has taken into account the time because traffic networks are highly dynamic and travelers have various range of accessibility over time with the same TTB, depending on the level of congestions in the networks (Tong et al., 2014). The classical space-time-prism concept is extended to dynamic space-time prism (DSTP) framework in which time-dependent travel time rather than constant travel time are used to calculate the accessibility within a transportation network. Similarly, a dynamic potential path area (DPPA) can be found given a specified departure time. With the concept of DPPA, the accessibility measure can be defined as (Wu and Miller, 2001):

Given the total time budget $T$ and departure time $\tau$, the dynamic opportunity set $M_o$ of valued activity locations from location $o$ is,

$$M_o = \{ a \in \Omega \mid t^\tau(o, a) + t^m_a + t'^\tau(a, o) \leq T \}. \quad \text{(Equation 1)}$$

Equation 1 can be interpreted as: a location $a$ is accessible only if the $T$ is greater than the total travel time $t^\tau(o, a) + t'^\tau(a, o)$ and minimum required activity time duration $t^m_a$, where the departure time from the activity location $\tau' = \tau + t^\tau(o, a) + t^m_a$.

Using Equation 1, the congestion effects can be explicitly reflected by incorporating time-dependent link travel times into the accessibility measure for different departure times. As illustrated in Figure 11, the prevailing travel speeds before the traveling to the activity location may be considerably different from the speeds after performing the activity due to recurrent or non-recurrent congestions.

It should be also pointed out that transportation network accessibility in general can be affected by many factors such as location attributes, road tolling policies, public
transportation policies, etc. However, in most literature in the past, travel time was primarily adopted for network accessibility evaluation and travel time is also used to evaluate the accessibility in this thesis.

Problem Formulation

Problem Statement

The target problem can be interpreted as: given the total construction budget constraint and available candidate locations, the park-and-ride facility optimal design problem aims to minimize the total travel costs for all travelers or maximize the number of travelers who can finish their trips within the time budget by constructing park-and-ride facilities with appropriate capacities at candidate locations. To formulate this problem, an intermodal urban traffic network is constructed in which traveler can reach their destinations through multiple travel modes. For instance, they may choose to take buses all the way to the event sites to avoid the parking pains, or they may choose to drive to destinations for more flexibility, or they may also choose a mixed option: drive to a park-and-ride facility first and then take buses to their destinations which is the target problem in the thesis.

In the context of this thesis, it is clear that constructing a special traffic network to represent multi-modal trips is the critical step. As shown in previous literature (Zhou et al., 2008), a multi-modal network can be modeled as a multi-layer network for dynamic traffic assignment with integrated management strategies, and travelers can choose different modes to finish their trips with certain mode-specific costs incurred. In addition,
in such an inter-modal network, road capacity constraints must be considered for all types of vehicles since they are moving at the same time.

To measure the performance of a particular park-and-ride facility design during special events, travelers’ various trip chains are assumed to start from particular origin locations, choose a travel mode, reach and stay at the activity locations for a period, and then return to their origin locations within their expected time budgets. If the road network is too congested or if the park-and-ride facilities have insufficient capacities, some travelers will have to either experience higher travel costs than their time budgets or give up the trips entirely, neither of which is desirable. Thus, it is important to formulate an integrated network design model to improve the overall traffic efficiency through optimizing the locations and capacities of park-and-ride facilities subject to the construction budget constraint and various flow and parking capacity constraints.

Mathematical Formulation

In a general form, the problem of park-and-ride facility optimal design for special events can be formulated as a particular optimization problem based on the space-time network model. Some simplifications are made as follows to ensure the problem within the power of analytical equations:

- Buses are considered to have unlimited capacities to carrying passengers;
- Passengers’ waiting time at park-and-ride facilities is assumed constant;

The notations of the mathematical formulation are listed in Table 4 in advance.
Table 4
Notations for Park-and-Ride Facilities Design Problem

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Set of all types of space-time arcs</td>
</tr>
<tr>
<td>$A^R, A^P, A^R, A^{RP}$</td>
<td>Set of space-time road traveling, transit service, parking lot, connection arcs</td>
</tr>
<tr>
<td>$A^W, A^A, A^V$</td>
<td>Set of space-time waiting, activity-performing at special event site, virtual traveling arcs</td>
</tr>
<tr>
<td>$c_{i,j,t,s}(p)$</td>
<td>Travel cost of arc $(i,j,t,s)$ for passenger $p$</td>
</tr>
<tr>
<td>$\text{cap}_{i,j}^p$</td>
<td>Maximum capacity of parking lot facility $(i,j)$ in terms of number of parking spaces</td>
</tr>
<tr>
<td>$E$</td>
<td>Set of transportation facility/service links in physical network</td>
</tr>
<tr>
<td>$E^R, E^P, E^T$</td>
<td>Set of road, available parking lot locations, transit services facilities in physical network</td>
</tr>
<tr>
<td>$H$</td>
<td>Set of time stamps in the planning horizon</td>
</tr>
<tr>
<td>$(i,t), (j,s)$</td>
<td>Indices of space-time vertexes, $(i,t), (j,s) \in Q$</td>
</tr>
<tr>
<td>$(i,j)$</td>
<td>Index of transportation facilities/links between adjacent nodes $i$ and $j$, $(i,j) \in E$</td>
</tr>
<tr>
<td>$(i,j,t,s)$</td>
<td>Index of space-time arcs indicating the actual movement at entering time $t$ and leaving time $s$ on link $(i,j)$, arc $(i,j,t,s) \in A$</td>
</tr>
<tr>
<td>$i, j, i', j'$</td>
<td>Indices of nodes, $i, j, i', j' \in N$</td>
</tr>
<tr>
<td>$o^p, \tau^p$</td>
<td>Indices of origin nodes, departing time of agent $p$, $o^p \in N$</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of passenger agents</td>
</tr>
<tr>
<td>$p$</td>
<td>Index of passenger agent, $p \in P$</td>
</tr>
<tr>
<td>$Q$</td>
<td>Set of vertexes in space-time network</td>
</tr>
<tr>
<td>$d_{ij}$</td>
<td>Construction cost for park-and-ride facility located on link $(i,j)$</td>
</tr>
<tr>
<td>$\mathcal{R}(p)$</td>
<td>Function to indicate if passenger $p$ can finish his trip within $TTB(p)$: $=1$ if $p$ finishes his trip before $TTB(p)$; $=0$ otherwise</td>
</tr>
<tr>
<td>$t,s,t',s'$</td>
<td>Indices of different time stamps, $t,s \in H$</td>
</tr>
<tr>
<td>$TTB(p)$</td>
<td>Total time budget for passenger $p$ in terms of number of time intervals</td>
</tr>
<tr>
<td>$TCB$</td>
<td>Total construction budget</td>
</tr>
<tr>
<td>$x_{i,j,t,s}(p)$</td>
<td>$= 1$, if a space-time arc $(i,j,t,s)$ is used in the tour for passenger $p$</td>
</tr>
<tr>
<td>$= 0$, otherwise</td>
<td></td>
</tr>
<tr>
<td>$y_{i,j}$</td>
<td>$= 1$, if parking lot $(i,j)$ is selected in the final decision to be constructed;</td>
</tr>
</tbody>
</table>
The decision variables, objective function and constraints of the mathematical formulation are described as follows:

**Decision variables**
- $x_{i,j,t,s}(p)$: Binary variable indicating passenger $p$ chooses link $(i, j)$, enters at time $t$ and leaves at time $s$; and
- $y_{i,j}$: Binary variable indicating if the parking lot $(i, j)$ is selected to be built

**Objective functions**
- Min $Z_1 = \sum_{p \in P} \sum_{(i,j,t,s) \in A} [c_{i,j,t,s}(p) \times x_{i,j,t,s}(p)]$ (Equation 2)
- Or Max $Z_2 = \sum_{p \in P} \Re(p)$ (Equation 3)

Objective function of $Z_1$ minimizes the total travel costs of all passengers during special event; objective function of $Z_2$ maximizes the accessibilities of all passengers.

**Constraints**

**Flow conservation constraints**
for all $p \in P$:

$$\sum_{(j,s) \in Q} x_{i,j,t,s}(p) - \sum_{(j,s) \in Q} x_{j,i,t,s}(p) =$$

$$\begin{cases} 
1, & i = o^p, t = \tau^p \\
-1, & i = o^p, t = \tau^p + TTB(p) \\
0, & \text{otherwise}
\end{cases}$$

(Equation 4)

**Activity-performing constraints at event site**

$$\sum_{(i,j,t,s) \in A^v \cup A^w} x_{i,j,t,s}(p) = 1, \text{ for all } p \in P$$

(Equation 5)

The space-time activity-performing arcs at special event site are virtual traveling arcs for passenger $p$. 

35
Road capacity constraints

A spatial queue mesoscopic traffic flow model (road capacity constraint) as well as the road temporary capacity are considered. That is, the total inflow satisfies:

\[
\sum_{p \in P} x_{i,j,t,t+s_{i,j}}(p) \leq \min\{cap_{i,j}^{in}(t), cap_{i,j}^{out}(t+s_{i,j}), L_{i,j} \times K_{i,j}^{jam} - A_{ij}(t) - D_{ij}(t)\} \quad \forall (i,j) \in E^p, t \in T
\]  

(Equation 6)

Where for link \((i, j)\), \(s_{i,j}\) is the free flow travel time, \(cap_{i,j}^{in}(t)\) is in-flow capacity at time \(t\), \(cap_{i,j}^{out}(t+s_{i,j})\) is out-flow capacity at time \(t + s_{i,j}\), \(L_{i,j}\) is the length and \(K_{i,j}^{jam}\) is the jam density.

The number of cumulative arrival \(A_{ij}(t)\) and departure agents, \(D_{ij}(t)\) on the link \((i, j)\) can be represented as:

\[
A_{ij}(t) = \sum_{t=0}^{t} \sum_{p \in P} x_{i,j,t,t+s_{i,j}}(p), \quad D_{ij}(t) = \sum_{t=0}^{t} \sum_{p \in P} x_{i,j,t-s_{i,j},t}(p)
\]  

(Equation 7)

Park-and-ride facility constraints

(i) Capacity associated with cars arriving and departing at parking lots: the cumulative number of arrival agents minus the cumulative number of departure agents could not exceed the space capacity of the parking lot.

\[
A_{ij}(t) - D_{ij}(t) \leq cap_{i,j}^{P} \times y_{i,j}, \quad for \ all \ (i,j) \in E^p, t \in T
\]  

(Equation 8)

(ii) The consistency constraints for using the same parking lot: each passenger agent should visit the same parking lot when parking and finding his/her car in the entire trip chain.

\[
\sum_{t \in H} x_{i,j,t,s}(p) = \sum_{t \in H} x_{j,i,t,s}(p), \quad for \ all \ (i,j) \in E^p
\]  

(Equation 9)
Total construction budget constraints

\[ \sum_{i,j} (y_{i,j} \times q_{i,j}) \leq TCB \]  

(Equation 10)

The total construction cost for selected parking lots should not exceed the total construction budget.

Solution Algorithm

In this section, a Lagrangian-relaxation (LR) based solution algorithm is described to reformulate and further decompose the primal problem.

Primal problem P1

P1: min \( Z_1 \) or max \( Z_2 \)

Subject to: Constraints (Equation 4) through (Equation 10) and binary constraints for variable vectors \( X = [x_{i,j,t,s}(p)] \) and \( Y = [y_{i,j}] \).

Through relaxing the capacity constraint (Equation 6), the relaxed problem P2 can be formulated as:

P2: \[ L = \sum_{p \in P} \sum_{(i,j,t,s) \in A} [c_{i,j,t,s}(p) \times x_{i,j,t,s}(p)] \]
\[ + \sum_{(i,j,t,s) \in A} \pi_{i,j,t,s} \times [A_{i,j}(t) - D_{i,j}(t) - cap_{i,j}] \quad \text{subject to the remaining constraints} \]  

(Equation 11)

Subject to the remaining constraints.

Note that both object function and constraints of P2 can be separated into two groups coupled through common Lagrangian multipliers with respect to variables X and Y. It is possible to decompose P2 into two relatively easy-to-solve problems:

\( P_X \): a constrained time-dependent routing problem for passengers subject to a multi-modal dynamic traffic assignment program subject to (Equation 4), (Equation 5), (Equation 8) and (Equation 9);
\( P_Y \): a knapsack problem subject to the total construction budget constraint (Equation 10). In a latest literature by Tong et al. (2015), a similar Lagrangian relaxation and decomposition approach was described in details.

The Lagrange multipliers \( \pi_{i,j,t,s} \) can also be interpreted as shadow price associated with capacity constraints.

The solution steps of the proposed algorithm can be listed as follows:

**Step 1: Initialization**

Set iteration number \( k = 0 \); the set of available parking lot locations are given in terms of links in set \( E^p \) and total construction budget \( TCB \).

Choose positive values to initialize the set of Lagrangian multipliers \( \pi_{i,j,t,s} \).

**Step 2: Solve the decomposed problems**

**Step 2.1:** Solve \( P_X \) using an enhanced multi-modal DTA simulator with a time-dependent least cost path algorithm and find a path solution \( X(p) \) for each agent \( p \). A spatial queue-based traffic flow simulator, namely DTALite (Zhou et al., 2014), is used to ensure the traffic inflow and spatial capacity constraints (Equation 6). Specifically, in a transportation network, a node is connected to different incoming links and outgoing links, and each link has two buffers in DNL, namely entrance buffer and exit buffer to facilitate traveling agents’ transfers between links. These two buffers on each link are commonly implemented as a first-in-and-first out (FIFO) queues. When the required link inflow and outflow capacities are available, an agent can move from the exit buffer of an upstream link to the entrance buffer of the downstream buffer.

To handle the remaining constraints, namely space-time flow balance constraints
(Equation 4) and activity-performing constraints (Equation 5), with the relaxed objective function (Equation 11), the time-dependent routing problem is solved for passenger agents with a set of constraints. Specifically, each arc \((i, j, t, s)\) in the available park-and-ride facilities (i.e., \(E^p\)) has an additional cost of \(\pi_{i,j,t,s}\) for the relaxed capacity constraints, which is equivalent to the estimated travel time penalties when agents use park-and-ride facilities. The space-time flow balance constraints are satisfied automatically in the routing algorithm, and the remaining activity-performing constraints at special event site can be handled through a simple decomposition to two trips, one from the origin to the special event site, and the other from the special event site back to the origin. Similarly, for Equation 5 and Equation 9, although it in essence defines multiple traveling sales man problems (in the defined space-time network, if one enters a park-and-ride facility at time \(t\), he must come back to the same park-and-ride facility later to pick up his car), the optimal route for a particular passenger can be enumerated through solving a series of shortest path problems. To be more specific, if a passenger chooses to use park-and-ride facility, \(P_i\), the optimal routing policy can be solved as:

1. Find the time-dependent shortest path from his origin to \(P_i\);
2. Add additional constant time (transit time between the selected park-and-ride facility and activity duration);
3. Find the time-dependent shortest path from \(P_i\) to his origin.

Since there are only a limited number of park-and-ride facilities constructing options, it is compared that the corresponding travel time among all park-and-ride facilities constructing options with the driving-only option. This approach provides the optimal routing policy for each passengers.
If the travel time budget constraint $TTB(p)$ is not met due to road traffic congestion or travel time penalty associated with $\pi_{i,j,t,s}$ at the parking lots, then the routing algorithm that aims to minimize the total disutility will default to the inaccessible virtual arc. It should be remarked that, even there are optional park-and-ride capacity available, some travelers (with the goal of accessibility maximization) could still select driving only mode, if the related road traffic condition is less congested.

**Step 2.2**: Solve $P_y$ using a dynamic programming algorithm, to find an optimal value for $Y$. The Lagrangian multipliers $\pi_{i,j,t,s}$ associated with the relaxed park-and-ride capacity constraints will encourage the decision makers to select the most cost-effective park-and-ride location/capacity allocation option to maximize the total cost (i.e. profit) to collect through a knapsack modeling framework, which is equivalent to max

$$\sum_{(i,j,t,s) \in A} p \{ \pi_{i,j,t,s} \times \left[ \text{cap}_{i,j} \times y_{i,j} \right] \}$$

subject to constraint (Equation 9).

**Step 3: Update Lagrangian multipliers**

Update Lagrangian multipliers $\pi_{i,j,t,s}$ using subgradient $\pi_{i,j,t,s} + d^k \times [A_{ij(t)} - D_{ij(t)} - \text{cap}_{i,j} \times y_{i,j}]$, where $d^k$ is the step length at iteration $k$.

**Step 4: Termination condition test**

If $k$ is less than a predetermined maximum iteration value, or the gap is smaller than a predefined toleration gap, terminate the algorithm; otherwise $k = k + 1$ and go back to Step 2.
In this chapter, the park-and-ride facility design problem is further formulated in a transformed network flow model. In general, constraints of optimization problems can be divided into two types: easy or hard. Hard constraints often make the problems difficult in solving on large scale. From the problem formulation in Section 3.3, constraint (Equation 4) is considered easy while (Equation 5) through (Equation 10) are considered hard constraints. A Lagrangian-relaxation-based heuristic solution algorithm is proposed to relax those hard capacity constraints. In this chapter, a new approach is proposed to transform the physical network into a new network in which the hard multi-modal constraints and park-and-ride capacity constraints can be automatically pre-built in the network while constructing the network flow model. As a result, those hard constraints are reflected in the objective functions rather than the constraints and such transformation will greatly reduce the complexity of this problem.

Relaxing the Multimodal and Park-and-Ride Facility Constraints in Transformed Networks

Original Park-and-Ride Network Model

Figure 12 shows a simple physical network for special events. travelers depart from their origin O1 and go to the event site D. They have three options to reach D: 1. They can drive to park-and-ride facility P1 and then take transit to go to D; 2. They can also drive to park-and-ride facility P2 and then take transit to go to D; or they can drive all the way to the event site D. After the special event ends, for those who chose park-
and-ride mode, they will have to go back to the same park-and-ride facility to pick up their cars and then drive home; for those who drove to D, they will drive back on the same road (To simplify, it is assumed that those park-and-ride routes are only open to park-and-ride travelers). In addition, both park-and-ride facility P1 and P2 have the limited capacity beyond which new vehicles must seek other parking locations.

Figure 12
Original Park-and-Ride Network

Transformed Park-and-Ride Network Model

Figure 13 shows the corresponding transformed network from the original network in Figure 12. The event site D is extended into three sub event sites: D01, D02 and D03 and travelers’ round trips are transformed into one-way trips (O1→D0X→O1’) where X=1, 2 or 3. Furthermore, the park-and-ride facility P1 and P2 are also transformed into (P1→P12) and (P2→P22). The lengths of parking links (P11, P12) and (P21, P22) are set as capacities of P1 and P2; the capacities of park links are set as
positively infinite; and the travel times on parking links represents the delay caused by
park-and-ride activities. The sub event sites are further extended to a pair of nodes and
the event links (D0X, D0X') (X=1, 2, 3) has $\Delta t$ travel time, representing the duration of
the event and unlimited capacity (assuming tickets are never over sold). In doing so,
park-and-ride constraints (Equation 8) and multi-modal constraints (Equation 9) are pre-
built into the network and only road link capacity, parking link capacity and flow
conservation constraints remain in the new problem.

![Diagram of Transformed Park-and-Ride Network](image)

Figure 13

Transformed Park-and-Ride Network

Mathematical Formulation for the Transformed Network

The objective of the new problem is slightly different from the one in Chapter 4 in
that the decision variable for building a park-and-ride facility is simplified to a prior
knowledge. The rationale is that, in most realistic cases, the decision makers can at most
build a few park-and-ride facilities (e.g., 3 to 5) once within the limited construction budget. Therefore it is most likely that we can lay out all the possible options in terms of the total number of facilities and their respective locations through subtle enumeration. For each construction option, the corresponding physical network can be transformed according to the method described in Section 4.1.
### Table 5

**Notations for Transformed Park-and-Ride Facilities Network Design Problem**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>Set of all links in the transformed network</td>
</tr>
<tr>
<td>$E^R, E^P$</td>
<td>Set of all links, parking links, event links in the transformed network</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of all arcs in space-time network</td>
</tr>
<tr>
<td>$V$</td>
<td>Set of all vertices in space-time network</td>
</tr>
<tr>
<td>$N$</td>
<td>Set of all nodes in the transformed network</td>
</tr>
<tr>
<td>$O, D, N^R, N^P, N^D$</td>
<td>Set of all origin nodes, destination nodes, road nodes, parking lot nodes and even nodes in the transformed network</td>
</tr>
<tr>
<td>$H$</td>
<td>Set of time stamps in the planning horizon</td>
</tr>
<tr>
<td>$T$</td>
<td>Time horizon</td>
</tr>
<tr>
<td>$P$</td>
<td>Set of travelers</td>
</tr>
<tr>
<td>$p$</td>
<td>Index of passenger agent, $p \in P$</td>
</tr>
<tr>
<td>$t, s, t', s'$</td>
<td>Indices of different time stamps, $t, s \in H$</td>
</tr>
<tr>
<td>$i, j, i', j'$</td>
<td>Indices of nodes, $i, j, i', j' \in N$</td>
</tr>
<tr>
<td>$o^p, \tau^p, d^p$</td>
<td>Indices of origin node, departing time and destination node of traveler $p$, $o^p \in O$</td>
</tr>
<tr>
<td>$(i, j)$</td>
<td>Index of transportation facilities/links between adjacent nodes $i$ and $j$, $(i, j) \in E$</td>
</tr>
<tr>
<td>$FFTT(i, j, t)$</td>
<td>Free-flow travel time on link $(i, j)$ at $t$</td>
</tr>
<tr>
<td>$(i, j, t, s)$</td>
<td>Index of space-time arcs indicating the actual movement at entering time $t$ and leaving time $s$ on link $(i, j)$, $(i, j, t, s) \in E$, $s = t + FTTT(i, j, t)$ and $s$ is also written as $s_{i,j}$ to indicate the link at some locations</td>
</tr>
<tr>
<td>$c_{i,j,t,s}(p)$</td>
<td>Travel cost of arc $(i, j, t, s)$ for passenger $p$</td>
</tr>
<tr>
<td>$cap^p_{i,j}$</td>
<td>Capacity of parking links</td>
</tr>
<tr>
<td>$cap^i_{i,j}(t), cap^o_{i,j}(t)$</td>
<td>Link inflow saturation rate and outflow saturation rate at $t$</td>
</tr>
<tr>
<td>$x_{i,j,t,s}(p)$</td>
<td>$= 1$, if a space-time arc $(i, j, t, s)$ is used in the tour for passenger $p$</td>
</tr>
<tr>
<td></td>
<td>$= 0$, otherwise</td>
</tr>
<tr>
<td>$\lambda_{(i,j,t)}, \mu{(i,j,t)}$</td>
<td>Lagrangian multipliers for travel links and transformed parking links</td>
</tr>
</tbody>
</table>
Decision variables

\(x_{i,j,t,s}(p)\): Passenger \(p\) chooses link \((i, j)\), enters at \(t\) and leave at \(s\);

Objective functions

\[
\text{Min } Z_1 = \Sigma_{0 \leq t \leq T} \Sigma_{p \in P} \Sigma_{(i,j) \in E} [c_{i,j,t,s}(p) \times x_{i,j,t,s}(p)]
\]

(Equation 12)

Constraints

Flow conservation constraints

At nodes where waiting is not allowed:

\[
\Sigma_{(i,j,t,s) \in A} x_{i,j,t,s}(p) - \Sigma_{(j,i,s,t) \in A} x_{j,i,s,t}(p) = \begin{cases} 
-1, & i = o^p, t = \tau^p \\
1, & i = d^p, t = T \\
0, & \text{otherwise}
\end{cases}
\]

(Equation 13a)

At nodes where waiting is allowed:

\[
\Sigma_{(i,j,t,s) \in A} x_{i,j,t,s}(p) - \Sigma_{(j,i,s,t) \in A} x_{j,i,s,t}(p) + x_{j,i,s-1,s}(p) - x_{j,i,s,s+1}(p) = \begin{cases} 
-1, & i = o^p, t = \tau^p \\
1, & i = d^p, t = T \\
0, & \text{otherwise}
\end{cases}
\]

(Equation 13b)

It should be noted that Equation 13 implies that if a traveler goes back home before the end of time horizon, he will wait at its destination \(d^p\) at no additional cost until the end of time horizon.

Road link capacity constraints

Road link capacity constraints can be divided into two types: temporal constraints, meaning that inflow rates should be always lower than link saturation rate on any link; spatial constraints, meaning queue length should be always shorter than link length (i.e., to prohibit queue spillback). Their mathematical formulations are:

Link temporal constraints:

\[ \sum_{p \in P} x_{i,j,t,s}(p) \leq \text{cap}^{int}_{i,j}(t), \forall (i,j) \in E, t \in [0,1, ..., T] \]  
(Equation 14)

Link spatial constraints (queue spillback prohibition):

\[ \left( \sum_{0 \leq \xi \leq t} \sum_{p \in P} x_{i,j,\xi,\xi+F_{FTT}(i,j,t)}(p) - \sum_{(j,j') \in A} \sum_{0 \leq \xi \leq t} \sum_{p \in P} x_{j,j',\xi,\xi+F_{FTT}(j,j',t)}(p) \right) \leq L_{i,j} \times k_{jam}, \forall (i,j) \in E, t \in [0,1, ..., T] \]  
(Equation 15)

Where: \( L_{i,j} \) is link length; \( k_{jam} \) is the jam density; \( \xi \) is an integer time index between 0 and \( t \);

Parking link length constraints

Parking link is defined as the links within the extended park-and-ride facilities, such as \((P_{11}, P_{12}), (P_{21}, P_{22})\) in Figure 13. Once a traveler chooses to part and ride, he will leave his car in the lot and take buses to go to event site. If it is assumed that the average ridership of vehicles is one passenger per vehicle, then the parking link length constraints can be interpreted as the cumulative (i.e., total) arrivals before the event must be always lower than park parking link lengths in terms of the number of vehicles.

\[ \sum_{0 \leq \xi \leq \tau} \sum_{p \in P} x_{i,j,\xi,\xi+F_{FTT}(i,j,t)}(p) \leq L_{(i,j)}, \forall L_{(i,j)} \in E^p \]  
(Equation 16)

Solution Algorithm

In a matrix form, the problem formulation in Section 4.2 can be expressed as follows:

P1: Min \( Z = CX \)  
(Equation 12)'

Subject to: \( AX = B \)  
(Equation 13)'

\( DX \leq E \)  
(Equation 14 to 16)'

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Similar with Chapter 3, the Lagrangian-relaxation-based optimization approach is used again. First, constraints \((CX \leq D)\) are relaxed via non-negative Lagrangian multipliers \(\lambda\) and form the relaxed problem P2.

\[
P2: \text{Min } \quad L = CX + \lambda(DX - E) \quad \text{(Equation 17)}
\]

Subject to: \(AX = B\) \quad \text{(Equation 18)}

More specifically, P2 can be formulated as:

\[
P2: \text{Min } \quad L = \sum_{0 \leq t \leq T} \sum_{(i,j) \in E} \sum_{p \in P} \left[c_{i,j,t,s}(p) \times x_{i,j,t,s}(p)\right] + \sum_{0 \leq t \leq T} \sum_{(i,j) \in E} \lambda_{(i,j,t)} \left[\sum_{p \in P} x_{i,j,t,s}(p) - \text{cap}^{in}_{i,j}(t)\right] + \sum_{(i,j) \in E^P} \mu_{(i,j)} \left[\sum_{0 \leq t \leq T} \sum_{p \in P} x_{i,j,t,s}(p) - L_{(i,j)}\right] \quad \text{(Equation 19)}
\]

Or

\[
P2: \text{Min } \quad L = \sum_{0 \leq t \leq T} \sum_{(i,j) \in E^P} \sum_{p \in P} \left[\left(c_{i,j,t,s}(p) + \lambda_{(i,j,t)}\right) \times x_{i,j,t,s}(p)\right] + \sum_{0 \leq t \leq T} \sum_{(i,j) \in E^P} \sum_{p \in P} \left[\left(c_{i,j,t,s}(p) + \lambda_{(i,j,t)} + \mu_{(i,j)}\right) \times x_{i,j,t,s}(p)\right] - \sum_{0 \leq t \leq T} \sum_{(i,j) \in E^P} \left(\lambda_{(i,j,t)} \times \text{cap}^{in}_{i,j}(t)\right) - \sum_{(i,j) \in E^P} \mu_{(i,j)} \times L_{(i,j)} \quad \text{(Equation 20)}
\]

Subject to: \(\text{Equation 13}\).

For simplicity, the queue spillback constraints are dropped because such a simplification will not substantially lower the validity of the overall solution since most of elements in queue spillback constraints in essence cancel out each other.

The optimization algorithm can be summarized as:

**Step 1: Initialization**

1.1 Set iteration number \(n = 0\);

1.2 Build the space-time network according to transformed time-dependent physical
1.4 Choose initial nonnegative values, such as 0, for all Lagrangian multipliers in 
\{\lambda(i, j, t), \mu(i, j, t)\};

Step 2: Solved relaxed problem P2

2.1 Given Lagrangian multipliers to get arc-cost in Equation 19, call the modified 
least-cost algorithm in Appendix A to solve P2 to determine the values for all route choice 
variables \(x_{ijts}^p\).

2.2 Calculate the objective function values of P2 using new values for \(\{x_{ijts}^f\}\) which 
gives a lower bound of system optimum;

2.3 Obtain a feasible solution by converting flow with hard capacity constraints using 
network loading tools, such as DTALite (Zhou and Taylor, 2014), to get a feasible solution 
to P1; this gives a upper bound of system optimum;

2.4 Calculate the gap value between upper bound and lower bound Equation 20; 
terminate the iterative process if \(n\) is greater than the maximum iteration or the relative gap 
is smaller than a specified threshold.

Step 3: Update Lagrangian multipliers using an approximate sub-gradient method

3.1 Calculate subgradients for all \(\lambda_{(i,j)}\) and \(\mu_{(i,j)}\) as:

\[
\nabla L_{\lambda_{(i,j)}} = \sum_{(i,j) \in E} \sum_{p \in P} x_{ijts}(p) - \text{Cap}^{in}(i, j, t), \forall (i, j) \in E, \forall t \in [0, T] ;
\]  
(Equation 20)

\[
\nabla L_{\mu_{(i,j)}} = \sum_{0 \leq t \leq T} \sum_{(i,j) \in E^p} \sum_{p \in P} x_{ijts}(p) - L_{(i,j)}, \forall (i, j) \in E^p
\]  
(Equation 21)

3.2 Updates all \(\lambda(i, j, t)\) and \(\mu(i, j)\) as:
\[ \lambda(i, j, t) = \max(0, \lambda(i, j, t) + \text{stepsize} \times \nabla L_{\lambda(i, j, t)}) ; \]  
\[ \mu(i, j) = \max(0, \mu(i, j) + \text{stepsize} \times \nabla L_{\mu(i, j)}) ; \]  

where step size is: \[ \frac{1}{n+1} ; \]

Go back to step 2.1 increasing iteration count to \( n + 1 \).

The rationale of calculating subgradients this way is that, if over congestions or queue spillbacks occur on some links, it is possible to reduce the allocated traffic in the next iteration by increasing the Lagrangian multipliers (i.e., penalties). If some links can accommodate more vehicles during certain periods, then the corresponding Lagrangian multipliers can be lowered in the next iteration to attract more vehicles on those underused links.

**Step 4: Termination Examination**

If the number of iterations \( n \) reaches a predefined maximum iteration or the gap values between P1 and P2 has been smaller than a specified tolerance threshold then STOP; otherwise, replace \( n \) by \( n+1 \) and go back to Step 2.
A Simple Three-Corridor Network Experiment

A simplified three-corridor network is constructed as presented in Figure 14 to illustrate the proposed space-time network for park-and-ride facilities. Specifically, in the network, node 1 represents the origins of travelers and node 12 represents the special event site (destination as well). The red lines represent roadway system that allows passenger cars driving through and the green lines represent dedicated public transit lines, which allows transit vehicles only. The links 4-8-7 and 5-9-7 allow passenger cars to drive to the park-and-ride facilities and then connect to public transit lines, whereas no access to transit link 7-13 and walking link 13-12.

Figure 14

Three-Corridor Network Representation
The traffic demand in the network can be divided into three types, which are driving-only, transit-only, and park-and-ride respectively. Initially, the background demand for each demand type is set to be 560 vehicles per hour. In order to illustrate the impact of background private car on the roadway network, two scenarios, the base and high demand, are tested.

Applying the proposed solution algorithms demonstrated in chapter 3, the above park-and-ride facilities design problem can be solved and achieved convergence by multiple computational iterations. Specifically, utilize a special version of DTALite (Zhou, et al., 2014), to obtain a feasible solution (i.e., upper bound) and propose a Lagrangian-relaxation based solution algorithms to obtain the lower bound of the optimum. The gap between lower bound and upper bound is reduced through (Dual-ascent) iterations until the gap is acceptable or the maximal number of iterations is reached. In the end, the corresponding feasible solution (lower bound) with the minimum gap is adopted to approximate the optimal solution.

The analytical results of two scenarios (scenario 1: 50 parking spaces in park-and-ride facilities; scenario 2: 500 parking spaces in park-and-ride facilities.) are showing in Table 6.
Table 6

Analytical Results of Three-Corridor Network Problem

<table>
<thead>
<tr>
<th>Park-and-ride capacity</th>
<th>100</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip mode</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Park-and-ride</td>
<td>100</td>
<td>247</td>
</tr>
<tr>
<td>Driving-only</td>
<td>900</td>
<td>753</td>
</tr>
<tr>
<td>Route flow (agents)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>127</td>
<td>120</td>
</tr>
<tr>
<td>900</td>
<td>157</td>
<td>138</td>
</tr>
<tr>
<td>Travel time (min)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>280min</td>
<td>1.61%</td>
<td>5.18%</td>
</tr>
<tr>
<td>330min</td>
<td>2.65%</td>
<td>6.02%</td>
</tr>
<tr>
<td>380min</td>
<td>51.43%</td>
<td>55.54%</td>
</tr>
<tr>
<td>430min</td>
<td>52.53%</td>
<td>56.64%</td>
</tr>
<tr>
<td>Accessibility of different time budgets</td>
<td>LB</td>
<td>UB</td>
</tr>
<tr>
<td>280min</td>
<td>1.61%</td>
<td>5.18%</td>
</tr>
<tr>
<td>330min</td>
<td>2.65%</td>
<td>6.02%</td>
</tr>
<tr>
<td>380min</td>
<td>51.43%</td>
<td>55.54%</td>
</tr>
<tr>
<td>430min</td>
<td>52.53%</td>
<td>56.64%</td>
</tr>
</tbody>
</table>

From the outputs of DTALite simulation, it can be seen that with the capacity of parking lots increasing, the number of people choosing park-and-ride mode increases from 50 to 396, and the average travel time reduces to 147.9 min. For the accessibility of different time budgets, the scenario with large parking lots capacity tends to better accessibility, for a travel time budget of 145 min, the accessibility increases from 52.53% to 56.64%.

Figure 15 indicates the relative gap and lower bound (LB) and upper bound (UB) with different scenarios of each iteration with proposed simulation algorithm. Specifically, the lower bound estimates are improved significantly after the first a few iterations, and the duality gap between the upper bound and the lower bound of the
optimum reduces dramatically to a relatively small difference after 10 iterations. In scenario 1, two park-and-ride facilities with 50 capacities are constructed, and the inaccessible agents is about 242. While in scenario 2, two park-and-ride facilities with 100 capacities are constructed, the inaccessible agents decrease to 208.

![Figure 15](image)

Relative Gap and LB and UB of Two Different Scenarios

A Realistic Case Study: International Horticultural Exposition 2019

In this thesis, a realistic case study is conducted to further examine the performance of the proposed algorithms. The International Horticultural Exposition will be hosted in Beijing, China in 2019 (hereafter referred to as Beijing Expo 2019). The exposition date will last from 29th April to 7th October 2019, more than 5 months in total. Beijing Expo 2019 is expected to have more than 100 official exhibitors (including
countries and international organizations), more than 100 other exhibitors (domestic provinces, cities and in China and abroad) and more than 16 million visitors as an initial traffic demand estimation, and the expected range of potential visitors will be 34 to 37 million according to the survey results.

Studies show that, according to the ideal departure model, the outer corridors within the scope of traffic impact area, including Jingzang Expressway, Jingxin Expressway, Xingyan Road and Jingzhang Highway (Yanqing branch) can accommodate about 252 thousand passenger cars in total per day. Thus there is a quite large gap between the existing road capacities and the expected traffic demand during the Beijing Expo 2019. Therefore, the best way to solve the problem of insufficient road capacity is to introduce park-and-ride alternatives to support the large volume of visiting demand.

The traffic impact analysis area of the Beijing Expo 2019 is illustrated in Figure 16. Within the study area, there are 66 OD zones, 1,519 nodes and 3,299 links with 22 different multimodal link types. The proposed analysis methodology in Chapter 3 is applied to the study area to examine the effectiveness of parking lots locations and capacity allocations, as well as space-time accessibility. Most of the study area is connected with Beijing central area and Zhangjiakou City through freeways, the two cities are considered as the primary traffic demand sources.
Traffic Network Representation of Yanqing District, Beijing, China

There are 2,127 existing transit lines within study area and adjacent cities, and 8 potential locations can be considered the candidate park-and-ride facilities. Based on the present situation, the key problems are to find optimal park-and-ride locations first and allocate capacities for each site and secondly to coordinate nearby transit lines to provide effective services.

Six scenarios are considered and optimized according to the proposed Lagrangian-relaxation-based solution algorithm, which take into considerations of governmental agency’s suggestions for parking lots locations and potential capacity constraints. The first scenario considers transit-only and driving-only modes but without park-and-ride mode. The second scenario considers a new bus rapid transit (BRT) line.
but without park-and-ride facility for transferring. For scenarios 3 to 6, different numbers of park-and-ride facilities are considered to be built under different total construction budget (TCB), and the park-and-ride facilities are served as transferring facilities to bus rapid transit (BRT) lines. In addition, in order to investigate the effectiveness of park-and-ride facilities on the riders’ accessibility in the space-time network, it is assumed that the total travel demand is 142,318 people (agents) per hour during peak hour period, and the demand types include driving-only, transit-only and park-and-ride.

The preliminary numerical results based on the initial OD demand estimates with limited survey data are listed in Table 7.
Table 7

Preliminary Numerical Results of Six Testing Scenarios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Driving-only</th>
<th>Transit-only</th>
<th>Park-and-ride</th>
<th>Accessibility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of agents</td>
<td>Travel time</td>
<td>Number of agents</td>
<td>Travel time</td>
</tr>
<tr>
<td>1</td>
<td>Without park-and-ride and BRT</td>
<td>80.1%</td>
<td>74.98</td>
<td>19.9%</td>
</tr>
<tr>
<td>2</td>
<td>Without park-and-ride, with BRT</td>
<td>61.6%</td>
<td>59.1</td>
<td>38.4%</td>
</tr>
<tr>
<td>3</td>
<td>With 5 park-and-ride facilities, with BRT</td>
<td>35.3%</td>
<td>22.31</td>
<td>18.1%</td>
</tr>
<tr>
<td>4</td>
<td>With 6 park-and-ride facilities, with BRT</td>
<td>32.6%</td>
<td>19.15</td>
<td>20.2%</td>
</tr>
<tr>
<td>5</td>
<td>With 7 park-and-ride facilities, with BRT</td>
<td>31.4%</td>
<td>18.57</td>
<td>22.3%</td>
</tr>
<tr>
<td>6</td>
<td>With 8 park-and-ride facilities, with BRT</td>
<td>30.8%</td>
<td>19.32</td>
<td>21.6%</td>
</tr>
</tbody>
</table>

The simulation results show that if all the visitors arriving the special event site by driving-only mode, the average travel time is 1 hour and 14.9 minutes, and there are only 23% of all visitors who can reach their accessibility goal, in other words, within their travel time budget. In scenario 2 with bus rapid transit links built, the average travel time is reduced to 59.1 minutes, and the percentage of visitors who reach their accessibility goal is dramatically increases to 47%. In this sense, building new bus rapid transit lines is necessary for accommodating such large number of travel demand.
At the second stage, the experiment examines different numbers of park-and-ride facilities to be built in each scenario. When 5 parking lots for the park-and-ride facilities are built, the average travel time will be reduced to 19.2 minutes significantly, and approximately 83% of visitors can reach their destination within their travel time budgets. In this scenario, more than 60% visitors will take public transit to the special event site. As examining scenarios 4 to 6, the number of park-and-ride facilities to be built ranging from 6 to 8, the results show that average travel time changes very little as well as the percentage of accessibility goal achieving. Thus a conclusion can be drawn that no obvious improvement can be made for more than 5 park-and-ride facilities built.

Based on the numerical experiments, it is clearly that optimized park-and-ride facilities locations and capacity allocation are both significantly helpful in terms of increasing visitors’ space time accessibility goals.
Summary and Conclusions

With the development of social and economic activities, there are more and more planned special events, such as conventions or exhibitions, in cities. Given that the travel demands for the events are typically much more than the capacities of the surrounding roadways and the nearby parking facilities cannot accommodate this surge of parking demand, special events workforce often consider build additional park-and-ride facilities in farther areas and open special bus lines to allow visitors to park and take buses to the event sites. Although it is ideal to build park-and-ride facilities with large capacities as many as possible, the workforce is often constrained by certain capital and land constraints while visitors also make the go-or-not decision based on their total time budgets. As a result, design park-and-ride facilities for special events is complicated and needs to seek leverage among many constraints.

In this thesis, the park-and-ride design problem is formulated as a network problem. Based on the space-time network models, the park-and-ride design problem is first formulated as a nonlinear programming problem to either minimal the total travel time (system optimum) or maximize the accessibility for travelers. Most of the constraints are hard except the flow-balance constraints.

A Lagrangian-relaxation-based solution algorithm was designed. Through reducing the gap between upper bound and lower bound of the optimum to an acceptable level, the quansi-optimal solution can be achieved. Specifically, the lower bound is the result of optimizing the relaxed problem (P2) while the upper bound is achieved based on DTALite, a dynamic network loading (DNL) tool which in essence relaxes the hard
constraints of First-In-First-Out (FIFO) and road capacity constraints in analytical
dynamic traffic assignment (DTA) formulations to obtain a feasible solution to the primal
problem (P1) based on the optimal solution to the relaxed problem (P2).

In addition to the standard mathematical programming formulation, certain
efforts are also devoted to how to reduce the complexity of park-and-ride facility design
problem. In Chapter 4, a transformed space-time network model is proposed to pre-build
the hard park-and-ride constraints into the network. Such transformation will not
compromise the fidelity of the problem formulation while it may considerably reduce
the number of hard constraints and so the problem complexity in some cases.

At last, two case studies are conducted: one is simplified network containing three
routes and the other is for a realistic park-and-ride facilities design for Beijing Expo
2019. Through comparison different scenarios, conclusions can be drawn that the
problem formulation based on the concept of space-time network and the proposed
solution algorithms in this thesis, it is possible to provide the most appropriate park-and-
ride facility design given the construction budgets, existing road capacities, proposed new
bus lines and average travel time budgets for visitors.

Further Work Recommendation

In the future, it is planned to design more efficient solution algorithms for the
relaxed problem (P2) and to further expand the concept of using transformed network
flow model to pre-build certain hard constraints in the network model. It is also planned
to develop certain computer programs to automatically transform the park-and-ride
facility design for special events from standard park-and-ride network models to the
transformed park-and-ride network models.
REFERENCES


Federal Highway Administration (FHWA), Management Travel for Planned Special Events, FHWA-OP-04-010 report, 2003)


APPENDIX

A DYNAMIC PROGRAMMING APPROACH TO SEARCH WAITING-
ALLOWABLE LEAST-COST PATH IN SPACE-TIME NETWORKS
In a space-time network \( STG = (V, E) \), denote \( lc(i, t) \) as the label cost of node \((i, t)\); \( Pred(i, t) \) as the predecessor node of node \((i, t)\); \( FFTT(i, j) \) as free flow travel time of physical link \( l = (i, j) \); \( c(f, i, j, t, s) \) as travel cost from \((i, t)\) to \((j, s), s = t + FFTT(i, j) \) and it is associated with the corresponding coefficient of \( x_{(i, j, t, s)} \) and \( n^U_i, n^D_i \) as the upstream node and downstream node of link \( l \); For vehicle \( f \), its least-cost path from \( o(f) \) to \( d(f) \) can be searched using the following waiting-allowable least-cost finding algorithm based on dynamic programming:

**Step 1: Initialization**

\[ lc(o(f), r(f)) := 0; \quad Pred(o(f), r(f)) := 0; \quad lc(j, t) := \infty \quad \text{for each node} \]
\[ (j, t) \in N - \{(o(f), r(f))\}; \quad \text{and LIST} := \{(o(f), r(f))\}; \]

**Step 2: Recursion**

For \( t = r(f) \) to \( T-1 \)

For each physical link \( l \in E \)

If \( lc(n^U_i, t) + c(f, n^U_i, n^D_i, t, t + FFTT(i, j)) < lc(n^D_i, t + FFTT(i, j)) \)

Then \( lc(n^U_i, t) + c(f, n^U_i, n^D_i, t, t + FFTT(i, j)) = lc(n^D_i, t + FFTT(i, j)) \)

\[ Pred(n^D_i, t + FFTT(i, j)) = (n^U_i, t) \]

End if

If \( n^U_i \) is a waiting node or an origin node

If \( lc(n^U_i, t + 1) > lc(n^U_i, t) + c(f, n^U_i, n^U_i, t, t + 1) \)
Then \( lc\left(n^U_i, t + 1\right) = lc\left(n^U_i, t\right) + c(f, n^V_i, n^U_i, t, t + 1) \)

\[
Pred\left(n^U_i, t + 1\right) = (n^V_i, t)
\]

End if

End if

End for each link

End for each stage

**Step 3: Trace back to get the least-cost path**

From the destination node \( (d(f), T) \), trace back using Pred \((j,t)\) until reaching the origin node \((i,0)\);

According to the algorithm structure, the upper bound of computing complexity for each vehicle \( f \) is:

\[
T \times (\text{No. of links} + \text{No. of waiting nodes})
\]