Conceptual Understanding of Multiplicative Properties

Through Endogenous Digital Game Play

by

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ABSTRACT

This study purposed to determine the effect of an endogenously designed instructional game on conceptual understanding of the associative and distributive properties of multiplication. Additional this study sought to investigate if performance on measures of conceptual understanding taken prior to and after game play could serve as predictors of game performance. Three versions of an instructional game, Shipping Express, were designed for the purposes of this study. The endogenous version of Shipping Express integrated the associative and distributive properties of multiplication within the mechanics, while the exogenous version had the instructional content separate from game play. A total of 111 fourth and fifth graders were randomly assigned to one of three conditions (endogenous, exogenous, and control) and completed pre and posttest measures of conceptual understanding of the associative and distributive properties of multiplication, along with a questionnaire.

The results revealed several significant results: 1) there was a significant difference between participants’ change in scores on the measure of conceptual understanding of the associative property of multiplication, based on the version of Shipping Express they played. Participants who played the endogenous version of Shipping Express had on average higher gains in scores on the measure of conceptual understanding of the associative property of multiplication than those who played the other versions of Shipping Express; 2) performance on the measures of conceptual understanding of the distributive property collected prior to game play were related to performance within the endogenous game
environment; and 3) participants who played the control version of Shipping Express were on average more likely to have a negative attitude towards continuing game play on their own compared to the other versions of the game.

No significant differences were found in regards to changes in scores on the measure of conceptual understanding of the distributive property based on the version of Shipping Express played, post hoc pairwise comparisons, and changes on scores on question types within the conceptual understanding of the associative and distributive property of multiplication measures.

The findings from this study provide some support for a move towards the design and development of endogenous instructional games. Additional implications for the learning through digital game play and future research directions are discussed.
DEDICATION

This thesis is dedicated to my beautiful wife Marian Doli Denham. Without your love, support, sacrifice, and never-ending patience I would not have been able to complete this journey.

This thesis is also dedicated to my maternal grandmother, Sylvia Marsh who laid the foundation for my interest in education.
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I dedicated this thesis to my wife, but I would also like to acknowledge her in this section. Marian, this degree is as much yours as it is mine. You sacrificed so much to go on this journey and I am forever indebted to you. I love you and I could not have made it to this point without my “rib”.

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Chapter 1

INTRODUCTION

GENERAL PROBLEM

Proficiency in multiplication is necessary for children to develop a robust number sense and vice versa. In order to be proficient in multiplication, children need to be placed in a learning environment that allows them to gain procedural fluency, conceptual understanding, and mastery of mathematical facts (National Research Council, 2001). The learning environment should show the connection between facts, concepts, and procedures as well. In addition, the environment itself should adapt to meet the instructional needs of each child (De Corte et al., 2003). The goal of this dissertation study is to investigate the effectiveness of a digital game that brings together these elements to promote conceptual understanding of two multiplicative properties.

There exists a strong theoretical case for the viability of digital games as environments for the kind of learning necessary for the promotion of conceptual understanding. Beginning with Malone’s (1981) work, educational researchers have sought to investigate the utility of digital games to impart learning. Prensky (2005) has written extensively about the need to teach “digital natives” through the technological tools that they use daily. Shaffer (2006) along with Gee (2003) asserts that a well-designed game provides an environment that is conducive to explore concepts, and participate in meaningful learning. Salen & Zimmerman
(2005) and Nelson, Erlandson & Denham (2011) have hypothesized about the potential of digital games to serve as assessment tools as well.

Previously conducted research on learning through digital game play has primarily focused on increasing player motivation and engagement (Malone 1981; Squire, 2005; Ke, 2008). This study differs from those, as the focus is on an investigation of the learning power associated with the use of a game environment to support conceptual understanding of the associative and distributive properties of multiplication and factual fluency of single-digit multiplication facts. Lampert (1986) cites the transition from intuitive knowledge, to computational knowledge, to concrete knowledge and principled knowledge of multiplication as the appropriate steps needed to achieve proficiency. Sherman et al. discourages the use of rote memorization as the sole instruction method as it hinders conceptual understanding by exposing students to a narrow view of multiplication and limits their understanding of multiplication to its facts. This narrow view of multiplication hinders success in higher-level math course when one’s ability to apply multiplicative reasoning requires a more comprehensive understanding of the operation.

In order to accomplish an approach to teaching multiplication that combines maintenance, remediation, and reinforcement of facts, along with interaction with multiplicative concepts, a digital game has been created for this study in which the associative and distributive properties of multiplication have been intrinsically integrated within the core mechanics of the game. Habgood (2005) attempted a similar study in the domain of division, with the main
difference being that he did not look at conceptual understanding. Habgood found that participants learning division in an endogenous digital game environment (where the instructional content is connected to the core game mechanics) significantly outperformed both those in an exogenous (where the instructional content is independent of the core game mechanics) and those in a control game environment on learning outcomes, game performance, and accuracy of answers.

**ISSUES IN K-12 MATHEMATICS EDUCATION**

There is a dearth of mathematically proficient students graduating from the American K-12 school system. The need for action to reverse this trend is so urgent that the proposed 2011 United States (U.S.) budget is allocating $3.7 billion in funding to various Science, Technology, Engineering and Mathematics (STEM) educational initiatives (White House, 2010). The federal government’s interest in the development of mathematically proficient students is motivated by a need for those entering the workforce to be equipped with more advanced STEM skills than is common currently. The Department of Labor’s Bureau of Labor Statistics has forecasted that the majority of rapidly growing occupations require strong skills in mathematics and science (White House, 2010). By addressing the need for a workforce proficient in STEM skills, the U.S. hopes to remain at the forefront of innovation and be able to compete in an ever-increasingly competitive global economy.

An additional impetus for addressing this issue is the performance of U.S. students in the Third International Mathematics and Science Study (TIMSS), which examined and compared the science and mathematics achievement of
fourth, eighth and twelfth graders around the world. Fourth graders in the U.S. scored above the international average in regards to general mathematics knowledge, but scored below average in eighth and twelfth grades. Most disturbing is the fact that the general knowledge mathematical items on the twelfth grade TIMMS, where U.S. students performed poorly, are designed to assess the mathematics skill level that a high school graduate would need to “function effectively in society as adults” (Gonzales, et al., 2004).

The results of the 2006 Program for International Student Assessment are equally troubling. Based on this assessment, the U. S. is ranked twenty-fourth out of the thirty most advanced countries in the world in mathematics proficiency (Gonzales, et al., 2004). A concerted focus on developing mathematical proficiency seems to be in the best interests of all those involved in K-12 mathematics education within the United States. One particular area in which to begin to address this issue is an area in which American Fourth graders performed poorly: number sense.

**NUMBER SENSE**

The development of a mature number sense is extremely important. The National Council of Teachers of Mathematics (NCTM, n. d.) defines number sense as a “person's general understanding of number and operations along with the ability to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for solving complex problems” (NCTM; Burton, 1993; Reys, 1991). Greeno (1991) presents a more comprehensive definition of number sense that will serve as the working
definition that this study will subscribe to:

“Number sense is an example of cognitive expertise—knowledge that results from extensive activity in a domain through which people learn to interact successfully with the various resources of the domain, including knowing what resources the environment offers, knowing how to find resources and use them in their activities, perceiving and understanding subtle patterns, solving ordinary problems routinely and generating new insights” (Greeno, 1991, p. 170).

Greeno extends his definition to assign three specific capabilities that are manifestations of number sense. The first, flexible numerical computation or the ability to understand the equivalences of numbers and being able to apply this knowledge in order to solve computational problems in an efficient manner. The second capability of number sense is numerical estimation. Numerical estimation refers to a consciousness of numerical approximations and their use in computational settings. The final capability of one who has number sense is the ability to make quantitative judgments and inferences about numerical values.

Although most children bring a natural understanding of numbers to kindergarten, formal schooling can cause disruptions in the development of a mature number sense (Fuson, Kalchmann, and Bradsford, 2005). Children have difficulty connecting their informally developed mathematical strategies and knowledge to formal settings (Carraher, Carraher, & Schliemann, 1985). While some children are able to make the connections later on, there are many who suffer from an emaciated number sense throughout their entire educational experience (Krasa & Shunkwiler, 2009).

Despite the efforts of formal schooling, certain preconceptions of
mathematics persist longer than they should. Fuson, Kalchmann, and Bradsford (2005) are of the opinion that learning environments should be created where preconceptions are addressed. The first preconception is that the sole function of mathematics is learning how to compute. The second preconception is that through the successful memorization and application of mathematical algorithm or “rules” one has achieved proficiency. Finally, the authors maintain that learning environments and teachers need to address the preconception that math is the domain of a select few. In other words, there are those who have the “natural” ability to do math and there are those who do not. The perpetuation of these preconceptions has the disastrous effect of stifling the development of a healthy number sense, and to a larger extent, mathematical proficiency.

An underdeveloped number sense contributes largely to weaknesses in mathematical proficiency. Large latencies are observed when asking a child with an immature number sense to recite numerical sequences, and to rapidly retrieve arithmetic facts directly from memory (Ashcraft, 1992; Baroody 1983; and Geary, 1999). It is clear that competence at lower mathematics is a prerequisite for competence at higher mathematics, right.

**MATHEMATICAL PROFICIENCY**

The process of achieving mathematical proficiency can be envisioned as a rope being braided. Individually each strand of a rope is weak and ineffective. Once these strands are intertwined, they create a strong rope that can support a large load. In the case of mathematics proficiency, the rope is made up of five strands: conceptual understanding, procedural fluency, adaptive reasoning,
strategic competence, and productive disposition (National Research Council, 2001).

Each of the five strands of mathematical proficiency serves a specific role. Each strand supports the others in order to carry the load, which in this case is mathematics. An exemplar of mathematical proficiency will be able to pull from each of these strands over a large range of mathematical concepts. In order for these five strands of mathematical proficiency to be woven together correctly though, the weaver must be skilled and the environment must be conducive to weaving.

NCTM has developed a set of curriculum focal points that can help with this “weaving” process. These focal points are designed to guide in the development of mathematical proficiency with K-8 students and “should be considered major instructional goals and desirable learning expectations, not as a list of objectives for students to master” (NCTM, 2006), p 10. One particular focal point that contributes greatly to the development of number sense (and ultimately mathematical proficiency) is the operation of multiplication. The NCTM designates the third and fourth grade as the years in which students should become proficient in multiplication. The Common Core State Standards for Mathematics (CCSSM) echoes the focus of the NCTM focal points. By the end of fourth grade the CCSSM call for students to be able to represent and solve problems involving multiplication and division, understand properties of multiplication, the relationship between multiplication and division, and multiply and divide by 100. By reaching the benchmarks for multiplication that the NCTM
and CCSSM have laid out, it is assumed that children will have a firm grasp on the facts, concepts, and procedures associated with multiplication. This study focuses on the development of proficiency in multiplication.

**MULTIPLICATION**

Multiplication has commonly been defined as the repeated addition of the same quantity (Anghileri, 2001). In order to be considered proficient in multiplication, a learner must be fluent in the multiplication facts, while possessing a conceptual understanding of the properties associated with multiplication (commutative, associative, distributive, identity, and multiplicative property of zero) and be able to solve multiplication expressions. If one is in possession of these skills, it will aid in the solving of complex problems that involve multiplication. It also aids in the understanding of the relationship between multiplication and division, the patterns associated with multiplication, and lays the foundation for algebraic reasoning.

Comparing the commonly held definition of multiplication (repeated addition) to the elements that are necessary for proficiency in multiplication (factual fluency and conceptual understanding), a more useful definition of multiplication should be applied. Lampert (1986) contends that a more precise definition of multiplication goes beyond the bounds of the repeated addition definition and accounts for the skills needed for proficiency in multiplication. In other words one does not just do multiplication but they also “know” multiplication. Lampert’s definition of multiplication is based on Kaput’s (1985) contention that the repeated addition definition limits multiplication to a counting
operation and is not applicable when one has to work with negative integers, rational numbers, algebraic quantities, continuous quantities, and ratios and proportions. In order to account for all of the ways in which multiplication can be applied it would be better to think of multiplication as the scaling or the stretching and shrinking of quantitative values based on knowledge of the facts and properties native to multiplication.

In comparison to single digit addition, there is significantly less research available on single-digit multiplication (Cooney, Swanson, & Ladd, 1988; National Research Council, 2001; Geary et al, 2008). What is known is that early on, children rely on rudimentary skills such as repeated addition, counting by n, equal grouping and skip counting. They then gradually transition to using self-created strategies and procedures, and begin to discover the patterns associated with multiplication. From there they transition to direct retrieval of multiplication facts from memory, which is referred to as automaticity (Geary, 1994; Siegler and Jenkins, 1989).

Unfortunately the direct retrieval of multiplication facts tends to be considered all that is important in regards to multiplication. Rote memorization techniques (i.e. multiplication tables, flash cards, rhythmic recitations) are commonly employed to teach the basic multiplication facts. While these instructional techniques are highly efficient and successful, they only support direct retrieval of multiplication facts, while failing to teach conceptual understanding, adaptive reasoning, and strategic competence (Sherman, et al, 2009). Geary et al. recommends multiplication be taught through an approach
which weaves together conceptual understanding and procedural knowledge (Geary et al., 2008) while Lampert (1986) recommends instruction that intertwines intuitive, computational, concrete, and principled knowledge. Lampert references Scribner (1984) study of milkmen who developed a context specific use of multiplication to determine the how best to pack dairy products and price the products based on the type of delivery container being used. This application of intuitive knowledge allowed the milkmen to multiply large quantities quickly through the use of an invented algorithm.

In contrast, computational knowledge of multiplication is the type of multiplication that is routinely found in formal school settings. The sequence of steps involved in multiplying 56 x 8 is an example of the application of computational knowledge. Students must first recognize that they need to convert this problem into a vertical problem:

\[
\begin{array}{c}
56 \\
\times 7 \\
\end{array}
\]

The student must then identify that they must work from the right hand column and multiply 7 x 6, which is 42. They must place the 2 below the 6 and 7 and know to carry the 4 above the 5 in the second column. Then they must know multiply 7 x 5, which gives them 35 and add the 4 that they carried over resulting in an answer of 392.

Concrete knowledge of multiplication “involves knowing how to manipulate objects to find an answer” (Lampert, 1986, p. 309). Concrete multiplication is demonstrated by students being able to understand the operation
of multiplication allows them to create equal groups of objects and then count the total number of objects in order to arrive at the correct answer.

Finally principled knowledge refers to the knowledge of multiplication that goes beyond solving multiplication problems with precision but being able to create mathematically sound techniques that can be applied in different situations. For example, someone with a principled knowledge of multiplication will understand that numbers within a multiplication can be decomposed by addition, operated on individually and then put back together to find the answer to the multiplication. For example, let's look at the problem we mentioned earlier: $56 \times 7$.

One with a principled understanding of multiplication could possibly look at this problem and solve it this way:

1. $56 = 50 + 6$
2. $50 \times 7 = 350$
3. $6 \times 7 = 42$
4. So $56 \times 7 = 350 + 42 = 392$.

This principled knowledge of multiplication can also be referred to conceptual understanding of multiplication.

**CONCEPTUAL UNDERSTANDING OF MULTIPLICATION**

Conceptual understanding of multiplication is important for overall mathematical proficiency. Fuson, Kalchman, and Bransford (2005) assert for one to be considered proficient in mathematics one must have “comprehension of mathematical concepts, operations, and relations” (Fuson, Kalchman, and Bransford p. 218). If this definition were repurposed for multiplication, it would
mean that one must have knowledge and understanding of the relationships, concepts, and properties native to the operation of multiplication. Lampert (1986) is of the belief that instruction should intensify the connections between intuitive computational, concrete knowledge and principled knowledge of multiplication in order to increase multiplicatively proficiency.

There are several concepts or properties of multiplication that students must acquire in order to be proficient. The first is the identity property, which states that any number multiplied by 1 results in the same number. Another property native to multiplication is the multiplicative property of zero. Plainly speaking the multiplicative property of zero states that any number multiplied by zero (or vice versa) is equal to zero. Another multiplicative property is the commutative property, which states the order of the numerals in a multiplication expression has no effect on the resulting product. The associative property of multiplication formalizes the fact that when multiplying three or more numbers, any grouping of numbers will result in the same product. For example \((A \times B) \times C = A \times (B \times C)\).

Finally the distributive property states that the sum of two numbers times a third number is equal to the sum of each addend times the third number. For example \(A \times (B + C) = A \times B + A \times C\).

Students should know these properties in order to be able to use them in other competencies, such as calculation or memorization of facts. For example, by understanding the multiplicative property of zero and the identity property, learners soon realize that there is no need to memorize any of the single-digit multiplication facts containing a zero or a one. In addition, when students truly
understand the commutative property, they will realize that they can be more efficient in the memorization of multiplication facts. Seven times three is the same quantity as three times seven, so one needs to just make a connection to the easy fact (three times seven) and the harder fact (seven times three).

Unfortunately this fact is not obvious to developmental learners and not true for all instances of multiplication (i.e., continuous quantities, ratios and proportions). This “principled” understanding of multiplication (Greeno, Riley, & Gelman, 1984) enables the learner to go beyond a procedural understanding of multiplication (being fully able to compute using multiplicative algorithms but with a lack of knowledge as to the meaning of what they are doing) and come to a place where the learner can “invent procedures that are mathematically appropriate and recognize that what he or she knows can be applied in a variety of different contexts” (Lampert, 1986, p. 309).

While there is much support for instruction that assists children’s understanding of the associative and distributive properties, there is little research being conducted on these properties. MacCuish (1986) conducted a study with elementary age students in which he asked them to solve computational problems that could be solved by the application of the distributive property. Students were given the solution to a multiplication problem (5 x 451 = 2255) and asked to solve a similar problem (5 x 452). MacCuish found that approximately 20% of the participants were able to correctly solve the problems through the application of the distributive property. Squire, Davies, and Bryant (2004) conducted a study in which they determined that only 5% of the fifth and sixth grade United Kingdom
students that they observed were able to solve more than half of word problems containing the distributive property that they were presented. In the study students were asked to solve word problems involving the distributive property:

“Cathy has a bar of chocolate that is 26 squares long and 21 squares wide. Altogether there are 546 squares. Sarah has a bar of chocolate 27 squares long and 21 squares wide. How many squares are there?”

That is disturbing in and of itself and indicates that much more work needs to be done to investigate children’s understanding of the associative and distributive properties.

Adaptive reasoning is critical as well. Adaptive reasoning refers to one’s “…capacity for logical thought, reflection, explanation, and justification” (Fuson, Kalchman, and Bransford, p. 218). National Research Council (2001) also defines adaptive reasoning as “the capacity to think logically about the relationships among concepts and situations and to justify and ultimately prove the correctness of a mathematical procedure or assertion. Adaptive reasoning also includes reasoning based on pattern, analogy or metaphor" (National Research Council, 2001, p. 170).

In an analysis of what is known about the difference between novices and experts, Hatano and Oura (2003) summarized results into six points. Of those six points, four have applicability to learning within schools. The first is that in order for one to be considered an expert, they must have and be able to demonstrate a comprehensive and robust framework of domain knowledge (Chi, Glaser, & Farr, 1988). Second, it takes focused and deliberate practice over a long period of time
for an expert to acquire proficiency in solving problems native to a particular domain (Ericsson, 1996; Lajoie, 2003). Third, once one becomes an expert, there is observable change in the socio-emotional composition of the person. Lastly, expertise is acquired when the learner has the support of people (teachers, tutor, peers, etc.) and artifacts (books, educational technology, software, etc.) (Shweder, Goodnow, Hatano, LeVine, Markus, & Miller, 1998).

The final piece of the puzzle is strategic competence. Strategic competence is the “ability to formulate, represent, and solve mathematical problems” (Fuson, Kalchman, and Bransford, p. 218). Children need to be exposed to many strategies to solving problems, as well as given the leeway to determine their own course of action. When presented with multiplication problems, a child with competence in strategies will have a large collection of tools from which to draw. Some of these strategies include: problem contextualization, informality of language use in problems, and use of manipulatives.

Physical representations (manipulatives) are also helpful as they help children to make connections between physical, concrete examples and the mental models that are being created (Griffin, 2001). The psychological functions of these tools are to provide a physical object in which learners explore in order to make the important connections between procedural concepts and conceptual understanding (Balka, 1993). In addition, theorist feel that manipulatives allow the learner to embody, ground and situate their mathematical learning, which in turns aids in the development of conceptual metaphorical maps which are can pulled from later on (Nunez, Edwards, & Matos, 1999). Unfortunately this course
of study is not always followed within formal settings. One reason could possibly be the lack of elementary school teachers who are proficient in mathematics.

**THE ROLE OF TEACHERS**

Most elementary school teachers are not content experts in mathematics. Issues associated with teaching multiplication correctly can sometimes be connected to a teacher’s own flawed understanding of multiplication and mathematics in general. In addition teachers may have low self-efficacy in relation to mathematics and can at times pass that on to their students (Middleton, & Spanias, 1999; National Research Council, 2001). High stakes standardized testing and district determined pacing also make it difficult for the typical classroom teacher to provide an environment conducive to acquiring proficiency in multiplication. Increased classroom sizes caused by adjustments made to offset budget cuts make the task even more difficult.

Difficulties in learning multiplication manifest themselves in other ways. Baroody & Coslick (1998) points to several instructional methods that inhibit the learning of multiplication. The first is the disregarding of a child’s intuitive knowledge. The second is instruction that does not seek to make a connection between the formal symbolic representation of multiplication and a child’s intuitive knowledge. The third and finally instructional impediment to learning multiplication is the tendency of teachers to shape a restricted conceptual understanding of multiplication. As stated earlier, a narrow view of multiplication would view 4 x 7 exclusively as 4 added seven times or four added seven times (Baroody & Coslick, 1998).
An additional impediment to the learning of multiplication is sometimes the child. Each child brings his or her own individual strengths and weaknesses to the classroom. The ideal teacher is able to quickly assess these strengths and weaknesses, and help guide their students to development of a robust number sense (Dehane, 1997). These teachers can then plot a course of action that will help each student continue progressing on an upward learning trajectory. Instead of taking this approach, some teachers may revert to instructional approaches that stress the flawed mathematics preconceptions mentioned previously (Bell, 1991). This only serves to limit the possibility of a child becoming proficient in multiplication. In order to address these issues, a learning environment in which the five strands of mathematical proficiency are fostered while adapting to the needs of individual learners is ideal. One such learning environment can be found within instructional digital games.

DEFINING DIGITAL GAMES AND PLAY

To begin the discussion of the use of instructional digital games to help foster learning of multiplication, it would first be appropriate to define two terms within the title of this study in order to clearly establish the reasoning for this study. There are many different definitions of these two terms, and for the sake of clarity it would be prudent to articulate the definition that will be applied in this study in order to situate the reader.

The first term to be defined is “game”. There are several widely used definitions of game. For that reason (and others) there is differing consensus on the definition of a game (Salen & Zimmerman, 2003). For example Avendon and
Sutton-Smith (1971) define a game as, “an exercise of voluntary control systems, in which there is a contest between powers, confined by rules in order to produce a disequilibrial”. Costikyan (2002, p. 24) in turn refers to a game as “an interactive structure of endogenous meaning that requires players to struggle toward a goal”. For the purposes of this study I will accept Schell’s definition of a game as my working definition. Schell (2008), simple defines a game as “a problem solving activity, approached with a playful attitude”.

The second term to be defined is play. What is play? Huizinga sought to examine play in his book *Homo Ludens* and through his examination of play Huizinga (1950) identified five definitive characteristic traits:

1) Play is free
2) Play is not real life
3) Play is separated from real life in terms of locality and duration
4) Play creates order
5) Play is connected to no material interest

While Huizinga’s five definitive characteristics of play are quite comprehensive, they are not concise enough to be a working definition. For the purposes of this study, I will use a modified version of Gilmore’s (1971) definition of play as a working definition: “Play refers to those (orderly) activities, which are accompanied by a state of comparative pleasure, exhilaration, power, and the feeling of self-initiative”. Through an analysis of the various definitions of games available in the literature, McGonigal (2011) and Schell (2008) independently extrapolated several characteristics of games. McGonigal (2008) suggests that
there are four traits that are obligatory for any activity to be considered a game. There must be a 1) goal, 2) a set of rules that are govern all interactions within the game, 3) a feedback system that provides instantaneous feedback on each player’s progress and 4) all players must participate under their own volition in addition to understanding and accepting the rules of game. Schell (2008) for his part refers to his list of game characteristics as game qualities. According to Schell there are ten game qualities: Games are entered willfully, have goals, have conflict, have rules, can be won or lost, contain interactivity, have challenge, create their own endogenous value, are engaging and are tight, formal systems (Schell, 2008).

There is an obvious overlap between McGonigal and Schell's list of game characteristics. For the purposes of my discussion of games I will refer to Schell's list of ten game qualities when discussing the characteristics of games that make them suited to potentially serve as learning environments.

DIGITAL GAMES AND LEARNING

There is a large body of research that supports the theoretical value and viability of the use of digital games within educational settings. The consensus of the games for learning community is that a well-designed game tailored to meet instructional objectives has the potential to help serve as a viable alternative learning environment to the ones typically found in most classrooms. These theorists believe that the native affordances of games, and specifically digital games, are conducive to creating an environment where learning can take place. (e.g. Gee 2003; Shaffer, 2006; Prensky 2005, Tobias and Fletcher, 2011).

Shaffer (2006) makes a strong case for the ability of computer games to
help children to learn. Shaffer theorizes that well-designed games allow the learner to participate in meaningful discovery learning, which is ideal for encoding information into long-term memory and developing a conceptual understanding of the content. Wong (1996) identifies four pertinent features of digital games that make them attractive as educational tools: instantaneous feedback, continual improvement, high response rates, and an unlimited ceiling on performance. While Wong’s four features of digital games are desirable characteristics of any good learning environment, games have the added ability to be distributed widely to a large audience and replicated consistently from person-to-person. That is not necessarily always feasible within a traditional instruction-lead environment.

According to Prensky (2005), the learners in today’s digital age are different from those in previous generations and require different tools to motivate them to learn. They prefer play over work, fantasy over reality, immediate payoff over patience, active learning over passive learning, to work in concert with their peers, and they view technology as a friend. Within a well-designed digital game there is enjoyment, involvement, structure, motivation, flow, outcomes, and constant feedback. Prensky has also theorized that there are five levels of learning that take place when one plays a digital game: learning how, learning what, learning why, learning where, and finally learning when and whether. Prensky also mentions that digital games encourage discovery learning, guided discovery, feedback, the ability to learn from mistakes, and task-based learning. In term of future directions, Prensky encourages future research that
combines game play, learning, and for a departure from primarily using the Internet as a venue for distance learning courses, but also as a means to distribute games that encourage learning.

Game designers can create games that encourage meaningful learning by putting thought into the context of the game, the participants of the game, the meaning of the game, the systems within the game, interactivity, and the choices that the user will make. Salen & Zimmerman (2005) discuss the learning and assessment value of choices made within games: what happened before the player was given the choice, how is the possibility of choice conveyed to the player, how did the player make the choice, what was the result of the choice, and how is the result of the choice conveyed to the player present some interesting considerations for meaningful learning. This combination of internal cognitive activities and external representations can take a game from being a game to being a meaningful game.

While theoretically sound, the games for learning movement have had difficulty producing empirical results that validate its conjectures. This is confirmed by Ke’s (2008) review of the literature on games for learning, which found little empirical evidence for the power of games for learning. One shortcoming of using games for learning is that because of all the stimuli inherent to games the learner can frequently get distracted from the actual learning task. Miller, Lehman, & Koedinger, (1999) conducted a study in which they had a condition that allowed the participants to play a game with a nebulous instructional goal. They found that these participants spent considerable more
time improving their ability to successfully play the game, than acquiring the tacit physics knowledge about electric fields that it was designed to teach.

Smith & Mann (2002) argue that because of the need for educational games to reach specific curricula goals, and to be able to directly assess the user, educational games take on a school like atmosphere. This mirrors Squire’s (2005) findings from his attempts to integrate digital games within a classroom. Squire sought to integrate a commercial video game (Civilization III) into the classroom. Some of the students in his study resisted the use of a digital game in class, while others were discouraged by the time it took them to actually learn to play the game. In addition Squire discovered because game play was a mandated part of the curriculum, it magnified the students’ resistance to the playing the game. It seems that the potential benefits of digital games as learning tools are diluted when adapted to meet educational assessment goals. Smith & Mann refer to this as the removal of the “gameness” from the game. It ceases to be a game and becomes just another school assignment.

There are additional possible explanations for the lack of empirical findings to support the direct learning gains from game play that can be tied to game design and development. One contributing factor could be the difficulty in creating a learning game comparable to commercial games in terms of production value and quality. This may dissuade researchers from participating in this arena. Another contributing factor could be as Papert (1998) describes as “Shaven Reversals” or the combining of the worst features of learning and game design.

Yet another reason for the lack of empirical studies that show direct
learning gains from digital game play could lie in the fact that what one learns in most games is specific to the game and it is difficult to transfer that knowledge outside of the game itself. Regardless of these shortcomings, there is still a solid theoretical framework for the potential for digital games to serve as an educational tool.

**INTRINSIC INTEGRATION**

One point of continual discussion within the games for learning community is whether instructional objectives should be met through endogenous (intrinsic) or exogenous (extrinsic) game play. Malone (1981) first tackled this issue by developing a theory that instructional games should account for the following motivational heuristics when being designed: challenge, fantasy and curiosity. Malone believes challenge is necessary to create an environment where the outcome is far from certain. While the goal is clear, the path to achieve a goal should require the user to expend energy to achieve it. For example, look at soccer. The goal is very simple: get the ball in the goal. The challenge comes in when one is told to accomplish the goal of the game without the use of ones hands on a field that is 110-120 yards long and 70-80 yards wide. An additional challenge is that there is a goalkeeper standing in front of the goal that can use her hands to stop the ball from going in the goal. Finally before one can even get into position to attempt to get the ball past the goal keeper, they have to get past the ten players that are on the goalkeeper’s side to prevent you from scoring on their goal, while simultaneously making sure that the other team doesn’t score in your own goal. By adding challenge to the game, the player remains engaged and
motivated to continue game play. Additional ways to add challenge are by adding levels of increasing complexity, having a variety of difficulty settings (beginner, intermediate, advanced) and having mini-goals within levels (complete a specific task within the shortest amount of time, attain a target score, etc.).

Malone believes that fantasy is an important heuristic for designing games because it allows users to connect to the game emotionally and metaphorically. Malone defines fantasy as the evoking of “mental images of physical objects or social situations that are not actually present” (Malone, 1984 p., 67).

The last of Malone’s three heuristics for game design is curiosity. Curiosity is important because it keeps the player engaged by introducing novelty and unpredictable interactions within the game environment. One specific means to accomplish this is through the use of two senses: hearing and sight. Audio and visual effects can be used to pique the curiosity of the player by representing sights and sounds the player is familiar with. Another means that Malone suggests to stimulate curiosity is through randomness. Human beings like patterns and predictability. Malone contends that randomness creates a sort of cognitive dissonance, which challenges players by making them feel like knowledge structure is “incomplete, inconsistent, or parsimonious” (Malone 1984, p. 67). This can be accomplished by introducing new features, tasks, or skills within the game play or requiring players to complete a previously mastered task under more challenging constraints.

Malone and Lepper (1987) expanded Malone’s theory on heuristics for designing games by adding four motivations: control, cooperation, competition,
and recognition. Within this expanded taxonomy of intrinsic motivations for learning in games, Malone and Lepper make the distinction between endogenous and exogenous games. Malone and Lepper define endogenous games as games in which have the following properties (Malone & Lepper, 1987, p. 240):

A) “The skill being learning and the fantasy depend on each other”

B) “There is an integral and continuing relationship between the fantasy context and the instructional content being presented.”

An excellent example of an endogenous game within the domain of mathematics that is closely related to this study is the game Motion Math. Within Motion Math (Adauto & Klein, 2010) content knowledge, which in this case is the estimation of rational numbers using a number, is required to progress within the game. The goal of Motion Math is to get a bouncing ball to correctly bounce on a specific point on the bottom of the screen. The bottom of the screen is a number line, and the ball identifies where on the number line the player should be aiming for by containing a fraction, percentage, decimal, or pie graph. Developed for the Apple iOS, Motion Math requires players to tilt the device (iPhone, iPad, or iPod Touch) in the direction that they want the ball to move. Successfully getting a pre-determined set of balls to land on the correct spot is needed to level up.
Malone and Lepper define an exogenous game as “one which the fantasy depends on the skill being learned but not vice-versa” (Malone & Lepper, 1987, p. 240). In other words the instructional content being taught is outside, or exogenous, to the actual game being played. Battersby (2010) in a review of intrinsic learning in games provides an excellent example of an exogenous game: Hangman. Success in Hangman is determined on one’s knowledge of spelling and vocabulary but that could be easily be switched to mathematics if one wanted to. The game mechanics would remain the same, as they are independent of the instructional content. Based on these two definitions and empirical studies, which they conducted on games for learning, Malone and Lepper contend, “in general, endogenous fantasies are both more interesting and more educational than exogenous fantasies.” (Malone & Lepper, 1987, p. 240).

Recently, Habgood, Ainsworth and Benford (2005) have sought to provide
an alternative theory to Malone and Lepper’s taxonomy for learning through endogenous digital games. They contend that the term “endogenous fantasy” is limiting in its scope, and that learning gains would be better accomplished through what Kafai (2001) calls intrinsic integration. The authors characterize intrinsic integration has having three distinct traits: Flow, core mechanics and representations. Flow, “a feeling of total concentration, distorted sense of time, and extension of self” are feelings that can be identified by anyone completely engaged on a task (Habgood, Ainsworth, & Benford, 2005, p. 492). Csikszentmihalyi’s flow theory states that the presence of “clear goals, achievable challenges, and accurate feedback are required to achieve a state of flow in an activity” (Csikszentmihalyi 1998 p. 34).

Core mechanics, is defined as the “mechanism through which players make meaningful choices and arrive at a meaningful play experience” (Salen & Zimmerman, 2004, p. 317). Habgood et al believes core mechanics are important for intrinsic integration because they help to create activities within the game that are relevant to the player. Core mechanics also help to create flow experiences and assisting in channeling many motivating by-products such as “challenge, control, cooperation, and competition” (Habgood, Ainsworth & Benford, 2005, p. 493). Finally the authors present representations as the final core trait of intrinsic integration. They point to empirical research which supports the supposition that the structures and interactions within an educational game will be more beneficial for learning if they are representative of the learning content (Ainsworth & Loizou, 2003, Miller, Lehman, & Koedinger, 1999, Papert & Talcott, 1997,
Reiber, 1996). By weaving interactions within the game with the metaphoric representations of the learning content, players will develop deeper conceptual understanding of the instructional content (Martin & Schwartz, 2005).

This theory of intrinsic integration was applied to the design and development of a game called Zombie Division (Habgood, 2005). The purpose of this third person action-adventure game was to integrate within the core game mechanics strategies for division of whole numbers. The goal of the game is to defeat skeletons walking ancient Greece, which are impediments to the completion of the player’s quest. Each skeleton has a different number on its chest and the player has to use a different sword attack based on that number. The skeleton is only defeated if the sword attack the player chooses to use results in the skeleton being divided without a remainder, while being limited to sword attacks two through ten. For example chopping the skeleton with 24 on its chest using the 4-sword attack would divide the skeleton into six pieces. Using the 5-sword attack would have not effect on the skeleton. Habgood addresses each of the three traits of intrinsic integration by using the action-adventure genre to create the flow experience, embodying the learning content through the use of the sword attack core game mechanic and by representing division metaphorically through the splitting of the skeletons once a sword attack is used.
In order to investigate the learning benefits of intrinsic integration, Habgood and Ainsworth (2007) conducted an empirical study in which three versions of Zombie division were compared: intrinsic (endogenous), extrinsic (exogenous), and control. In the intrinsic condition the skeletons would only be able to be defeated by using the aforementioned sword attack core mechanic. In the extrinsic condition the skeletons had the sword attack that would defeat them labeled on their chest and at the conclusion of each level, the player had to take a multiple choice test based on the division problems they have solved during the level. Within the control condition the game play was exactly the same as the extrinsic condition with the exception that the player did not have to take a test at the end of each level.
Two studies were conducted using these three versions of Zombie Division. In study one the researchers sought to determine the difference in participants’ learning gains from playing each version of the game for a period of two hours while receiving support from their classroom teacher. While all participants improved over the length of the study, the participants in the intrinsic condition significantly outperformed those in the extrinsic condition on post and delayed posttests.

In the second study the researchers sought to determine the impact of intrinsic integration on choice of game. Participants were introduced to the intrinsic and extrinsic versions of the game within the same sitting and shown how to switch between each version of the game through a menu command. The students were allowed to play the game or choose another activity entirely but were allowed to go back and forth between activities. Students played for a total of approximately 45-50 minutes over four different sessions. After the fourth session the participants were interviewed and asked to share with the researchers any differences in game mechanics they observed between the two versions of the game. Each child was asked which version they preferred and why. As a group, the children also discussed which version of the game was most enjoyable and which one they felt they learned the most from. Using time on task game logs, the researchers were able to ascertain that students played the intrinsic version of the game seven times longer (75.5 minutes, SD = 35.5) than the extrinsic version (10.28 minutes, SD = 10.28). This difference was significant (t = 7.38, p < .01, r = .89). Coding of the group interview indicated that the children spoke more
positively of the intrinsic version of the game and displayed a keen insight into the differences between the two versions.

These results speak to the validity of the theory of intrinsic integration within instructional game design. Habgood & Ainsworth (2011) encourage the use of this approach as it encourages the development of games that challenge children to explore mathematical strategies within a motivating environment. While the results of this experiment are promising, one would wonder if the learning effects would be bolstered by the inclusion of intelligent game play adaptation within the game architecture.

**STUDY PURPOSE**

This study sought to determine the benefits (if any) of an endogenously (intrinsic integration) designed digital instructional game compared to an exogenous version of the same game, and the comparative ability of each game type to create conditions necessary for the development of mathematical proficiency and number sense. Specifically, this study focuses on the mathematical operation of multiplication and the effectiveness of a digital game to support conceptual understanding of the multiplicative associative and distributive properties.

*Study Importance*

The results of this study carry significance for the field of mathematics education by exploring the aforementioned need for embodied, situated learning environments that consistently support the acquisition of mathematical proficiency. In addition this study begins to address the lack of empirical work
being done on the conceptual understanding of the associative and distributive properties of multiplication. By focusing on the simultaneous exploration of multiplicative conceptual understanding and factual fluency within a digital environment, this study seeks to at the least replicate the results observed by Woodward (2006) during his instructor led intervention.

Woodward conducted a review of the literature related to intervention strategies for teaching mathematics facts, and conducted an experimental study comparing an integrated approach (combination of teaching strategies using rectangular arrays and number lines and timed practice drills) with timed practice drills of multiplication facts. 58 fourth graders participated in the study with 15 of the students possessing math disabilities. While both groups improved in the ability to recall multiplication facts, the participants in the integrated approach performed better than the timed practice drill group on posttest and maintenance test measures that were designed to assess the application of multiplication facts to extended facts and approximation tasks. Based on these results, Woodward concludes that the "integrated approach and timed practice drills are comparable in their effectiveness at helping students move toward automaticity in basic facts" (Woodward, 2006, p. 287).

Denham & Nelson (2011) conducted investigation of the integration of Woodward’s combined drill and strategy within an exogenous instructional video game, *Escape From Goldac*. In the game, the learner played a character (Lerpz) that was sentenced to spend the rest of his life on a far away planet after being wrongly convicted of a crime. There is a spaceship that Lerpz can use to escape
the planet, but the fuel that he needs to power the spaceship is locked inside various force fields. The only way that Lerpz can turn off each force field is by correctly determining the combination to the force field. Players of the game get the correct combination by solving a varying number of multiplication facts in a set amount of time. When a player’s Lerpz avatar collided with a force field, an input device appeared in the top right hand portion of the screen, which presented the student with the time left to unlock the force field, a multiplication expression to solve, and a space to display their answer. A keypad was available to enter their answer and to submit that answer to be checked. In a timed-drill plus strategy instruction (integrated) version of the game, players had to unlock the force fields and they were also provided with strategies for each family of multiplication facts.

![Figure 3: Goldac Screenshot](image)

Participants could access these hints at anytime, as they were integrated into the Heads-Up-Display. Selecting a hint would pause the game and allow players time
to digest the information. Each strategy contained a sample problem and solution, along with a visual representation of the strategy. All strategy screens contained a reminder on the bottom that provided information on the commutative, identity and multiplicative property of zero.

![Goldac Hints Screenshot](image)

*Figure 4: Goldac Hints Screenshot*

After collecting a set amount of fuel cells, the force field surrounding the spaceship was unlocked and players were able to escape. Participants in the timed-drill version of the game did not have access to the strategies for each family of multiplication facts.
Table 1.

<table>
<thead>
<tr>
<th>Strategy Text</th>
<th>Numerical Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0</strong> Any number times zero equals zero.</td>
<td>8 x 0 = 0</td>
</tr>
<tr>
<td>This is called the <strong>Multiplicative Property of Zero.</strong></td>
<td>0 x 8 = 0</td>
</tr>
<tr>
<td><strong>1</strong> Any number times one equals that number.</td>
<td>7 x 1 = 7</td>
</tr>
<tr>
<td>This is called the <strong>Identify Property of Multiplication.</strong></td>
<td>1 x 7 = 7</td>
</tr>
<tr>
<td><strong>2</strong> Add the number to itself</td>
<td>2 x 4 = 4 + 4 = 8</td>
</tr>
<tr>
<td></td>
<td>4 x 2 = 4 + 4 = 8</td>
</tr>
<tr>
<td><strong>3</strong> When multiplying by three, skip count</td>
<td>3 x 4 = 3 + 3 + 3 + 3 = 12</td>
</tr>
<tr>
<td><strong>4</strong> Double and Double Again</td>
<td>4 x 3 = 6</td>
</tr>
<tr>
<td></td>
<td>3 x 2 = 6</td>
</tr>
<tr>
<td></td>
<td>6 x 2 = 12</td>
</tr>
<tr>
<td><strong>5</strong> When multiplying a number by five the answer is always half the number times ten</td>
<td>5 x 4 = 10 x 2 = 20</td>
</tr>
<tr>
<td></td>
<td>4 x 5 = 2 x 10 = 20</td>
</tr>
<tr>
<td><strong>6</strong> When multiplying a number six, first multiply that number by 2, and then by 3.</td>
<td>4 x 6 = 8</td>
</tr>
<tr>
<td></td>
<td>Multiply 4 x 2 = 8</td>
</tr>
<tr>
<td></td>
<td>Multiply 8 x 3 = 24</td>
</tr>
<tr>
<td><strong>7</strong> Using neighboring facts when multiplying by seven</td>
<td>7 x 5 = (6 x 5) + 5 = 35</td>
</tr>
<tr>
<td></td>
<td>2 x 7 = (2 x 6) + 2 = 14</td>
</tr>
<tr>
<td><strong>8</strong> When multiplying a number by 8, double the number three times.</td>
<td>3 x 8 = 3 x 2 = 6</td>
</tr>
<tr>
<td></td>
<td>6 x 2 = 12</td>
</tr>
<tr>
<td></td>
<td>12 x 2 = 24</td>
</tr>
<tr>
<td><strong>9</strong> When multiplying by nine remember the add one and minus one pattern.</td>
<td>9 x 2 = 18</td>
</tr>
<tr>
<td></td>
<td>9 x 3 = 27</td>
</tr>
</tbody>
</table>

The purpose of this study was to see if Woodward’s result could be replicated (in regards to retrieval times for multiplication facts and conceptual understanding of multiplication) through the medium of digital game play. While there was a significant reduction in retrieval times, \( t(29) = 4.401, p < .01 \) and a significant increase on the measure of conceptual understanding on average, \( t(28) = 3.008, p < .01 \) for all players, there was no difference found between those students in the timed-drill condition and those in the combined drill and strategy condition. Denham and Nelson concluded that the exogenous design of
the game was not conducive to the exploration of the concepts native to multiplication. In addition, they observed that the ordering of tasks within the game did not help students to address the facts and concepts that they needed to spend the most time with. The current study seeks to address these shortcomings, and also to help define the importance of adaptivity and the intrinsic integration of instructional content within an instructional game.

An additional contribution of the current study to the field of mathematics education is its contribution to the literature on associative and distributive properties. Geary et al. (2008) state there is an inadequate amount of research being conducted on children’s understanding of the associative and distributive properties of multiplication.

Likewise, the results of this study may carry importance for the field of digital games for learning. As stated earlier, educational researchers who have previously conducted research in this field have had difficulty finding and replicating learning gains that can be directly attributable to time spent playing instructional digital games. Most research has found strong evidence to support the statement that instructional digital games are motivating and engaging. This is promising, but in order for this field of study to progress, empirical work must be done to validate the theoretical learning benefits of instructional digital games.
RESEARCH QUESTIONS

1. What effect, if any, does the intrinsic integration of instructional content within the core game mechanics of a digital game have on players’ conceptual understanding of the multiplicative associative and distributive properties?

2. Does performance on measures of conceptual understanding of the multiplicative associative and distributive properties predict performance within the game environment?

HYPOTHESIS

Based on the previous discussion, the following hypothesis will be investigated:

1. There are significant differences between participants on the conceptual understanding of the associative property measure based on whether they played the endogenous, exogenous, or control version of Shipping Express. 
   H1: Endogenous > Exogenous > Control.

2. There are significant differences between participants on the conceptual understanding measure of the distributive property based on whether they played the endogenous, exogenous, or control version of Shipping Express. 
   H2: Endogenous > Exogenous > Control.

3. One can predict how well a participant will perform within the game based on their performance on measures of conceptual understanding taken prior to game play within the endogenous game environment.
Independent Variables

1. Version of Shipping Express Played
   a) Game, with intrinsically integrated (an endogenous representation of concepts integrated into the core game mechanics) conceptual learning about distributive and associative properties of multiplication.
   b) Game, with non-integrated (an exogenous presentation of concepts) conceptual learning about distributive and associative properties of multiplication.
   c) Game, with no presentation of instruction related to conceptual learning about distributive and associative properties of multiplication.

2. Level reached in game environment

Dependent Variables

1. Conceptual understanding of associative property of multiplication measure.

2. Conceptual understanding of distributive property of multiplication measure.
PARTICIPANTS

A study was conducted in the spring of 2012 to investigate the aforementioned hypotheses. There were 111 participants in this study. The participants were fourth and fifth graders enrolled at an elementary school in the southwest region of the United States. In light of their age, no monetary stipend was provided. It was assumed that based on their age, participants were still in the process of developing their multiplication skills, or had just been recently introduced to single-digit multiplication. There were 54 males and 45 females (based on those who reported gender) in this study. In terms of ethnicity, 46.4% of participants were white, 5.4% African-American, 8% Hispanic, 5.4% Native American, 0.9% Asian, 16.1% other, and 17.8% choose not identify ethnicity. Institutional Review Board approval was acquired in order to protect the rights of the researcher and participants.

DIGITAL GAME-BASED LEARNING ENVIRONMENT

Participants within this study played one of three versions of a digital game designed for this study entitled Shipping Express.

1. Endogenous (N = 38)
2. Exogenous, (N = 37)
3. Control (N = 36)

Within Shipping Express, the participants take on the role of the dock manager for a shipping company. As the dock manager, they are tasked with loading trucks of
various sizes with boxes to be delivered around their city. As each truck pulls up, it indicates the number of boxes needed in order for it to leave the facility. Within each level, players have a set amount of trucks to send out within a time period, with bonus time being added for each correct answer. Players have an unlimited amount of attempts to complete a level. As players complete a level they are put up for a promotion in order to be transferred to larger facility and get an increase in pay. Each subsequent level/facility requires players to send out more and more trucks. In later levels the trucks will increase in frequency of appearance, providing a further challenge. Players beat the game when they have successfully completed a shift at the main shipping facility of Shipping Express.

**GAME VERSIONS**

For the purposes of this study, three versions of Shipping Express were developed. The main version of Shipping Express is the endogenous version, from which the exogenous version was built. Within the endogenous version of Shipping Express, game play is governed by the aforementioned properties of multiplication (associative and distributive). All actions within the game require an understanding of these properties as well as knowledge of multiplication facts. In the endogenous version of the game, multiplication is attached to a set of mouse actions. The figures below illustrate will be used to explain how interactions within the endogenous version of Shipping Express are governed.

Figure 5 is an example of an early level within Shipping Express. Within this level the player must tap one of the numbers located at the bottom of the screen in order to generate the number of boxes to fill a trick. For example if the
player wanted to fill the truck waiting for nine boxes they could either tap the number 9, which would generate a 9 x 1 array of boxes and place them in the truck or click the number 3 three times to generate a 3 x 3 array of boxes.

Figure 5: Multiplication Diagram for Shipping Express

Figure 6 shows the screen layout for the levels that require players to apply the associative property. In these levels the players still use the numbers located on the bottom of the screen, but they are used to input three numbers that when multiplied together will generate the number of boxes needed to fill a particular truck. For example if a player wanted to fill the truck waiting for 24 four boxes they could press 3, 2, then 4. This would generate a rectangular array containing 24 boxes. They can enter these values in any order that they choose to in order to generate the correct number of boxes.
Finally, Figure 7 shows how the layout for the levels within Shipping Express in which the distributive property must be applied. This level’s game mechanic is similar to the game mechanic for the levels focused on the associative property. The only difference being that instead of multiplying three numbers to generate the correct number of boxes the players must first add two number and then multiply the sum by another number. For example in order to fill a truck with 56 boxes, one possible solution could be to tap 3 and then 5. Moving from left to right this would fill the first two spots in the expression. Then they would tap the 7 filling the last space in the expression. The sum of 3 and 5, which is 8, multiplied by 7 would generate an 8 x 7 array of boxes to be placed in the truck waiting for 56 boxes.
Each property is introduced as participant’s progress through levels in the game. The associative property is introduced in level five and the distributive property is introduced within level eight. There are ten total levels within the game.

In the exogenous version of Shipping Express, the boxes on the dock floor have the multiplication pairs displayed on them, which corresponds to the number of boxes that a particular truck needs in order to leave the dock. For example in the exogenous version of Shipping Express, if a truck needs 16 boxes to leave the facility, the player looks for boxes that have either 4 x 4, or 8 x 2 displayed on the top of the boxes and then drags and drops that box into the truck. This game mechanic has no connection to multiplication or the associative and distributive properties. The core game mechanic in the exogenous version of Shipping Express could easily be substituted with other content areas, for example matching Spanish and English words, where the truck displayed a word in
Spanish. The goal of such a game would be to find the stack of boxes that were labeled with the equivalent English word and place that stack of boxes within the truck.

*Figure 8: Exogenous Shipping Express Gameplay Screenshot*

At the beginning of every level of the exogenous version of Shipping Express, the participants were prompted with short descriptions of the associative and distributive properties, which they were asked to read. Participants were instructed on how to access these prompts during game play and encouraged to access them whenever they felt the need.
Figure 9: Exogenous Shipping Express Hints Screenshot

The control version of Shipping Express is exactly like the exogenous version of Shipping Express with the exception being the lack of elements necessary to constitute it as a game. The control version of Shipping Express gives none of the audio feedback on correctness or incorrectness of answer present in the exogenous and endogenous versions of the game. Unlike the exogenous version of the game, participants seeing the control version of Shipping Express were not provided with prompts related to the associative and distributive properties prior to and during gameplay.

The tables below show the differences between the three versions of Shipping Express in regards to gameplay and how the endogenous, exogenous, and control versions of the game handle integration of mathematical skills and concepts.
Table 2

Differences in mathematical integration within Shipping Express

<table>
<thead>
<tr>
<th>Feature</th>
<th>Endogenous</th>
<th>Exogenous</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Multiplication (Levels 2-4)</td>
<td>Players tap numbers at the bottom of the screen to generate the boxes needed</td>
<td>Players match fact pairs to the values on the truck</td>
<td>Same as Exogenous</td>
</tr>
<tr>
<td>Associative Property (Levels 5-7)</td>
<td>Players fill in an expression in order to generate the boxes needed</td>
<td>Players receive pre-level instruction on associative property</td>
<td>None</td>
</tr>
<tr>
<td>Distributive Property (Levels 8-10)</td>
<td>Players fill in an expression in order to generate the boxes needed</td>
<td>Players receive pre-level instruction on associative property</td>
<td>None</td>
</tr>
</tbody>
</table>

Table 3

Differences in game mechanics

<table>
<thead>
<tr>
<th>Feature</th>
<th>Endogenous</th>
<th>Exogenous</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Game mechanic introduction</td>
<td>Players are given instruction on each game mechanic prior to game play.</td>
<td>Players are given instruction on the game mechanic prior to game play.</td>
<td>Players are given instruction on the game mechanic prior to game play.</td>
</tr>
<tr>
<td>Feedback on Correct Answers</td>
<td>For each correct answer players receive bonus level time and a bell rings.</td>
<td>For each correct answer players receive bonus level time and a bell rings.</td>
<td>For each correct answer players does not receive bonus level time, a bell doesn’t ring, but they receive a new truck.</td>
</tr>
<tr>
<td>Feedback on Incorrect answers</td>
<td>For each incorrect answer, players lose a chance and the truck blows it’s horn.</td>
<td>For each incorrect answer, players lose a chance and the truck blows it’s horn.</td>
<td>For each incorrect answer, does not lose a chance, the truck does not blow its horn, and a new truck is not received.</td>
</tr>
</tbody>
</table>
MEASURES

Three measures were administered to assess the participants within this study.

2. Conceptual Understanding of Distributive Property of Multiplication Measure.
3. Gameplay Survey

Participants were measured on their conceptual understanding of the associative and distributive properties. Through extensive research and consultation with experts within the field of mathematics education, it was concluded that there was no pre-existing, reliable and valid measure for conceptual understanding of multiplication the associative and distributive property of multiplication. This is due impart to the paucity of research on children’s understanding of the aforementioned properties (Geary, et. al, 2008). In order to address this measurement and assessment gap, this study serves as an initial step in the construction of a measure conceptual understanding of the associative and distributive properties of multiplication. To that end participants were asked to answer questions designed to identify their level of conceptual understanding of the associative and distributive properties of multiplication.
Below are examples of three types of questions from the conceptual measure of the associative property (the full measure can be found in appendix A & B):

*Solve each problem below. Show your work in the space provided below and explain how you arrived at your answer.*

a. $6 \times (1 \times 4) = \underline{\hphantom{000}}$

b. $8 \times (4 \times 2) = \underline{\hphantom{000}}$

Fill in the blanks on the following equations to make them true.

a. If $18 = (3 \times 3) \times 2$ then $24 = (\underline{\hphantom{00}} \times \underline{\hphantom{00}}) \times \underline{\hphantom{00}}$

b. If $25 = 5 \times (5 \times 1)$ then $20 = \underline{\hphantom{00}} \times (\underline{\hphantom{00}} \times \underline{\hphantom{00}})$

Circle all of the equations that you think are true. Explain why you think the equations you circle are true.

1. $3 \times (5 \times 7) = (3 \times 5) \times 7$
2. $3 + (5 + 7) = (3 + 5) + 7$
3. $3 - (5 - 7) = (3 - 5) - 7$
4. $3 \div (5 \div 7) = (3 \div 5) \div 7$

Below are sample examples of the three types of questions from the conceptual measure for the distributive property (the full measure can be found in appendix A & B):

*Solve each problem. Show your work in the space provided below and explain how you arrived at your answer.*

a. $8 \times (5 + 4) = \underline{\hphantom{0000}}$

b. $2 \times (3 + 5) = \underline{\hphantom{0000}}$
Fill in the blanks on the following equations to make them true.

a. If 20 = (2 + 3) x 4, then 12 = (___ + ___) x ___

b. If 27 = (8 + 1) x 3, then 45 = (___ + ___) x ___

Circle all of the equations that you think are true. Explain why you think the equations you circle are true.

1. \((4 + 2) \times 7 = (4 \times 7) + (2 \times 7)\)

2. \((4 + 2) - 7 = (4 - 7) + (2 - 7)\)

3. \((4 + 2) \div 7 = (4 \div 7) + (2 \div 7)\)

In terms of the associative property, the intent of the first set of problems is to ascertain if students are able to solve numerical expressions of this property and how they arrive at their answers. For the second set of problems the intent is to determine if learners can apply their understanding of the associative property in the decomposition of a value to the product of three multiplicands, whose product is equivalent to the given value. The third set of items are designed to see if students understand that within the operations of addition and multiplication, the grouping of values within an expression has no bearing on the resulting product or sum, and that the two expressions are in fact equivalent.

The goals for the test items related to the distributive property are similar to those used for the associative property. The initial set of problems were designed to determine if students could evaluate or simplify distributive problems in the given form and make explicit the steps that they toke to reach that value. The second set of the problems were designed to determine is learners apply their understanding of the distributive property in the decomposition of a value to the
sum of two products equivalent to the given value. As in the third set of items within the associative property measure, the third set of items in the distributive property measure were designed to see if students understand the application of the distributive property in terms of equivalent expressions.

In both instances the inference is being made that learner performance on each set of items indicates some level of conceptual understanding of the aforementioned properties.

For the associative property measure, a total of 8 items were administered. Participants were asked to answer four questions related to the first question type, three questions related to the second question type, and one question related to the third question type. Each problem set is worth four points for a maximum total of 12 points. Items within the associative property measure were scored using the rubric below and the guidelines found in Appendix F:
Table 4

Associative Property Measure Scale

<table>
<thead>
<tr>
<th></th>
<th>Computational</th>
<th>Concrete (Decomposition)</th>
<th>Principled (Equivalence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Demonstrates effective consistency in accurately solving numerical equations.</td>
<td>Demonstrates consistency in accurately decomposing values using the associative property.</td>
<td>Accurately identifies demonstrate associative equivalence.</td>
</tr>
<tr>
<td>3</td>
<td>Demonstrates adequate consistency in solving numerical equations.</td>
<td>Demonstrates adequate consistency in decomposing values using the associative property.</td>
<td>Adequately identifies associative equivalence.</td>
</tr>
<tr>
<td>2</td>
<td>Demonstrates some consistency in solving numerical equations.</td>
<td>Demonstrates some consistency in decomposing values using the associative property.</td>
<td>Demonstrates some consistency in identifying associative equivalence.</td>
</tr>
<tr>
<td>1</td>
<td>Demonstrates difficulty in accurately solving numerical equations.</td>
<td>Demonstrates difficulty in decomposing values using the associative property.</td>
<td>Demonstrates difficulty in identifying associative equivalence.</td>
</tr>
</tbody>
</table>

For the distributive property measure, a total of 8 items were administered. Participants were asked to answer four questions related to the first question type, three questions related to the second question type, and one question related to the third question type. Each problem set is worth four points for a maximum total of 12 points. Items within the distributive property measure were scored using the rubric below and the guidelines found in Appendix F.
### Table 5

**Distributive Property Measure Scale**

<table>
<thead>
<tr>
<th>Computational</th>
<th>Concrete (Decomposition)</th>
<th>Principled (Equivalence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Demonstrates effective consistency in accurately solving numerical equations.</td>
<td>Demonstrates consistency in accurately decomposing values using the distributive property.</td>
<td>Accurately identifies distributive equivalence.</td>
</tr>
<tr>
<td>3 Demonstrates adequate consistency in solving numerical equations.</td>
<td>Demonstrates adequate consistency in decomposing values using the distributive property.</td>
<td>Adequately demonstrate distributive equivalence.</td>
</tr>
<tr>
<td>2 Demonstrates some consistency in solving numerical equations.</td>
<td>Demonstrates some consistency in decomposing values using the distributive property.</td>
<td>Demonstrates some consistency in identifying distributive equivalence.</td>
</tr>
<tr>
<td>1 Demonstrates difficulty in accurately solving numerical equations.</td>
<td>Demonstrates difficulty in decomposing values using the distributive property.</td>
<td>Demonstrates difficulty in identifying distributive equivalence.</td>
</tr>
</tbody>
</table>

In both instances the inference is being made that learner performance on each set of items indicate some level of conceptual understanding of the aforementioned properties. Learners were assessed on accuracy, and on the processes and reasoning used to arrive at their answers.

**Gameplay Survey.** Survey data were collected from participants in order to better understand their thoughts and feelings about game play and instructional material. The survey asked participants to respond in agreement or disagreement to statements related to the instructional content of the game, game play, and future usage. All responses were given a score between 1 and 5, with 1 representing Strongly Disagree and 5 representing Strongly Agree (the full
measure can be found in Appendix C). Below are several sample items from the gameplay survey:

- This game challenged me to remember my multiplication facts.
- After playing the game I feel that I remember my multiplication facts better than before I played the game.
- If I could I would continue to play this game on my own.

PROCEDURES

This study was conducted within a computer lab setting at a public school. Prior to participating in the study, participants returned a consent form signed by their parent or legal guardian. Participants were briefed on the purpose of the study, what their participation entailed and their rights as a participant. After this briefing, participants were asked to sign an additional individual consent form. Participants who successfully completed the consent section of the study were assigned a pseudonym ID and then assessed on their conceptual understanding of the associative and distributive properties using the measure found in Appendix A. After the collection of the measures, each participant was randomly assigned to one of three treatments: Endogenous, Exogenous, and Control. Participants used their ID to log into their version of Shipping Express in order to track their interaction within the environment. Each group was briefed on the rules of the game and provided instruction on game controls. Participants then spent fifty minutes playing their assigned version of the game. At the end of that time participants were reassessed using the associative and distributive property measures, which can be found in Appendix B. This measure contained items that
are isomorphic to the items used within the measures taken prior to game play. Additionally a game play survey was administered to provide information on usability and highlight possible areas of improvement. Figure 11 shows the sequence of tasks within this study.

Pre-test ➔ Game Demo ➔ Gameplay Sessions ➔ Post-test ➔ Gameplay Survey

*Figure 10: Study Sequence of Tasks*
RESULTS

RELIABILITY AND VALIDITY OF MEASURES

Cronbach’s alpha was used to determine the internal consistency estimate for the reliability of the measures of conceptual understanding with the target audience. For the measure of conceptual understanding of the associative property of multiplication, the reliability of the forms of the scale is .76, indicating acceptable reliability. For the measure of conceptual understanding of the distributive property of multiplication, the reliability of the forms of the scale is .81, indicating good reliability.

Two internal consistency estimates were computed for the gameplay survey: a split-half coefficient expressed as a Spearman-Brown corrected correlation and coefficient alpha. For the split-half, the scale was split into two halves such that the two halves could be as equivalent as possible. In splitting the items, the sequencing of the items were taken into account as well as whether items assessed thoughts on instructional content in the game or thoughts about the aesthetics of the game. The first half included items, 1, 2, 4, 5, and 6, while the other half included 3, 7, 8, 9, and 10. The values for coefficient alpha and the split-half coefficient alpha were the same, .71, each indicating acceptable reliability.

CONCEPTUAL UNDERSTANDING OF ASSOCIATIVE PROPERTY

A one-way analysis of variance (ANOVA) was conducted to determine if participants’ performance on the pretest of conceptual understanding of the
associative property differed across conditions. The results showed that there were no significant differences between the endogenous, exogenous, and control conditions on this measure, $F(2, 101) = 2.58, p = .08$.

An ANOVA was conducted to evaluate the relationship between playing Shipping Express and the change in scores on the conceptual measure of the associative property of multiplication. The independent variable, the version of Shipping Express, included three levels: endogenous, exogenous, and control. The dependent variable was the change in scores on the conceptual measure of the associative property of multiplication taken before playing Shipping Express and after. The ANOVA was significant $F(2, 100) = 3.34, p < .05$. The strength of the relationship between the version of Shipping Express and the change in scores on the conceptual measure of the associative property of multiplication, as assessed by $\eta^2$, was moderate, with the game version factor accounting for 6% of the variance of the dependent variable. Figure 11 displays the changes in scores on the measure of conceptual understanding of the associative property based on the version of Shipping Express played.
Follow-up tests were conducted to evaluate pairwise differences among the means. Because the variance among the three groups ranged from 5.43 to 16.56, I chose not to assume the variances were homogenous and conducted post hoc comparisons with the use of the Dunnett’s C test, a test that does not assume equal variances among the three groups. There was no significant difference between the group that played the exogenous version of Shipping Express and the group that played the control version of Shipping Express, and no significant difference found between the groups who played the endogenous and control versions of Shipping Express. The group that played the endogenous version of Shipping Express showed a greater increase in scores on the conceptual understanding of associative property of multiplication in comparison to the
exogenous group. The 95% confidence intervals for the pairwise differences, as well as the means and standard deviations for the three versions of Shipping Express are reported in Table 6.

Table 6

95% Confidence Intervals of Pairwise Differences in Mean Changes in Scores on Measure of Conceptual Understanding of the Associative Property of Multiplication

<table>
<thead>
<tr>
<th>Game Version</th>
<th>M</th>
<th>SD</th>
<th>Endogenous</th>
<th>Exogenous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous</td>
<td>.87</td>
<td>2.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exogenous</td>
<td>-1.05</td>
<td>4.07</td>
<td>-0.02 to 3.86</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>.33</td>
<td>2.72</td>
<td>-2.06 to .99</td>
<td>-3.86 to .02</td>
</tr>
</tbody>
</table>

Table 7 contains the number of potential trials that participants where the associative property was required to be used to complete a level within the endogenous level of Shipping Express.

Table 7

Number of Trials for Associative Property per Level

<table>
<thead>
<tr>
<th>Game Version</th>
<th>Number of Trials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 5</td>
<td>10</td>
</tr>
<tr>
<td>Level 6</td>
<td>12</td>
</tr>
<tr>
<td>Level 7</td>
<td>15</td>
</tr>
</tbody>
</table>

CONCEPTUAL UNDERSTANDING OF DISTRIBUTIVE PROPERTY

A one-way analysis of variance (ANOVA) was conducted to determine if participants’ performance on the pretest of conceptual understanding of the distributive property differed across conditions. The results showed that there
were significant differences between the endogenous, exogenous, and control conditions on this measure, $F(2, 104) = 4.11, p < .05$.

A one-way analysis of covariance (ANCOVA) was conducted. The independent variable, version of Shipping Express played, included three levels: endogenous, exogenous, and control. The dependent variable was the gain scores on the measure of conceptual understanding of the distributive property of multiplication and the covariate was the total score on the pretest. A preliminary analysis evaluating the homogeneity-of-slopes assumption indicated that the relationship between the covariate and the dependent variable did not differ significantly as a function of the independent variable, $F(2, 89) = 2.12, MSE = 4.32, p = .13$, partial $\eta^2 = .05$. The ANCOVA was not significant, $F(2, 88) = 2.83, MSE = 4.23, p = .06$. The strength of the relationship between the version of Shipping Express played and the dependent variable moderate, as assessed by partial $\eta^2$, with the version of Shipping Express accounting for 6\% of the variance of the dependent variable, holding constant the scores on the pretest of conceptual understanding of the distributive property of multiplication. Table 8 contains the means and standard deviations for changes in score on conceptual understanding of the distributive property of multiplication. Figure 12 displays the changes in scores on the measure of conceptual understanding of the distributive property based on the version of Shipping Express played.
Figure 12. Changes in scores on conceptual understanding of the distributive property measure based on version of Shipping Express played.

Table 8
**Means and Standard Deviations for Changes in Score on Conceptual Understanding of the Distributive Property of Multiplication**

<table>
<thead>
<tr>
<th>Game Version</th>
<th>M</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous</td>
<td>.11</td>
<td>1.57</td>
</tr>
<tr>
<td>Exogenous</td>
<td>-.75</td>
<td>2.40</td>
</tr>
<tr>
<td>Control</td>
<td>-.94</td>
<td>2.20</td>
</tr>
</tbody>
</table>

An ANOVA was conducted to evaluate the relationship between participants who reached level 8 in all versions of the game (level 8 is the level when the distributive property is introduced within the game) and the gain scores on the conceptual measure of the distributive property of multiplication. The
ANOVA was not significant, $F(2, 42) = 1.09, p = .35$. The strength of the relationship between the game version conditions, and the gain scores on the conceptual measure of the distributive property as assessed by $\eta^2$ was moderate, with version of Shipping Express played accounting for 5% of the variance of the dependent variable. Table 9 contains the means and standard deviations for changes in score on conceptual understanding of the distributive property of multiplication for those who reached level 8.

Table 9

*Means and Standard Deviations for Changes in Score on Conceptual Understanding of the Distributive Property of Multiplication (Level 8)*

<table>
<thead>
<tr>
<th>Game Version</th>
<th>$M$</th>
<th>$SD$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous</td>
<td>.09</td>
<td>1.57</td>
<td>22</td>
</tr>
<tr>
<td>Exogenous</td>
<td>.25</td>
<td>2.40</td>
<td>12</td>
</tr>
<tr>
<td>Control</td>
<td>-.73</td>
<td>2.20</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 10 contains the number of potential trials that participants where the distributive property was required to be used to complete a level within the endogenous level of Shipping Express.

Table 10

*Number of Trials for Associative Property in Endogenous Condition*

<table>
<thead>
<tr>
<th>Game Version</th>
<th>Number of Trials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 8</td>
<td>10</td>
</tr>
<tr>
<td>Level 9</td>
<td>12</td>
</tr>
<tr>
<td>Level 10</td>
<td>15</td>
</tr>
</tbody>
</table>
PERFORMANCE WITHIN THE GAME ENVIRONMENT

A linear regression analysis was conducted to evaluate the prediction of how well participants would perform within the endogenous game environment based on their performance on the pretest of conceptual understanding of the associative property. The regression equation for predicting the performance in the game environment is

\[ \text{Level Reached in Game} = 0.244 \times \text{Assoc. Pretest Score} + 6.907. \]

The 95% confidence interval for the slope, -0.255 to 0.713, contains the value zero, and therefore the associative property pretest score was not significantly related to performance within the game environment. The hypothesis that those who score higher on the associative property pretest would reach a higher level within the endogenous version of Shipping Express was not validated. A post-hoc power analysis was conducted for this measure. Based on an observed \( R^2 \) of 0.05, a sample size of 38, with one predictor, the observed statistical power was 0.28.

A linear regression analysis was conducted to evaluate the prediction of how well participants would perform within the endogenous game environment based on their performance on the pretest of conceptual understanding of the distributive property. The regression equation for predicting the highest level reached within the game is

\[ \text{Level Reached in Game} = 0.719 \times \text{Distributive Pretest Score} + 1.792 \]

The 95% confidence interval for the slope, 0.07 to 1.37, does not contain the value of zero, and therefore the highest level reached in the game is significantly related to scores on the pretest of conceptual understanding of the distributive property.
This means that those who score well on the distributive property pretest will reach a higher level within the endogenous version of Shipping Express.

Accuracy in predicting the highest level reached in the game was moderate. The correlation between the highest level reached in the game and score on the distributive property pretest was .39. Approximately 15% of the variance of the highest level reached in the game variable was accounted for by its relationship with scores on the distributive property pretest.

Post hoc analysis was conducted to see if a relationship existed between changes in scores on each of the question types within the conceptual measure of the associative property of multiplication and the version of Shipping Express that was played. The independent variable, the version of Shipping Express, included three levels: endogenous, exogenous, and control. The dependent variable was the change in scores on conceptual measure of the associative property of multiplication taken before playing Shipping Express and after based on question type.

A one-way multivariate analysis of variance (ANOVA) was conducted to determine the effect of three versions of Shipping Express (endogenous, exogenous, control) on the three dependent variables, changes in scores on question type one, two, and three from the conceptual understanding of the associative property of multiplication measure. No significant differences were found among the three versions of Shipping Express on the dependent measures. Table 11 contains the means and standard deviations on the dependent variables for the three groups.
Table 11
Means and Standard Deviations for Changes in Score on Question Types Within the Conceptual Understanding of the Associative Property of Multiplication

<table>
<thead>
<tr>
<th>Game Version</th>
<th>Question Type 1</th>
<th>Question Type 2</th>
<th>Question Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous</td>
<td>0.45</td>
<td>1.48</td>
<td>-0.15</td>
</tr>
<tr>
<td>Exogenous</td>
<td>0.00</td>
<td>1.73</td>
<td>-0.68</td>
</tr>
<tr>
<td>Control</td>
<td>0.25</td>
<td>1.56</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

A one-way multivariate analysis of variance (ANOVA) was conducted to determine the effect of three versions of Shipping Express (endogenous, exogenous, control) on the three dependent variables, changes in scores on question type one, two, and three from the conceptual understanding of the distributive property of multiplication measure. No significant differences were found among the three versions of Shipping Express on the dependent measures. Table 12 contains the means and standard deviations on the dependent variables for the three groups.

Table 12
Means and Standard Deviations for Changes in Score on Each Question Type Within the Conceptual Understanding of the Distributive Property of Multiplication

<table>
<thead>
<tr>
<th>Game Version</th>
<th>Question Type 1</th>
<th>Question Type 2</th>
<th>Question Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous</td>
<td>0.07</td>
<td>0.84</td>
<td>-0.24</td>
</tr>
<tr>
<td>Exogenous</td>
<td>0.15</td>
<td>1.18</td>
<td>0.00</td>
</tr>
<tr>
<td>Control</td>
<td>-0.14</td>
<td>1.53</td>
<td>-0.36</td>
</tr>
</tbody>
</table>
SURVEY DATA

The table below contains the means per question per condition of the responses to the survey questions. The questions asked on the survey can be found in Appendix C. The charts showing the frequency distribution for all questions based on version of game played can be found in Appendix D.

Table 13

Mean Item Response of Survey Questions.

<table>
<thead>
<tr>
<th>Game Version</th>
<th>Endogenous</th>
<th>Exogenous</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>3.38</td>
<td>3.40</td>
<td>3.66</td>
</tr>
<tr>
<td>Question 2</td>
<td>2.41</td>
<td>2.40</td>
<td>2.07</td>
</tr>
<tr>
<td>Question 3</td>
<td>3.56</td>
<td>3.20</td>
<td>3.34</td>
</tr>
<tr>
<td>Question 4</td>
<td>3.59</td>
<td>3.43</td>
<td>3.41</td>
</tr>
<tr>
<td>Question 5</td>
<td>3.56</td>
<td>3.13</td>
<td>3.24</td>
</tr>
<tr>
<td>Question 6</td>
<td>3.44</td>
<td>3.17</td>
<td>3.17</td>
</tr>
<tr>
<td>Question 7</td>
<td>3.72</td>
<td>3.57</td>
<td>3.66</td>
</tr>
<tr>
<td>Question 8</td>
<td>3.28</td>
<td>3.37</td>
<td>2.97</td>
</tr>
<tr>
<td>Question 9</td>
<td>2.91</td>
<td>2.93</td>
<td>2.72</td>
</tr>
<tr>
<td>Question 10</td>
<td>2.69</td>
<td>3.17</td>
<td>2.38</td>
</tr>
</tbody>
</table>

A one-way ANOVA was conducted to evaluate the relationship between participant’s responses to the survey questions and the version of the game that they played. The ANOVA was only significant for question 10 (If I could, I would continue to play this game on my own), $F(2, 92) = 4.19, p < .05$. The strength of the relationship between the version of the game played and responses to question 10 of the survey as assessed by $\eta^2$, was moderate, with the game version factor accounting for 8% of the variance of the dependent variable.

Follow-up tests were conducted to evaluate pairwise differences among the means. Because the variance between the groups ranged from 1.38 to 1.98, we
chose not to assume that the variances were homogenous and conducted post hoc comparisons with the use of Dunnett’s C test, a test that does not assume equal variances among the three groups. There was no significant difference in the means between the three groups. The 95% confidence intervals for the pairwise differences, as well as the means and standard deviations for the gameplay groups are reported in Table 14.

Table 14

<table>
<thead>
<tr>
<th>Game Version</th>
<th>M</th>
<th>SD</th>
<th>Endogenous</th>
<th>Exogenous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous</td>
<td>2.69</td>
<td>1.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exogenous</td>
<td>3.23</td>
<td>1.41</td>
<td>-1.34 to .27</td>
<td></td>
</tr>
<tr>
<td>Control</td>
<td>2.37</td>
<td>.91</td>
<td>-33 to .96</td>
<td>.11 to 1.59</td>
</tr>
</tbody>
</table>

Full responses to questions about ways to improve the each version of Shipping Express can be found in Appendix E.
DISCUSSION

DISCUSSION OF THE MAIN PURPOSE

The purpose of this study was to evaluate the impact of applying an endogenous approach to the design of an instructional digital game. In order to investigate the impact of this approach to game design, three hypotheses were proposed. The first hypothesis was that a digital game for learning where the instructional content was endogenous to the game mechanics, would have an impact on conceptual understanding of the associative property of multiplication, compared to a similar game in which the instructional content was exogenously related to game mechanics. The second hypothesis was similar to the first hypothesis, with the main difference being a focus on an impact on conceptual understanding of the distributive property of multiplication. The third hypothesis sought to determine if one could predict how well a participant would perform within Shipping Express based on their performance on measures collected prior to game play.

The study had three significant findings: 1) there was a significant difference between participants’ gains on the measure of conceptual understanding of the associative property of multiplication, based on the version of Shipping Express they played. Participants who played the endogenous version of Shipping Express had on average greater gains in scores on the measure of conceptual understanding of the associative property of multiplication than those
who played the other versions of Shipping Express; 2) performance on the measures of conceptual understanding of the distributive property collected prior to game play were positively related to performance within the endogenous game environment; and 3) participants who played the control version of Shipping Express were on average more likely to have a negative attitude towards continuing game play on their own compared to the other versions of the game.

No significant differences were found in regards to changes in scores on the measure of conceptual understanding of the distributive property based on the version of Shipping Express played, post hoc pairwise comparisons, and changes on scores on question types within the conceptual understanding of the associative and distributive property of multiplication measures. Within this chapter implications of these findings for the field of game-based learning, and future directions will be discussed.

**Can endogenous games promote conceptual understanding?** The first significant finding of the study was that the application of an approach to game design, in which the instructional content is intrinsically integrated within the game mechanics, enhances conceptual understanding of the associative property of multiplication compared to having instructional content extrinsic to game mechanics, but not when compared to the control condition. On average the exogenous version of Shipping Express actually resulted in poorer performance on the measure of conceptual understanding of the associative property of multiplication collected after game play in comparison to the endogenous and control conditions. In other words the exogenous game design hurt conceptual
understanding of the associative property relative the endogenous and control condition. It seems that by interrupting game play at the start of each level to provide instruction on the associative and distributive properties of multiplication, learners were discouraged from sharpening their conceptual understanding in relation to procedural fluency, decomposition, and principled knowledge. Another possible explanation for the effect that the exogenous game had on conceptual understanding could be that the game mechanic did not allow learners to immediately make connections between the text-based instruction they received at the start of each level and the items on the post measures. Those who played the endogenous version of Shipping Express had to directly apply their knowledge of the associative and distributive properties in order to be successful in the game, while those in the exogenous version were not required to. This lack of interaction with the aforementioned properties could have resulted in decreased motivation and the inability to enter a state of flow. Habgood & Ainsworth (2011) attribute learning gains from intrinsic integration to increased motivation and the encouragement of a flow state within learners. This flow state has the potential to support changes in conceptual understanding by promoting “persistence, more focused attention, increased arousal, increased affect, and alternative strategies” (Habgood & Ainsworth, 2011, p. 30).

Another possible contributor to the significant findings related to performance on the measure of conceptual understanding of the associative property could be the use of external representations within the game environment which were congruent to the concept of interest. The endogenous version of
Shipping Express had learners build a rectangular array of boxes to be shipped by applying the associative property of multiplication. This is consistent with the findings of Segal (2011) who found that congruency between digital representations within game environments assists learners in the development of more robust mental representations. In other words, the learners who played the endogenous version of Shipping Express had a strong metaphorical representation of the associative property of multiplication to use in the solving of problems than those who played the other versions of the game.

There were no significant differences found in the change in scores on the measure of conceptual understanding of the distributive property of multiplication based on the version of Shipping Express participants played. One possible explanation for this could be that on average the highest level reached by those in the endogenous was level 8. Level 8 is the first level in which they are introduced to the distributive property game mechanic. In contrast the associative property game mechanic is introduced in level 5 and continues for two more levels. This means that on average those in the endogenous solved more problems related to the associative property than the distributive property.

Another possible contributor to the non-significant change in scores on the distributive property measure was the difficulty that students had in mentally transitioning from applying the associative property to applying the distributive property to solve problems within the game. A large number of students complained the game was indicating that their solutions were incorrect once they got to level eight. Upon further investigation it was found that they were still
applying the associative property and looking at the problems incorrectly. In other words the students were still trying to multiply three numbers together to create their boxes, instead of multiplying a value by the sum of two numbers. For example a student trying to fill a truck with 24 boxes would mistakenly tap 3, 2, 4 thinking they were multiplying. In fact they were generating 20 boxes instead of 24. Students were reminded of the change in game mechanic and seemed not to have any problem with solving problems, but time constraints prohibited them from completing subsequent levels that involved the distributive property.

When conducting research on games for learning within formal school settings, researchers should be conscious of the fixed time constraints that will be placed on them. Due to the increased emphasis on high stakes standardized testing, schools administrators and teachers are leery of sacrificing instructional time in order to participate in a research study. If given access to a K-12 population, researchers should be prepared to maximize the fixed amount of time in which they are allotted to collect data and implement interventions. In the case of those conducting research within the games for learning community, this means designing a game environment in which learners are able to sufficiently interact with the instructional content within the provided time constraints. If not a situation may arise, as in this study, in which the learners are not afforded enough time to fully engage with the concept(s), procedure(s), processes, or construct(s) of interest.

**Does performance on measures of conceptual understanding predict game performance?** The second significant finding of the study was that
performance on the measure of conceptual understanding of the distributive property taken prior to game play was strongly correlated to performance in the endogenous version of the game. This result should be interpreted cautiously. The implication for this finding is promising as it speaks to the use of digital game environments as a tool of assessment. Theorists within the field of games for learning community (Bowman, 1982, Squire, 2003, Nelson et al, 2011, Shute, 2011) have held the belief that the affordances of digital games lend them to be leveraged as tools for assessment. The general agreement between these theorists is that games allow us to measure things which are difficult to assess such as teamwork, critical thinking, adaptive reasoning, systems-thinking, and conceptual understanding, more authentically and less obtrusively based on the actions that a players takes within a particular game environment (Shute, 2011). One important step in taking this idea of games as an assessment tool from theory into application is empirically showing that a game environment is able to perform measurements as well as a valid and reliable paper or computer based measure.

While the assessment used to measure conceptual understanding of the associative and distributive properties of multiplication in this study were used based on face validity, the fact that the distributive property measure predicted the highest level participants would achieve in the endogenous version of the game holds promise. More promising would have been if data analysis found a significant correlation between participant’s performance on the posttest and the highest level they achieved in the game as well.
FINDINGS FROM SURVEY

Does the design of the game have any bearing on participant’s motivation? There is no more important factor that can inhibit or enhance learning than motivation (Gee, 2003). With this in mind, the fact that the majority of mean response related to the survey questions were neutral and showed no significant difference between participants (with the exception of question 10) based on the version of the game they played was troubling.

These findings are in conflict with Habgood & Ainsworth (2011) who found that when given a choice between playing an intrinsic (endogenous) or extrinsic (exogenous) version of the game he designed to teach division, the participants played the intrinsic version of his game on average seven times longer than the extrinsic version. While Habgood and Ainsworth allowed participants to play both versions of the game and this study only allowed participants to play one version of the game, it was anticipated that the integration of instructional content within game mechanics would result in learners being more interested in continuing their game play experience after the study concluded. It appears that the endogenous version of Shipping Express was not more motivating to learners than the other versions.

For those involved in designing games for learning, there should be a constant focus on ensuring the instructional material is being presented in a motivating manner. If not it will be difficult to acquire consistent learning gains from the use of a digital game. In other words those “games that can’t be learned, or where the learning is not motivating, don’t get played” (Gee, 2008, p. 12). Care
should be taken to design games that will leverage the internal motivators of the audience for which the game is being designed.

Schell (2008) encourages game designers to build games through the lens of the player. For example Schell points out that males and females differ in the things that they look for in a game:

<table>
<thead>
<tr>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mastery</td>
<td>Emotion</td>
</tr>
<tr>
<td>Competition</td>
<td>Real World</td>
</tr>
<tr>
<td>Destruction</td>
<td>Nurturing</td>
</tr>
<tr>
<td>Spatial Puzzles</td>
<td>Dialog and Verbal Puzzles</td>
</tr>
<tr>
<td>Trial and Error</td>
<td>Learning by Example</td>
</tr>
</tbody>
</table>

While these two lists are neither exhaustive nor inclusive, they do point to differences between genders in terms of the game design elements they find motivating. One could also make the case for differences between potential game audiences based on other demographic factors such as age, ethnicity, income, etc.

Trying to design an instructional game that accounts for all of these external factors would be difficult.

A more inclusive approach to designing motivating game environments has been proposed by Hunicke, LeBlanc & Zubek (2004). The authors have developed the MDA framework (Mechanics, Dynamics, and Aesthetics) to serve as a lens from which game designers can view their games. The Aesthetics portion of the MDA framework is comprised of a taxonomy of elements which the authors believe make a game motivating or “fun”; sensation, fantasy,
narrative, challenge, fellowship, discovery, expression, and submission. Game designers of instructional games can use this MDA framework along with the recommendations made by Schell (2008), Habgood & Ainsworth (2011), and Koster (2005) to make sure they fine tune their games to address the areas of in which they are aesthetically flawed. Considerable time should be allotted within the game design schedule to play testing with the population the game is being designed for, in order to bring to the surface any aesthetic flaws. In addition multiple play testing sessions should be employed in order for game designers to measure their progress in addressing aesthetic flaws.

STUDY LIMITATIONS

This study focused specifically on mathematical proficiency in regards to multiplication. Complete proficiency in multiplication lies beyond the scope of this project, so this study focused on the conceptual understanding of the associative and distributive properties of multiplication. In addition this study focused on the strengths and limitations of exogenous and endogenous instructional games to aid in the achievement of the aforementioned goals.

In addition to the limitations of the study in terms of scope, there were additional weaknesses:

• Only 50-55 minutes of game play
• Endogenous version of the game was not seen as more motivating than the other versions.
• Study took place at the end of the school year.
• Lack of pre-existing valid and reliable measures
FUTURE DIRECTIONS

There is a significant amount of research that still needs to be conducted on the design of endogenous games for instructional purposes. In relation to Shipping Express, considerable time should be spent redesigning the game in order to ensure that it is more motivating to learners, while continuing to promote conceptual understanding of the associative and distributive properties of multiplication. Additionally a decision needs to be made whether to intrinsically integrate the associative and distributive properties of multiplication within the same game or in separate games. This would help to ensure that learners have ample time to interact with both properties.

Further works needs to be done on determining the best interface for implementing Shipping Express. Segal (2011) work on the use of touch screen devices as a means of Gestural Conceptual Mapping (GCM) in the instruction number line estimation and basic addition shows promise as a more robust interface for the intrinsic integration of instructional content within Shipping Express. Segal found that learners who played a game designed to teach number estimation and basic addition through the use of GCM on an touch screen device, performed better on post intervention learning outcomes than those who used a point and click mouse. Segal believes that touch screen devices allow for the stronger mental representations because they allow the learner to better embody the instructional content through the use of physical touch.

Finally research should be conducted on the use of games for learning as a tool of assessment. Paper-based test of mathematical competence, for example the
associative and distributive property, are suitable for measuring inert knowledge. Were they fall short is measure whether student’s can apply their knowledge within context. To accomplish this currently, one must make use of expensive assessments and inefficient measures such as clinical interviews. Games on the other hand provide an affordable, practical alternative to assessing students non-inert knowledge based on their situated, contextualized, and/or embodied competence. In the case of Shipping Express this would first require a more definitive determination of the validity and reliability of the paper based measures of conceptual understanding. Once that has been accomplished, learner’s performance within the game environment must be shown to be parallel to their performance on the validated and reliable paper based measure. This is a difficult task but any work done towards the use of digital games as tools for assessment would be a beneficial contribution to the games for learning community.
REFERENCES


of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.


White House, Executive Office of the President, Office of Science and


APPENDIX A

PRETEST
USER ID: _________________________

Write your answer on the line beside each problem. Make sure to show your work.

9 \times (6 + 3) = \underline{\quad} \quad \quad \quad \quad 7 \times (2 + 7) = \underline{\quad}

4 \times (6 + 2) = \underline{\quad} \quad \quad \quad \quad 6 \times (0 + 3) = \underline{\quad}

Fill in the blanks on the following equations to make them true:

If 20 = (2 + 3) \times 4, then 12 = (\underline{\quad} + \underline{\quad}) \times \underline{\quad}

If 27 = (8 + 1) \times 3, then 45 = (\underline{\quad} + \underline{\quad}) \times \underline{\quad}

If 14 = (3 + 4) \times 2, then 21 = (\underline{\quad} + \underline{\quad}) \times \underline{\quad}

Circle all of the equations that you think are true. Explain why you think they are true.

4. \( (4 + 2) \times 7 = (4 \times 7) + (2 \times 7) \)

5. \( (4 + 2) - 7 = (4 - 7) + (2 - 7) \)

6. \( (4 + 2) \div 7 = (4 \div 7) + (2 \div 7) \)
Write your answer on the line beside each problem. Make sure to show your work.

6 \times (1 \times 4) = \underline{\hspace{2cm}} \quad 8 \times (3 \times 6) = \underline{\hspace{2cm}}

8 \times (7 \times 0) = \underline{\hspace{2cm}} \quad 1 \times (2 \times 2) = \underline{\hspace{2cm}}

Fill in the blanks on the following equations to make them true:

If 18 = (3 \times 3) \times 2 then 24 = (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}

If equal 25 is equal to 5 \times (5 \times 1) then 20 = (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}

If equal 30 is equal to 5 \times (2 \times 3) then 42 = (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}) \times \underline{\hspace{1cm}}

Circle all of the equations that you think are true.

Explain why you think they are true.

1. \ 3 \times (5 \times 7) = (3 \times 5) \times 7
2. \ 3 + (5 + 7) = (3 + 5) + 7
3. \ 3 - (5 - 7) = (3 - 5) - 7
4. \ 3 ÷ (5 ÷ 7) = (3 ÷ 5) ÷ 7
APPENDIX B

POSTTEST
Write your answer on the line beside each problem. Make sure to show your work.

7 \times (4 + 2) = ________
5 \times (2 + 5) = ________

3 \times (6 + 2) = ________
4 \times (7 + 3) = ________

Fill in the blanks on the following equations to make them true:

If 20 = (2 + 3) \times 4, then 42 = (____ + _____) \times ____
If 27 = (8 + 1) \times 3, then 54 = (____ + _____) \times ____
If 14 = (3 + 4) \times 2, then 72 = (____ + _____) \times ____

Circle all of the equations that you think are true. Explain why you think they are true.

7. (6 + 3) \times 4 = (6 \times 4) + (3 \times 4)
8. (6 + 3) - 4 = (6 - 4) + (3 - 4)
9. (6 + 3) \div 4 = (6 \div 4) + (3 \div 4)
Write your answer on the line beside each problem. Make sure to show your work.

8 \times (2 \times 3) = \underline{\hspace{2cm}} \quad 7 \times (3 \times 3) = \underline{\hspace{2cm}}

10 \times (2 \times 5) = \underline{\hspace{2cm}} \quad 3 \times (4 \times 2) = \underline{\hspace{2cm}}

Fill in the blanks on the following equations to make them true:

If 18 = (3 \times 3) \times 2 then 63 = (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \times \underline{\hspace{2cm}}

If equal 25 is equal to 5 \times (5 \times 1) then 36 = (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \times \underline{\hspace{2cm}}

If equal 30 is equal to 5 \times (2 \times 3) then 81 = (\underline{\hspace{2cm}} \times \underline{\hspace{2cm}}) \times \underline{\hspace{2cm}}

Circle all of the equations that you think are true.

Explain why you think they are true.

1. $8 \times (3 \times 2) = (8 \times 3) \times 2$
2. $8 + (3 + 2) = (8 + 3) + 2$
3. $8 - (3 - 2) = (8 - 3) - 2$
4. $8 \div (3 \div 2) = (8 \div 3) \div 2$
APPENDIX C

GAMEPLAY SURVEY
ID NUMBER: ________________________________

Gender (Check one)

______ Male

______ Female

Age _______________

Race (Check one)

White _______ White, Non Hispanic _______ African America ______

Hispanic _______ Asian-Pacific Islander _______ Native American_______

Other _______

Please answer the following question based on your time spent playing Shipping Express. Place a check mark within one box for each question.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree or Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
</table>

This game helped me to learn about multiplication.
The multiplication problems were difficult.
I enjoy learning through video games.
This game challenged me to remember my multiplication facts.
After playing the game I feel that I remember my multiplication facts better than before I played the
I would like to use other games in order to help me in math.

Please answer the following question based on your time spent playing Shipping Express. Place a check mark within one box for each question.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree or Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>The game is easy to play.</td>
<td>I would recommend this game to my friends.</td>
<td>The game play felt realistic.</td>
<td>If I could I would continue to play this game on my own.</td>
<td></td>
</tr>
</tbody>
</table>

Did you beat the game?

_______ Yes

_______ No What level did you get to?

If you could improve anything about the game, what would it be?

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________
Question 1: This game helped me to learn about multiplication.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 2: The multiplication problems were difficult.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 3: I enjoy learning through video games.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 4: This game challenged me to remember my multiplication facts.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 5: After playing the game I feel that I remember my multiplication facts better than before I played the game.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 6: I would like to use other games in order to help me in math. (1 = Strongly Disagree, 5 = Strongly Agree)

Histogram for GAMEVESION= Endogenous

Histogram for GAMEVESION= Exogenous

Histogram for GAMEVESION= Control
Question 7: The game is easy to play.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 8: I would recommend this game to my friends. (1 Strongly Disagree, 5 = Strongly Agree)
Question 9: The game play felt realistic.
(1 Strongly Disagree, 5 = Strongly Agree)
Question 10: If I could I would continue to play this game on my own.
(1 Strongly Disagree, 5 = Strongly Agree)
If you could improve anything about the game, what would it be?

**Endogenous Version**

- I thing the game is good the way it is.
- Multiplication facts
- Make it fun and extraordinary
- That you have to get an evil trash guy trying to steal the boxes
- I would make it so that when it resets after you hit the reset button it would change back to the first parenthesis
- I would do nothing
- To make it more realistic. Make more options of games not just making the boxes in the truck.
- When you get an answer wrong and it make a noise like your right. Make it make a wrong sound.
- Nothing
- I would improve the multiplication facts
- I would a race car and when you answer right it moves
- I would improve that this game is easy and fun
- Different levels
- Different levels
- Nothing
- No math
- It wouldn’t be anything
- For it to be easy
- Nothing
- That it is fun and you can learn your multiplication and other things
- I would put a bad guy or something to get the person playing to go faster
- Make more interesting games
- This game is good enough
- Nothing really
- More variety
- Animation, graphics
- I don’t know. I would change that I would need the scores
- I would make it so that you couldn’t play when it is paused

**Exogenous Version**

- I love the game. I wish I could take it home and play it.
- I would make the game a little easier
- Tip exactly to help with the problem you are working on
- I would add more levels to this game
- More levels
- Nothing
- I wouldn’t improve anything. I like it the way it is
- I would probably give more time when people are trying to figure out which truck to put it in.
- I will say that I really think that this game is awesome and I wish to have it on all computers.
- I wouldn’t improve anything.
- I would let students use the keyboard more than dragging it.
- I will learn my multiplication tables and study more.
- I should really work hard at it so that I can go to 5th grade and learn my math problems.
- I would leave it alone because it has math that can make you learn and get frustrated sometimes.
- I would make the game more adventurous.
- It would be noises.
- I would improve the random white fuzz background.
- The time. I would have timed them.
- It is good as it is.
- Nothing really because I love it and people would learn while having fun.
- It could be harder.
- I would not improve anything because I love how it is right now.
- To put more action, fantasy, and 3D art like Zelda.
- I would make it more interesting and more fun.
- Put more guns and bad guys.
- Nothing really.
- Nothing.
- Nothing it’s great.
- I would improve the graphics and background.
- It is good for a multiplication game.
- It would a lot slower.

*Control Version*

- I would not change the game.
- It will be multiplication.
- At one time it would not let me do anything.
- I don’t think it really needs to improve.
- Nothing.
- I guess it is fine as it is.
- The multiplication problems wouldn’t be timed.
- I wouldn’t change anything.
- Less math.
- I will probably play this again.
- I don’t think I would change anything. I wouldn’t change anything because I liked the game how it was.
- Um nothing except for the fact that we are timed.
- I would not improve anything.
- I would like to be able to do it fast because of guessing.
- That you could move a truck by doing multiplication facts to a factory
- Nothing
- It is bad!
- Nothing
- That you wouldn’t get rushed and that it was actually a real game
- Nothing really, this game is as good as it’s going to get. This will improve the minds of young children like me.
- I would fix all the bugs
- If i could fix anything on this game it would be to not be timed
- To make it a lot funner
- I would improve sometimes when you lose a level it doesn’t say replay
- If i could improve anything about the game it would be the background. Like just change the graphics.
- I would make sure that all answers appear. I had problems where the answers weren’t a possible answer
- Well I guess it’s like any other so I would try to make it unique
- Nothing have time bar
**Guideline for Scoring Conceptual Understanding Measure of the**

**Associative Property of Multiplication**

Question Type One:

- A score of 4 should be given if all four questions were answered correctly.
- A score of 3 should be given if three questions were answered correctly.
- A score of 2 should be given if two questions were answered correctly.
- A score of 1 should be given if one or none of the questions were answered correctly.

Question Type Two:

- A score of 4 should be given if all four questions were answered correctly.
- A score of 3 should be given if three questions were answered correctly.
- A score of 2 should be given if two questions were answered correctly.
- A score of 1 should be given if one or none of the questions were answered correctly.

Question Type Three:

- A score of 4 should be given if multiplicative and addition associative equations were identified and explanation provided.
- A score of 3 should be given if multiplicative associative equation was identified and explanation provided.
- A score of 2 should be given if multiplicative associative equation was identified and no explanation provided.
- A score of 1 should be given if multiplicative associative equation was not identified.
Guideline for Scoring Conceptual Understanding Measure of the Distributive Property of Multiplication

Question Type One:
- A score of 4 should be given if all four questions were answered correctly.
- A score of 3 should be given if three questions were answered correctly.
- A score of 2 should be given if two questions were answered correctly.
- A score of 1 should be given if one or none of the questions were answered correctly.

Question Type Two:
- A score of 4 should be given if all four questions were answered correctly.
- A score of 3 should be given if three questions were answered correctly.
- A score of 2 should be given if two questions were answered correctly.
- A score of 1 should be given if one or none of the questions were answered correctly.

Question Type Three:
- A score of 4 should be given if distributive equation was identified and correct explanation provided.
- A score of 3 should be given if distributive equation was identified and partial correct explanation provided.
- A score of 2 should be given if distributive equation was identified and no explanation provided.
- A score of 1 should be given if distributive equation was not identified.
Andre Denham was born in Kingston, Jamaica, but spent his formative years in Queens, New York. He completed his secondary education at the prestigious Brooklyn Technical High School located in Ft. Greene, NY. After graduating from Brooklyn Tech, he attended Oakwood College (now Oakwood University) where he graduated with a B. A. in Mathematics. After graduating from Oakwood he spent several years as a mathematics and computer applications instructor at Oakwood Adventist Academy. While at Oakwood Academy he received a M. A. in Curriculum and Instruction with an emphasis in Educational Technology from La Sierra University. Andre left his position at Oakwood Academy to pursue a PhD. in Educational Technology at Arizona State University. During his time at Arizona State, he served as a graduate research assistant and teaching associate, which aided in the formation of his research agenda and teaching philosophy. His research focuses on the development of conceptual understanding of mathematical properties through digital gameplay, the use of mobile devices as instructional and performance support tools for model-eliciting activities, and intelligent tutoring systems. Beginning in the fall of 2012, he will be an assistant professor in the College of Education at the University of Alabama.